Bruhn Errata

Bill Gran

GRAN Corporation

A Companion to

Analysis and Design of Flight Vehicle Structures

by Elmer F. Bruhn, PhD

Bruhn Errata

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by

BILL GRAN

President, CEO & Super Genius

GRAN ***CORPORATION***

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Preface

Mestrius Plutarchus (Plutarch)

"The mind is not a vessel to be filled but a fire to be kindled."

Albert Einstein

“Anyone who has never made a mistake has never tried anything new.”

“Example isn't another way to teach, it is the only way to teach.”

“It is a miracle that curiosity survives formal education.”

Thomas Huxley

“Try to learn something about everything and everything about something.”

Bertrand Russell

“Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry.”

Sir Isaac Newton

"If I have seen farther than others it is because I have stood on the shoulders of giants."

Forward

"Be honest … work hard … love everyone."

Martin Gran

"Love all, trust a few."

William Shakespeare

Legal Department

While every effort has been made to assure that the information contained in the book is accurate and correct, it is only intended to provide general information for educational use. It is not intended to be a substitute for the reader's own research and judgment and the author assumes no liability for damages or losses caused by, directly or indirectly, the information contained within.

Dedication

Dedicated to the memory of James R. Gran, Martin Gran, Marie Gran, Meredith “Swede” Gran, Gordon Gran, Connie Gran, Grace Gran, Lavonne Gran … my friends and classmates Robin Bennett, Gary Golubski, Ed Lehman, Mike Lewis, Roger Merrick, Julie Reiter, Steve Sanger, Leon Schoenthaler, Sean Shea, Dave Wetzel … inventors and authors John V. Atanasoff, Clifford E. Berry, Elmer F. Bruhn, Hardy Cross, Louis G. Dunn, Paul Kuhn, David J. Peery, William F. McCombs, Ernest E. Sechler, Fred Seely, Francis R. Shanley, James Smith, Stephen P. Timoshenko and all of the giants in mathematics, science, engineering and computer science.

“The whole earth is the tomb of heroic men and their story is not given only on stone over their clay but abides everywhere without visible symbol, woven into the stuff of other men’s lives.”

Pericles

Acknowledgments

Special thanks to Dr. Charles (Ben) Basye, Dr. William F. Riley, Dr. Loren W. Zachary, Dr. Charles R. Mishke, Dr. Howard N. Shapiro, Sherman Burns, Glen Crawford, Keith Francois, Sverre Gran, Gary Holst, Lou Mavros, Bill Merrick, Rex Sanger, Fred Taylor … the many contributors such as Jim Baldwin, Robert Booth, Chris Boshers, Frank Dylla, Rudy Haitchi, Ned Hanson, Chuck Lilly, Craig Plunkett, Jeff Schmidt, Clint Smith, Curt Wigant, Joe Zuklic … my extended family from Norway to California … all of the fine folks in and around Maquoketa and the state of Iowa. Takk så mye!

Finally, an ever grateful “thousand thanks” to all veterans …

TUSEN TAKK and **GOD BLESS AMERICA!**

Contents at a Glance

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Introduction

Analysis and Design of Flight Vehicle Structures by Elmer Franklin Bruhn may as well have been written in Latin when I purchased my first copy more than twenty-five years ago. It is arguably the most comprehensive book on aircraft stress analysis ever written.

While many of the methods in “Bruhn” have been overshadowed by Finite Element Analysis, they provide valuable insight into the fundamentals of aircraft design and an appreciation for the men and women that designed aircraft before John Vincent Atanasoff and Clifford Berry invented the electronic digital computer at Iowa State University. You won't find many aircraft stress analysts using the Moment Distribution (Hardy Cross) Method, Slope Deflection Method or the Methods of Elastic Weights, Dummy Unit-Loads, Moment Areas, Virtual Work or Influence Coefficients. While some of the methods are more than fifty years old, the lessons are timeless.

This book is a compilation of the mistakes, omissions and typographical errors my teammates and I have discovered in the last two decades. Keep in mind that “Bruhn” was written before electronic calculators and spreadsheets were invented. Note the slide rule disclaimer on page A13.13, column 2:

“The calculations in this example being done on a slide rule cannot provide exact checks.”

The examples in “Bruhn” are condensed. The book could easily be expanded to fill a five volume set. I would organize it differently ... but as Marge and Homer Simpson might say, "It's easy to criticize ... and FUN TOO!"

If you insist on punishing yourself and throwing your life away by pursuing a career in aeronautical or aerospace engineering ... study *Analysis and Design of Flight Vehicle Structures* by Elmer F. Bruhn and the first edition of *Aircraft Structures* by David J. Peery. A lot of subjects are still Greek to me, but in the last thirty years I have learned this much pseudo-Latin:

ILLEGITIMI NON CARBORUNDUM.

Mange Takk!

1.0 Analysis and Design of Flight Vehicle Structures

Page A3.7 Moment of Inertia of an Airplane

Column 1 N.A.O.A. *should be* N.A.C.A. Thanks to Dr. Howard W. Smith.

Page A3.11 Properties of a Two-Cell Wing Beam Section

Table 9, Page A3.11

Stringer Number	Stringer Area A_{str} (in^2)	Effective Skin Area (in^2)	Total Area A (in^2)	y (in)	Ay (in^2)	Ay^2 (in^3)	x (in)	Ax (in^2)	Ax^2 (in^3)	I_{xy} = A x y (in^4)
1	0.110	0.031	0.141	4.00	0.563	2.252	-33.15	-4.665	154.640	-18.659
2	0.110	0.031	0.141	6.05	0.851	5.151	-29.28	-4.120	120.642	-24.928
3	0.300	0.080	0.380	7.00	2.660	18.620	-24.85	-9.443	234.659	-66.101
4	0.130	0.038	0.168	7.37	1.241	9.147	-21.18	-3.567	75.543	-26.287
5	0.130	0.038	0.168	7.55	1.271	9.599	-16.60	-2.795	46.404	-21.106
6	0.130	0.038	0.168	7.50	1.263	9.473	-12.60	-2.122	26.735	-15.914
7	0.130	0.038	0.168	7.30	1.229	8.974	-8.60	-1.448	12.455	-10.572
8	0.130	0.038	0.168	6.90	1.162	8.018	-4.00	-0.674	2.694	-4.648
9	0.240	0.050	0.290	6.50	1.885	12.253	-0.35	-0.102	0.036	-0.660
10	0.070	0.100	0.170	-3.30	-0.561	1.851	-33.25	-5.653	187.946	18.653
11	0.070	0.100	0.170	-4.90	-0.833	4.082	-29.28	-4.978	145.744	24.390
12	0.130	0.150	0.280	-5.95	-1.666	9.913	-24.85	-6.958	172.906	41.400
13	0.110	0.200	0.310	-7.40	-2.294	16.976	-18.70	-5.797	108.404	42.898
14	0.110	0.200	0.310	-8.13	-2.520	20.490	-12.42	-3.850	47.819	31.302
15	0.110	0.200	0.310	-8.62	-2.672	23.034	-6.10	-1.891	11.535	16.300
16	0.240	0.110	0.350	-8.87	-3.105	27.537	-0.35	-0.123	0.043	1.087
		Σ	**3.693**		**-1.525**	**187.368**		**-58.184**	**1348.205**	**-12.843**

$$\bar{x} = -\frac{58.184}{3.693} = -15.753\ in \qquad \bar{y} = -\frac{1.525}{3.693} = -0.413\ in$$

$$I_x = A\ y^2 - A\ \bar{y} = 187.368\ in^4 - 3.693\ in^2\ \left(-0.413\ in\right)^2 = 186.738\ in^4$$

$$I_y = A\ x^2 - A\ \bar{x} = 1{,}348.205\ in^4 - 3.693\ in^2\ \left(-15.753\ in\right)^2 = 431.604\ in^4$$

$$I_{xy} = I_{xy} - A\ \bar{x}\ \bar{y} = -12.843\ in^4 - 3.693\ in^2\ \left(-15.753\ in\right)\left(-0.413\ in\right) = -36.867\ in^4$$

$$\tan 2\phi = \frac{2\ I_{xy}}{I_y - I_x} = \frac{2\ \left(-36.867\ in^4\right)}{431.604\ in^4 - 186.738\ in^4} = -0.301$$

2ϕ = -0.292 radians = 16.758 ° ϕ = -0.146 radians = -8.379 ° $\sin\phi$ = -0.146 $\cos\phi$ = 0.989

$$I_{xp} = I_x\ \left(\cos\phi\right)^2 + I_y\ \left(\sin\phi\right)^2 - 2\ I_{xy}\ \sin\phi\ \cos\phi$$

$$I_{xp} = 186.738\ in^4\left(0.989\right)^2 + 431.604\ in^4\left(-0.146\right)^2 - 2\ \left(-36.867\ in^4\right)\left(-0.146\right)\left(0.989\right)$$

$$I_{xp} = 181.308\ in^4$$

Page A4.3 Motion of Rigid Bodies

Column 1, Equation 3 $v - v_0^2 = 2\ a\ s$ *should be* $v^2 - v_0^2 = 2\ a\ s$

Thanks to SparWeb on the www.eng-tips.com website.

Page A4.13 Dynamic Effect of Air Gusts

Column 2, paragraph 2 “a air load” *should be* “an air load”

Thanks to Dr. Howard W. Smith.

Page A5.22 Equations for a Compressive Axially Loaded Strut

Column 2

$$i = \frac{1}{P}\left(\frac{M_2 - M_1}{L} - \frac{w\,L}{2} + w\,x - \frac{C_1}{j}\cos\frac{x}{j} + \frac{C_2}{j}\sin\frac{x}{j}\right)$$

should be

$$\theta_1 = \frac{1}{P}\left(\frac{M_2 - M_1}{L} - \frac{w\,L}{2} + w\,x - \frac{C_1}{j}\cos\frac{x}{j} + \frac{C_2}{j}\sin\frac{x}{j}\right)$$

Thanks to Dr. Howard W. Smith.

Page A5.23 Beam-Columns

Table A5.I, Case III, Distributed Load, No End Moments

I use the following

$C_1 = w\, j^2 \tan\left(\frac{L}{2\, j} \right)$ $\quad C_2 = w\, j^2$ $\quad f\left(x \right) = -\, w\, j^2$ instead of

$C_1 = \dfrac{w\, j^2 \left[\cos\left(\frac{L}{j} - 1 \right) \right]}{\sin \frac{L}{j}}$ $\quad C_2 = -\, w\, j^2$ $\quad f\left(x \right) = -\, w$

Table A5.I, Case V, Concentrated Side Load, No End Moments

Equation for Point of Maximum Bending Moment

$$\tan \frac{x}{j} = \frac{C_1}{C_2}$$

Maximum Span Bending Moment

$$M_{\max} = \left(C_1^{\,2} + C_2^{\,2} \right)^{\frac{1}{2}}$$

Should be	Maximum Bending Moment	Point of Maximum Bending Moment
$If\ b > \dfrac{\pi\, j}{2}$	$M_{\max} = \dfrac{W\, j \sin\left(\frac{a}{j} \right)}{\sin\left(\frac{L}{j} \right)}$	$at\ x = L - \dfrac{\pi\, j}{2}$
$If\ b < \dfrac{\pi\, j}{2}$	$M_{\max} = \dfrac{W\, j \sin\left(\frac{a}{j} \right)}{\sin\left(\frac{L}{j} \right)} \sin\left(\dfrac{b}{j} \right)$	$at\ x = a$

Page A5.28 Biplane Wing Spar Example

Column 2, “Substituting in equation (A)” $\tan\frac{x}{j}=\dfrac{D_2-D_1\cos\frac{L}{j}}{D_1\sin\frac{L}{j}}$

$$=-\frac{50180-(-55505\,x\,-.26981)}{-55505\,x\,.96290}=\frac{-65156}{-53441}=1.2192$$ *should be*

$$=-\frac{50{,}198-[-55{,}523(-0.26952)]}{-55{,}523(0.962996)}=\frac{-65{,}162}{-53{,}469}=1.2187$$

Figure A5.67

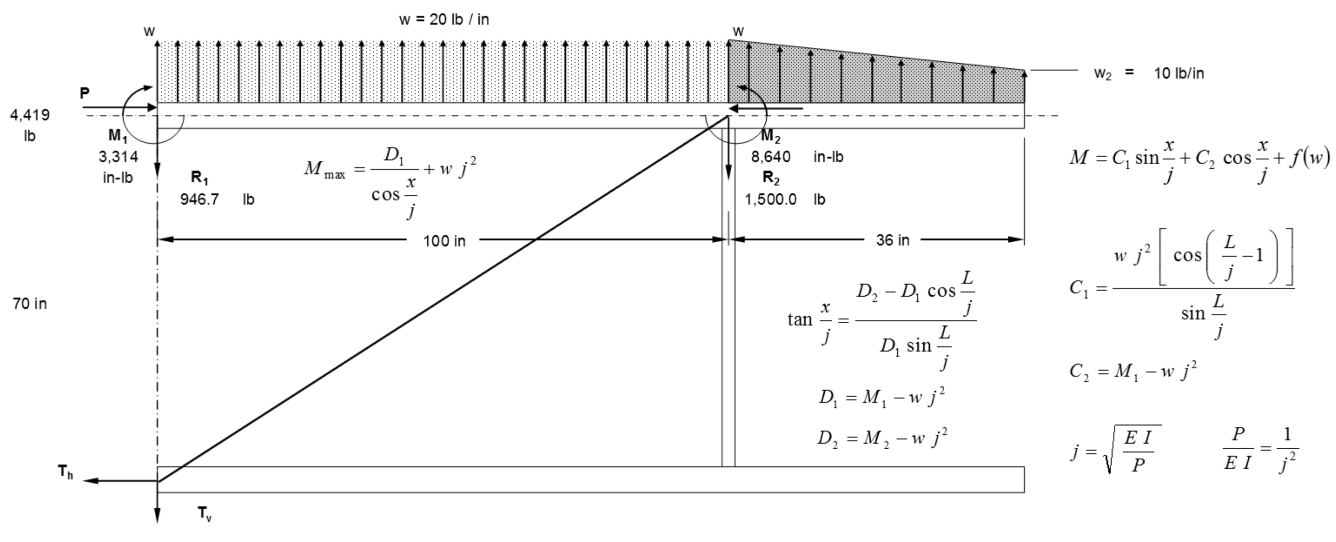
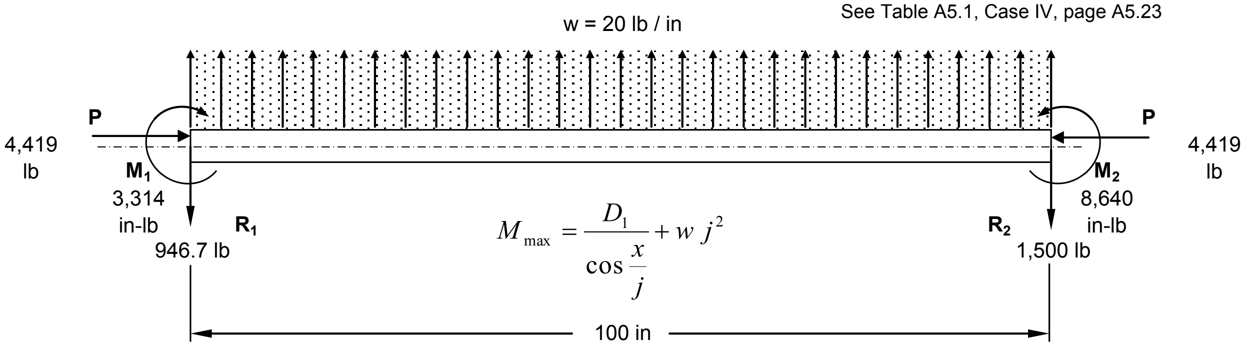
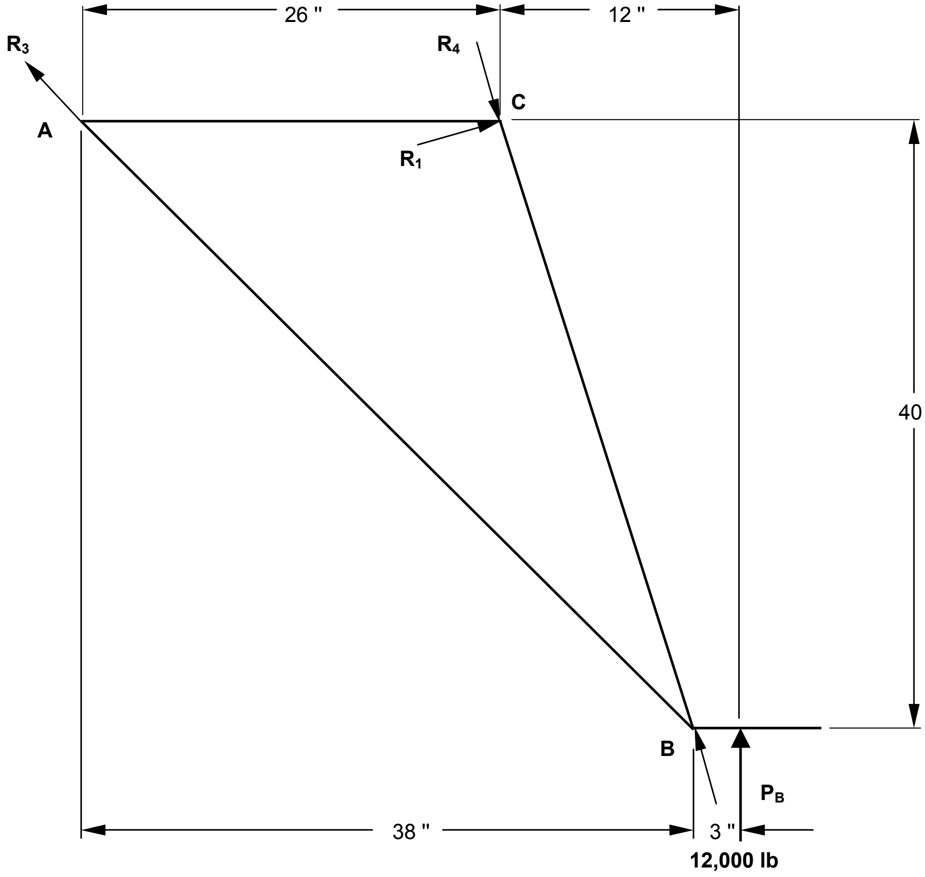


Figure A5.68


See Table A5.1, Case IV, page A5.23

Page A5.29 Beam-Column, Landing Gear

Bruhn, Example Problem 2, page A5.29



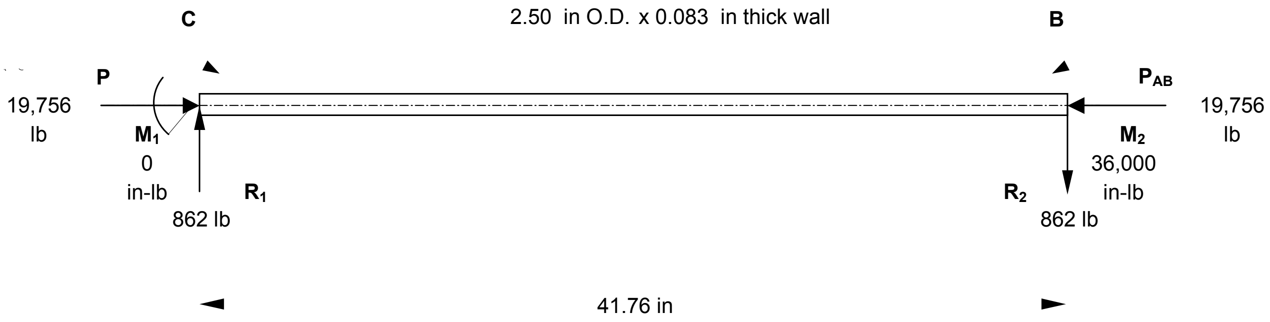
$$M = C_1 \sin\frac{x}{j} + C_2 \cos\frac{x}{j} + f(w)$$

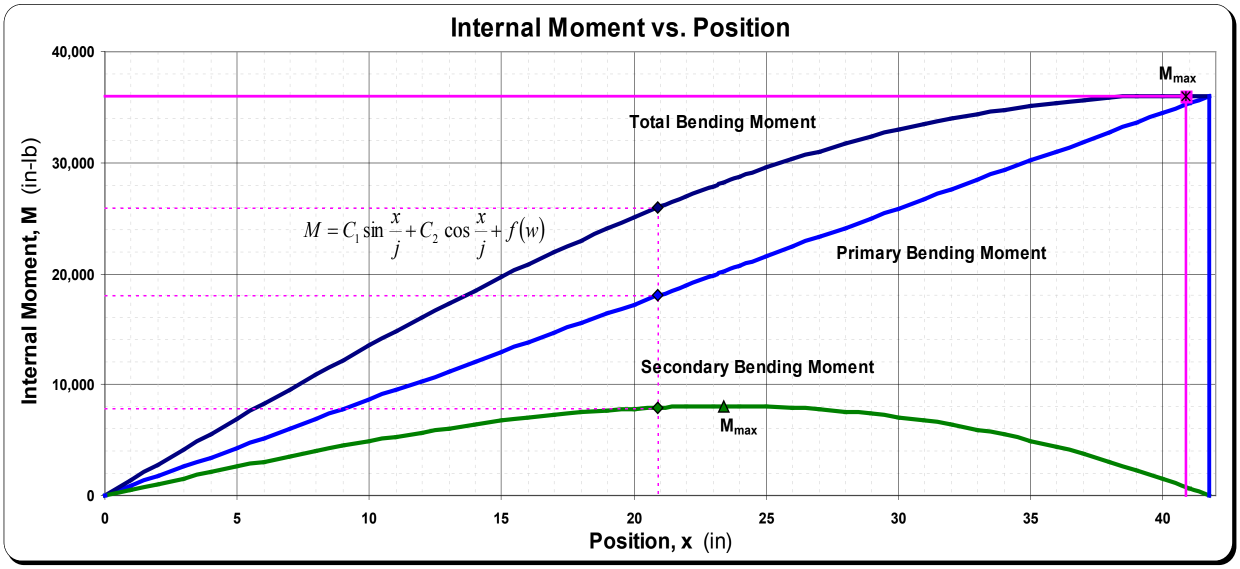
$$\tan\frac{x}{j} = \frac{M_2 - M_1 \cos\frac{L}{j}}{M_1 \sin\frac{L}{j}} \qquad If\ M_1 = 0, \quad x = \frac{\pi\, j}{2} \qquad M_{\max} = \frac{M_2}{\sin\frac{L}{j}}$$

Beam-Column, Landing Gear

Bruhn, Example Problem 2, page A5.29

Figure A5.70





Beam-Column, Landing Gear

Bruhn, Example Problem 2, page A5.29

INPUT

Design Load	P_B	12,000	lb
Young's Modulus	E	2.90E+07	psi
Diameter of Tube	D	2.50	in
Wall Thickness	t	0.083	in
Dimension	a	26	in
Dimension	b	12	in
Dimension	c	40	in
Offset of P	d	3	in
Moment - Point C	M_1	0	in-lb

OUTPUT

Maximum Bending Moment

	M_{ctr}	36,022	in-lb

Location of Maximum Moment on BC

If $M_1 = 0$, $x = \pi j / 2$	x	40.85	in

Total Bending Moment at Midpoint of BC

at L / 2	M_{ctr}	25,913	in-lb

Primary Bending Moment at Midpoint of BC

at L / 2, M = P d / 2	M_x	18,000	in-lb

Secondary Bending Moment at Midpoint of BC

	$M_{secondary}$	7,913	in-lb

Transverse Deflection at Midpoint of BC

	x_{max_M}	0.401	in

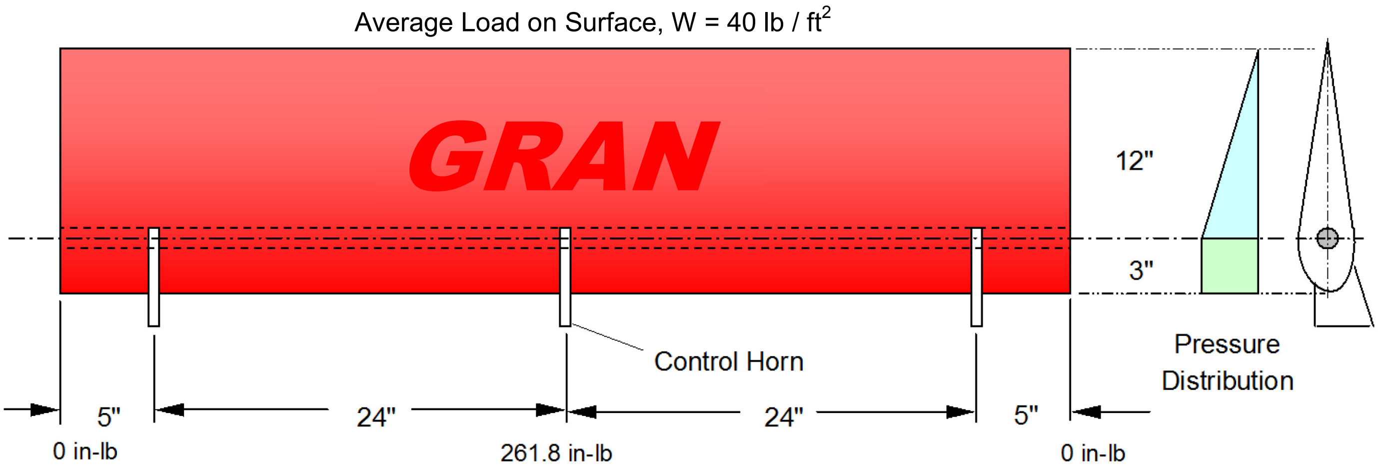
DATA

R_{3y}	6,923.1	lb	R_{4y}	18,923.1	lb	P_B	12,000	lb	ΣF_y	0	lb
R_{3x}	6,576.9	lb	R_{4x}	5,676.9	lb	$R_{3x} - R_{4x}$	900.0	lb	ΣF_x	0	lb
R_3	9,549.1	lb	R_4	19,756.3	lb	R_1	862.0	lb	ΣM_B	0	in-lb

M_1	0	in-lb	C_1	36,022		L	41.76	in	x	20.881	in
M_2	36,000	in-lb	C_2	0		j	26.007		j	26.007	
P_{AB}	19,756	lb	I	0.4608	in^4	L / j	1.606		x / j	0.80289	

Page A6.2 Transmission of Power by a Cylindrical Shaft

Elmer F. Bruhn, *Analysis and Design of Flight Vehicle Structures*, page A6.2



INPUT

L_1	24	in
L_2	5	in
D	1.25	in
t	0.049	in
G	3.80E+06	psi
W	40	lb/ft^2

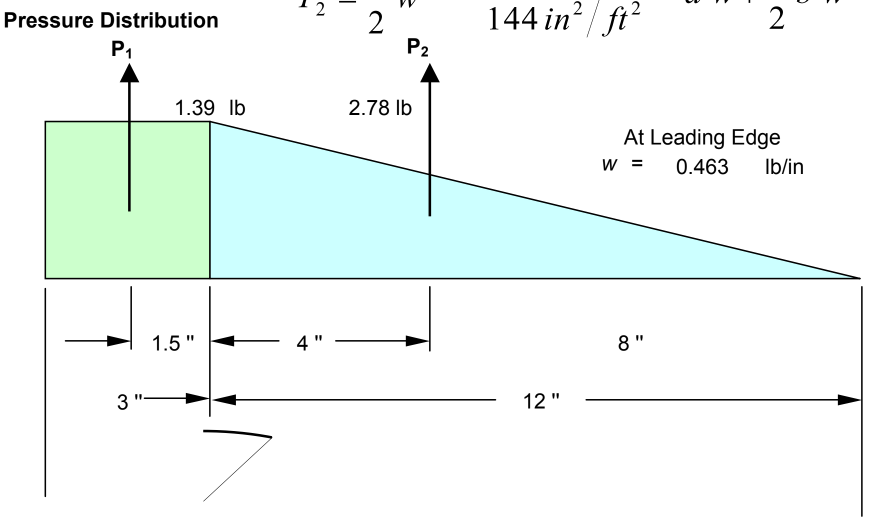
Dimension, a

a	3	in

Dimension, b

b	12	in

$$P_2 = \frac{b}{2}\,w \qquad \frac{W\,(a+b)}{144\,in^2/ft^2} = a\,w + \frac{1}{2}\,b\,w$$



Total Load on One Inch Strip

4.17 lb/in

Load at Leading Edge, w (lb/in)

4.17 lb/in

w = 0.463

OUTPUT

Angle of Twist	θ	0.0150	radians
Angle of Twist	θ	0.86	degrees

Angle of Twist is Based on Average Torque, T_{max} / 2

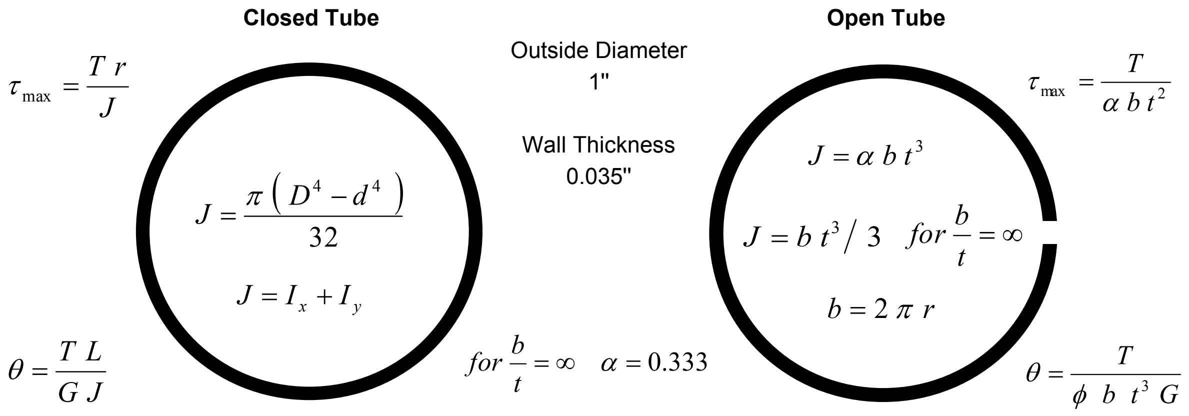
Running Load	*w*	0.463	lb/in
Running Torque	T / in	9.03	in-lb/in

Maximum Torque	T_{max}	261.8	in-lb
Max. Shear Stress	τ_{max}	2,450	psi

Note: Average Torque is Maximum Torque / 2

Page A6.5 Torsional Stiffness

Elmer F. Bruhn, *Analysis and Design of Flight Vehicle Structures*, page A6.5



$$\tau_{\max} = \frac{T\,r}{J} \qquad J = \frac{\pi\left(D^4 - d^4\right)}{32} \qquad J = I_x + I_y \qquad \theta = \frac{T\,L}{G\,J}$$

$$\tau_{\max} = \frac{T}{\alpha\,b\,t^2} \qquad J = \alpha\,b\,t^3 \qquad J = b\,t^3 / 3 \quad for\ \frac{b}{t} = \infty \qquad b = 2\,\pi\,r \qquad \theta = \frac{T}{\phi\ b\ t^3\ G}$$

$$for\ \frac{b}{t} = \infty \quad \alpha = 0.333$$

Closed Tube $J = \dfrac{\pi\left(D^4 - d^4\right)}{32}$

Open Tube $J = \dfrac{b\,t^3}{3}$

INPUT

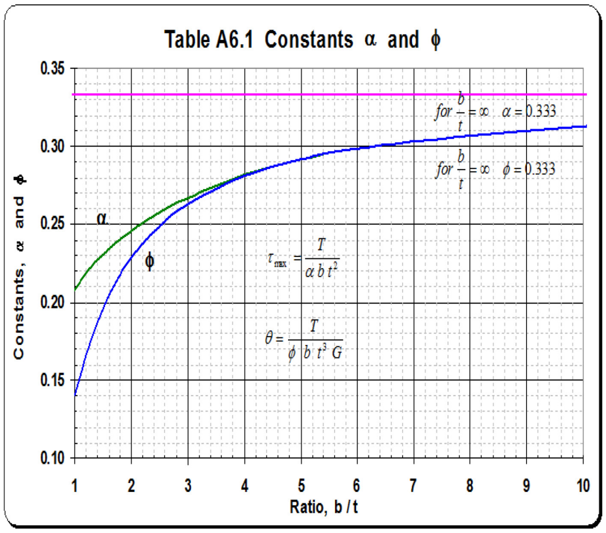
Outside Diameter	**D**	**1.00**	in
Wall Thickness	**t**	**0.035**	in
Shear Modulus	**G**	**3.80E+06**	psi

DATA

Inside Diameter	**d**	**0.930**	in
Moment of Inertia	**I**	**0.01237**	in^4
Width	**b**	**3.142**	in
Ratio	**b / t**	**89.8**	

OUTPUT

Constant, α	**α**	**0.333**	
Constant, ϕ	**ϕ**	**0.333**	
Closed Tube	**J**	**0.024735**	in^4
Open Tube	**J**	**0.000045**	in^4
Torsional Stiffness	**Ratio**	**550.9**	to 1

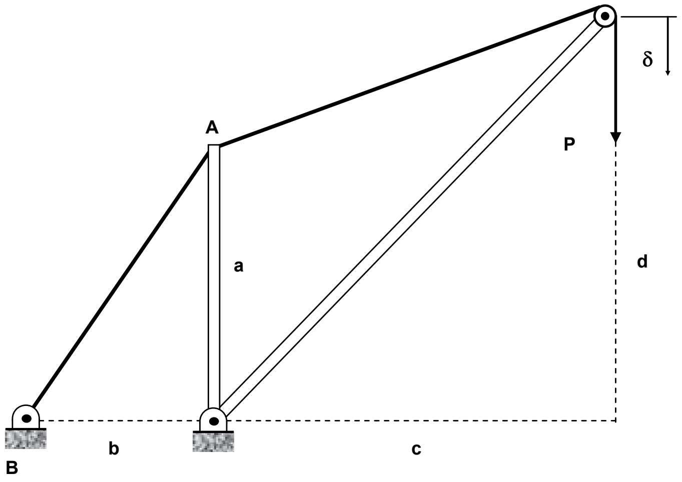


See also Raymond J. Roark, *Formulas for Stress and Strain*, Table 10.1, Case 17 and Case 18.

Thanks to Frank Dylla.

Page A7.7 Castigliano's Theorem

Note: Do NOT use Castigliano's theorem for non-linear problems.



Member	**Length L** (feet)	**Area A** (in^2)	**Modulus E** (psi)	**A E**	**Load, S** (lb)	**S^2 L / A E** ($x\ P^2\ x\ 10^{-6}$)
OA	40	4.70	2.90E+07	136.3	-1.5 P	**0.660**
AB	50	0.875	1.35E+07	11.813	2.5 P	**26.455**
AC	63.25	0.875	1.35E+07	11.813	1.581 P	**13.385**
OC	84.85	4.70	2.90E+07	136.3	-2.121 P	**2.801**
					Σ	**43.302** P^2 x 10^{-6}

Strain Energy

$$U = \frac{1}{2}\frac{S^2\ L}{A\ E} = \frac{43.30\ \ P^2\ \ x\,10^{-6}}{2}\ \ lb-ft$$

Castigliano's Theorem

$$\delta_P = \frac{\partial U}{\partial P} = 2\left(\frac{43.30\ P\ x\,10^{-6}}{2}\right) = 43.3\ x\,10^{-06}\ P\ \ feet$$

Example

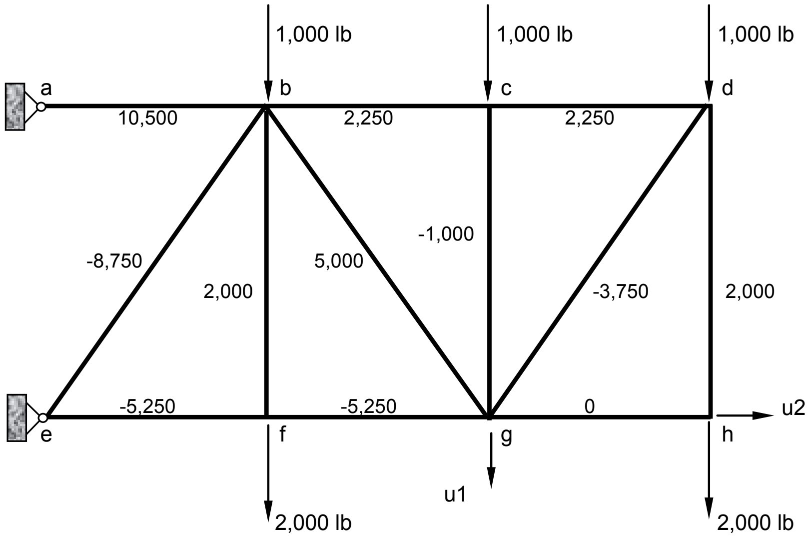
For P = 10,000 lb

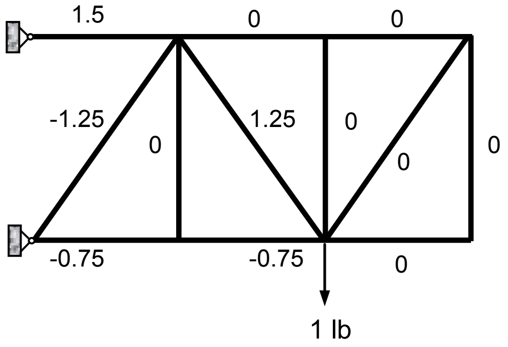
Deflection δ = 10,000 lb (4.33 x 10^{-5}) feet = 0.433 feet

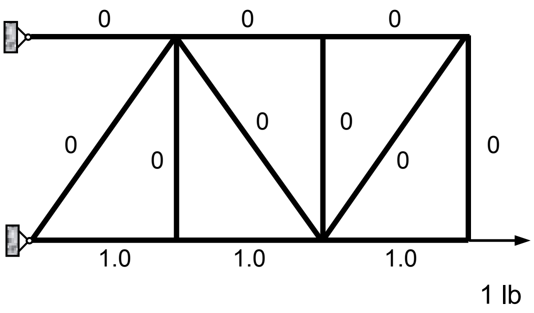
δ = 0.433 feet (12 inches / foot) = 5.2 inches

Page A7.11 Truss with Pinned Joints

Elmer F. Bruhn, *Analysis and Design of Flight Vehicle Structures*, page A7.11.







Check the last two columns in Table A7.3 for slide rule errors.

Member	Length, L (in)	A	AE ($\times 10^{-6}$)	L/AE ($\times 10^{-6}$)	Load, S (lb)	Unit Load, u_1	Unit Load, u_2	$\frac{S\, u_1\, L}{AE}$ ($\times 10^{3}$)	$\frac{S\, u_2\, L}{AE}$ ($\times 10^{3}$)
ab	30	0.456	4.785	6.270	10,500	1.50	0	9.405	0
bc	30	0.293	3.074	9.759	2,250	0	0	0	0
cd	30	0.293	3.074	9.759	2,250	0	0	0	0
ef	30	0.511	5.366	5.591	-5,250	-0.75	1.0	2.097	-2.796
fg	30	0.511	5.366	5.591	-5,250	-0.75	1.0	2.097	-2.796
gh	30	0.331	3.480	8.621	0	0	1.0	0	0
be	50	0.967	10.150	4.926	-8,750	-1.25	0	5.131	0
bg	50	0.331	3.480	14.368	5,000	1.25	0	8.552	0
dg	50	0.511	5.365	9.320	-3,750	0	0	0	0
bf	40	0.293	3.074	13.012	2,000	0	0	0	0
cg	40	0.293	3.074	13.012	-1,000	0	0	0	0
dh	40	0.293	3.074	13.012	2,000	0	0	0	0
							Σ =	27.3	-5.6

$$\delta_1 = \sum \frac{S\ u_1\ L}{A\ E} = \frac{27.3}{1{,}000} = 0.027\ inch \qquad \delta_2 = \sum \frac{S\ u_2\ L}{A\ E} = -\frac{5.6}{1{,}000} = -0.006\ inch$$

Page A7.14 Virtual Work

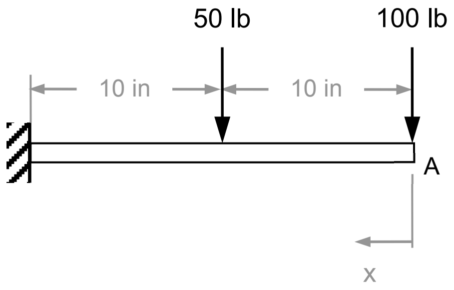
I prefer to use G for shear modulus rather than E_s .

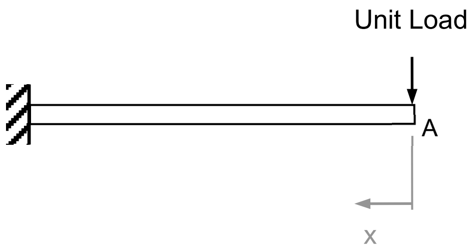
$$\delta_{total} = \int_0^L \frac{V\,v}{A\,G}\,dx \qquad \text{etc.}$$

Page A7.15 Dummy Unit Load

Example Problem 19

I've added the x coordinates to Figure A7.22.





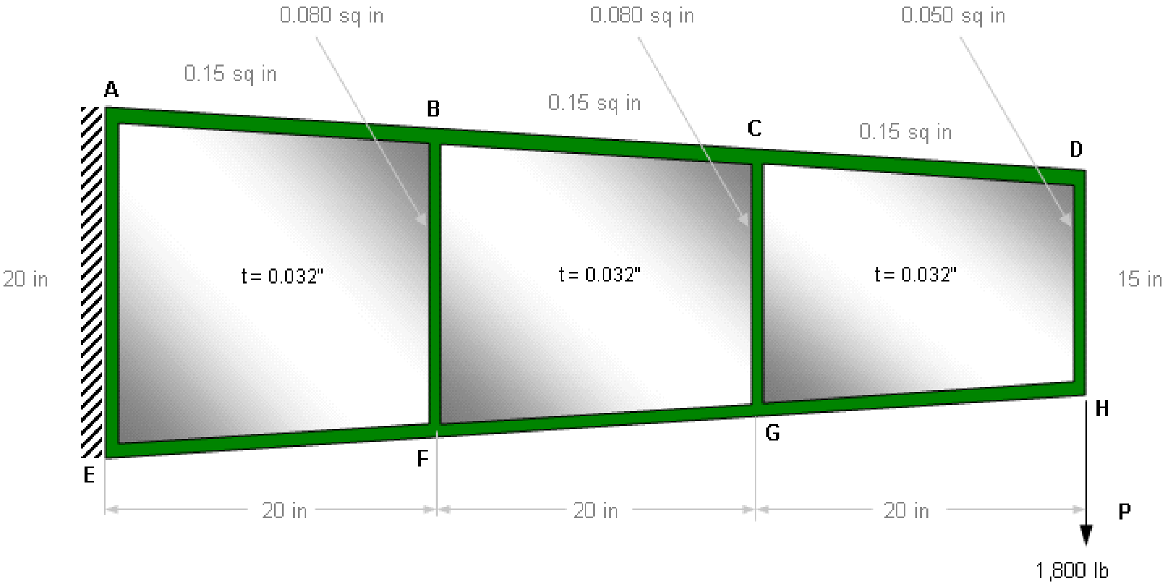
Again, I prefer G to E_s .

$$\delta = \int \frac{V\,v}{A\,G}\,dx \qquad \delta = \int \frac{T\,t}{G\,J}\,dx$$

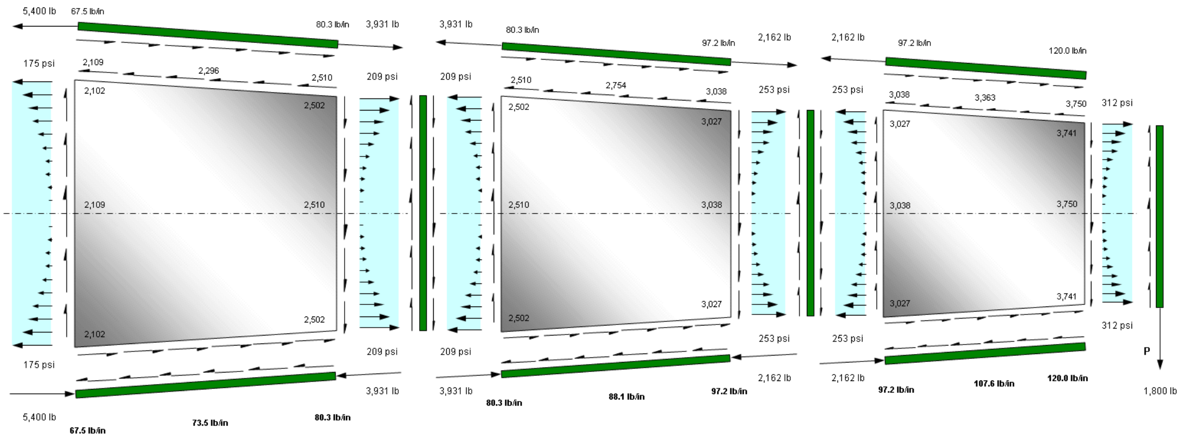
Page A7.16 Tapered Shear Beam

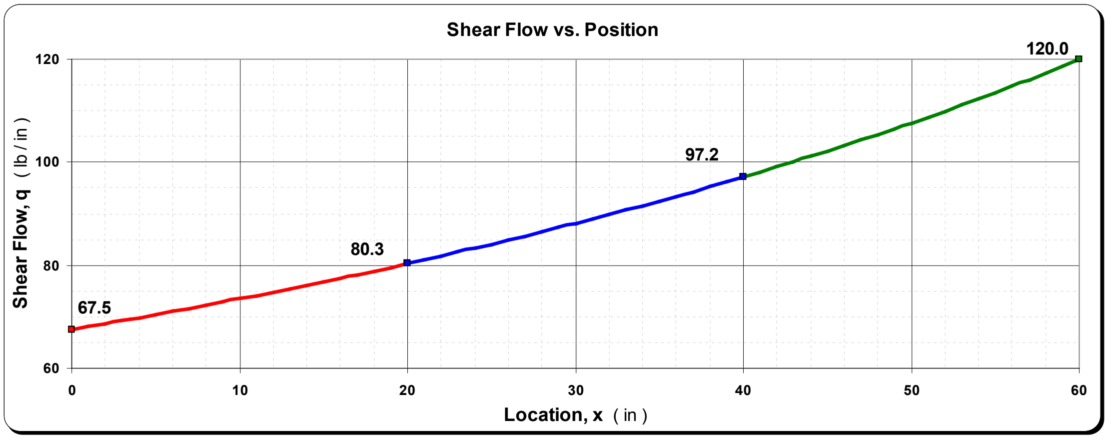
Method of Virtual Work

Determine the deflection at Point G.



Free Body Diagram





Shear Flow

	Position x (in)	Height h (in)	h_o (in)	Shear Flow q_o (lb / in)	h_o / h	q_{avg} (lb / in)	f_s (psi)
	0	20	18.333	80.33	0.917	**67.5**	2,109
	1	19.92	18.333	80.33	0.920	68.1	2,127
	2	19.83	18.333	80.33	0.924	68.6	2,145
	3	19.75	18.333	80.33	0.928	69.2	2,163
	4	19.67	18.333	80.33	0.932	69.8	2,181
	5	19.58	18.333	80.33	0.936	70.4	2,200
	6	19.50	18.333	80.33	0.940	71.0	2,219
	7	19.42	18.333	80.33	0.944	71.6	2,238
	8	19.33	18.333	80.33	0.948	72.2	2,257
Bay 1	9	19.25	18.333	80.33	0.952	72.9	2,277
	10	19.17	18.333	80.33	0.956	73.5	2,296
	11	19.08	18.333	80.33	0.961	74.1	2,317
	12	19.00	18.333	80.33	0.965	74.8	2,337
	13	18.92	18.333	80.33	0.969	75.4	2,357
	14	18.84	18.333	80.33	0.973	76.1	2,378
	15	18.75	18.333	80.33	0.978	76.8	2,399
	16	18.67	18.333	80.33	0.982	77.5	2,421
	17	18.59	18.333	80.33	0.986	78.2	2,443
	18	18.50	18.333	80.33	0.991	78.9	2,465
	19	18.42	18.333	80.33	0.995	79.6	2,487
	20	18.333	18.333	80.33	1.000	**80.3**	2,510
	20	18.333	16.667	97.20	0.909	**80.3**	2,510
	21	18.25	16.667	97.20	0.913	81.0	2,532
	22	18.17	16.667	97.20	0.917	81.8	2,556
	23	18.09	16.667	97.20	0.921	82.5	2,579
	24	18.00	16.667	97.20	0.926	83.3	2,603
	25	17.92	16.667	97.20	0.930	84.1	2,627
	26	17.84	16.667	97.20	0.934	84.9	2,652
	27	17.75	16.667	97.20	0.939	85.7	2,677
	28	17.67	16.667	97.20	0.943	86.5	2,702
Bay 2	29	17.59	16.667	97.20	0.948	87.3	2,728
	30	17.50	16.667	97.20	0.952	88.1	2,754
	31	17.42	16.667	97.20	0.957	89.0	2,780
	32	17.34	16.667	97.20	0.961	89.8	2,807
	33	17.25	16.667	97.20	0.966	90.7	2,834
	34	17.17	16.667	97.20	0.971	91.6	2,861
	35	17.09	16.667	97.20	0.975	92.5	2,889
	36	17.01	16.667	97.20	0.980	93.4	2,918
	37	16.92	16.667	97.20	0.985	94.3	2,947
	38	16.84	16.667	97.20	0.990	95.2	2,976
	39	16.76	16.667	97.20	0.995	96.2	3,005
	40	16.667	16.667	97.20	1.000	**97.2**	3,038
	40	16.667	15.00	120	0.900	**97.2**	3,038
	41	16.59	15.00	120	0.904	98.1	3,066
	42	16.51	15.00	120	0.909	99.1	3,097
	43	16.42	15.00	120	0.913	100.1	3,128
	44	16.34	15.00	120	0.918	101.1	3,160
	45	16.26	15.00	120	0.923	102.2	3,193
	46	16.17	15.00	120	0.927	103.2	3,226
	47	16.09	15.00	120	0.932	104.3	3,259
	48	16.01	15.00	120	0.937	105.4	3,293
Bay 3	49	15.92	15.00	120	0.942	106.5	3,328
	50	15.84	15.00	120	0.947	107.6	3,363
	51	15.76	15.00	120	0.952	108.7	3,398
	52	15.67	15.00	120	0.957	109.9	3,434
	53	15.59	15.00	120	0.962	111.1	3,471
	54	15.51	15.00	120	0.967	112.3	3,508
	55	15.42	15.00	120	0.972	113.5	3,546
	56	15.34	15.00	120	0.978	114.7	3,585
	57	15.26	15.00	120	0.983	116.0	3,624
	58	15.18	15.00	120	0.988	117.2	3,664
	59	15.09	15.00	120	0.994	118.5	3,704
	60	15.00	15.00	120	1.000	**120.0**	3,750

68 = (18.333 / 20)^2 x 80.33

74 = 18.333 / 20 x 80.33

80 = (16.667 / 18.333)^2 x 97.20

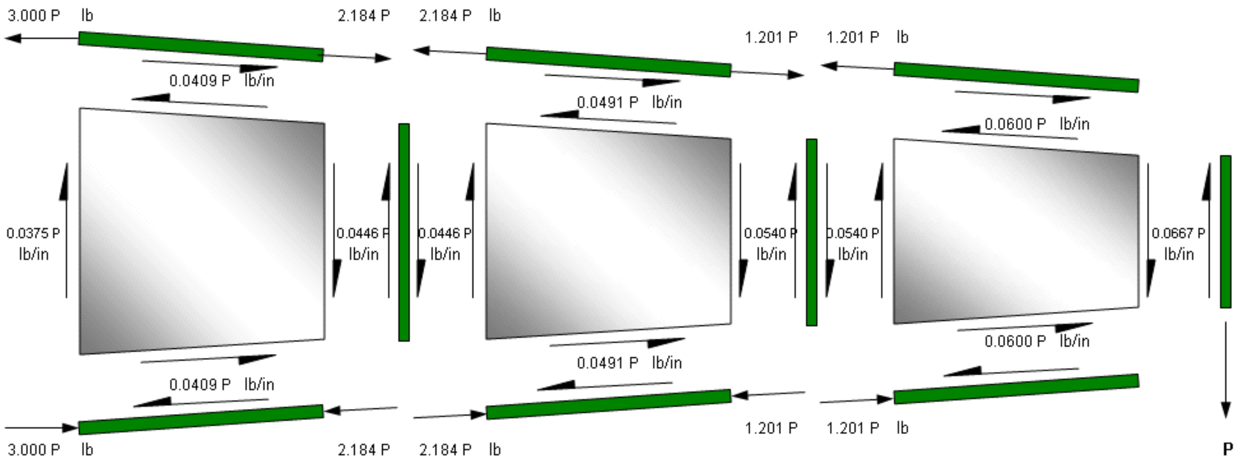
88 = 16.667 / 18.333 x 97.20

97 = (15 / 16.667)^2 x 120

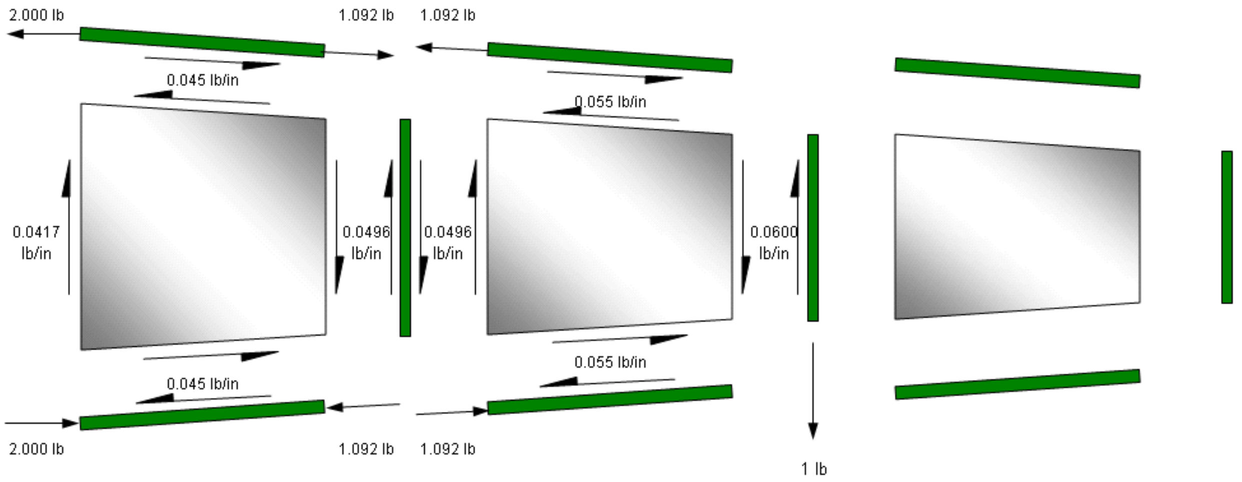
108 = 15 / 16.667 x 120

120 = 1,800 lb / 15 inches

Real Loads



Virtual Loads



Data

Member (Flange)	**L** (in)	**A** (in^2)
AB	20.017	0.15
BC	20.017	0.15
CD	20.017	0.15
EF	20.017	0.15
FG	20.017	0.15
GH	20.017	0.15
DH	15.000	0.05
CG	16.667	0.08
BF	18.333	0.08

E = 10.0 E6 G = 3,846,154 psi P = 1,800 lb

Spar Caps and Stiffeners

1	2	3	4	5	6	7	8	9	10
Member (Flange)	S_i	S_j	u_i	u_j	$\frac{2u_i + u_j}{6}$	$\frac{u_i + 2u_j}{6}$	**L / A E** (x 10^6)	Columns (2)(6)+(3)(7)	Columns (8) x (9)
AB	3 P	2.184 P	2.000	1.092	0.849	0.697	13.34	4.069 P	54.30 P
BC	2.184 P	1.201 P	1.092	0	0.364	0.182	13.34	1.013 P	13.52 P
CD	1.201 P	0 P	0	0	0	0	13.34	0 P	0 P
EF	-3 P	-2.184 P	-2.000	-1.092	-0.849	-0.697	13.34	4.069 P	54.30 P
FG	-2.184 P	-1.201 P	-1.092	0	-0.364	-0.182	13.34	1.013 P	13.52 P
GH	-1.201 P	0 P	0	0	0	0	13.34	0 P	0 P
DH	1 P	0 P	0	0	0	0	30.00	0 P	0 P
CG	0 P	0 P	1.000	0	0.333	0.167	20.83	0 P	0 P
BF	0 P	0 P	0	0	0	0	22.92	0 P	0 P
								Σ	135.64 P x 10^{-6}

Spar Webs

1	2	3	4	5	6
Member (Web)	Shear Flow **q avg** (lb / in)	Shear Flow **$\bar{q}$ avg** (lb / in)	Panel Area **A** (in^2)	**1 / G t** (x 10^6)	Columns (2)(3)(4)(5)
A-B-E-F	0.0409 P	0.0455	383.33	8.125	5.792 P
B-C-F-G	0.0491 P	0.0545	350.00	8.125	7.615 P
C-D-G-H	0.0600 P	0	-	-	0 P
	Real	Virtual		Σ	13.41 P x 10^{-6}

Deflection at Point G

$$\delta_G = \int \frac{S\, u\, dx}{A\, E} + \iint \frac{q\, \bar{q}}{G\, t}\, dx\, dy$$

$$\int_0^L \frac{S\, u\, dx}{A\, E} = \frac{L}{A\, E} \left(\frac{S_i\, u_i}{3} + \frac{S_i\, u_j}{6} + \frac{S_j\, u_i}{6} + \frac{S_j\, u_j}{3} \right)$$

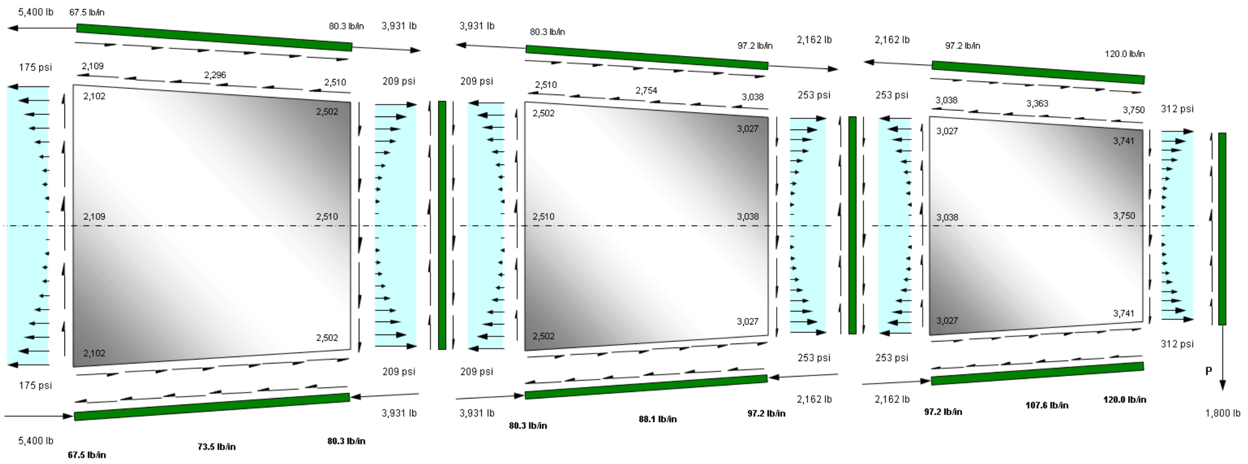
$$\iint \frac{q\, \bar{q}}{G\, t}\, dx\, dy \approx q_{avg}\, \bar{q}_{avg}\, \frac{A}{G\, t}$$

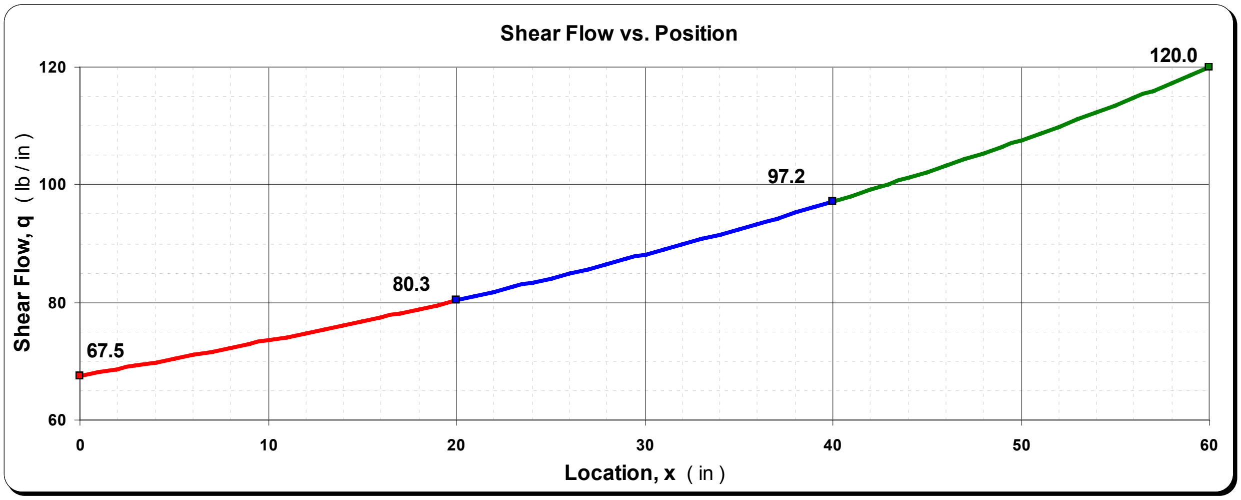
δ_G = (135.64 P + 13.41 P) 10^{-6} = 139.04(1,800 lb) 10^{-6} = 0.268 inch **δ_G = 0.268 inch**

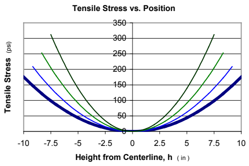
See *Analysis and Design of Flight Vehicle Structures*, pages A7.16 and A15.27.

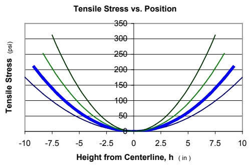
Shear Flow in Tapered Webs

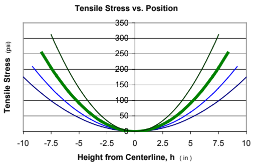
See *Aircraft Structures*, First Edition, pages 197-200

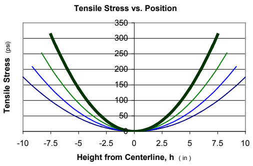












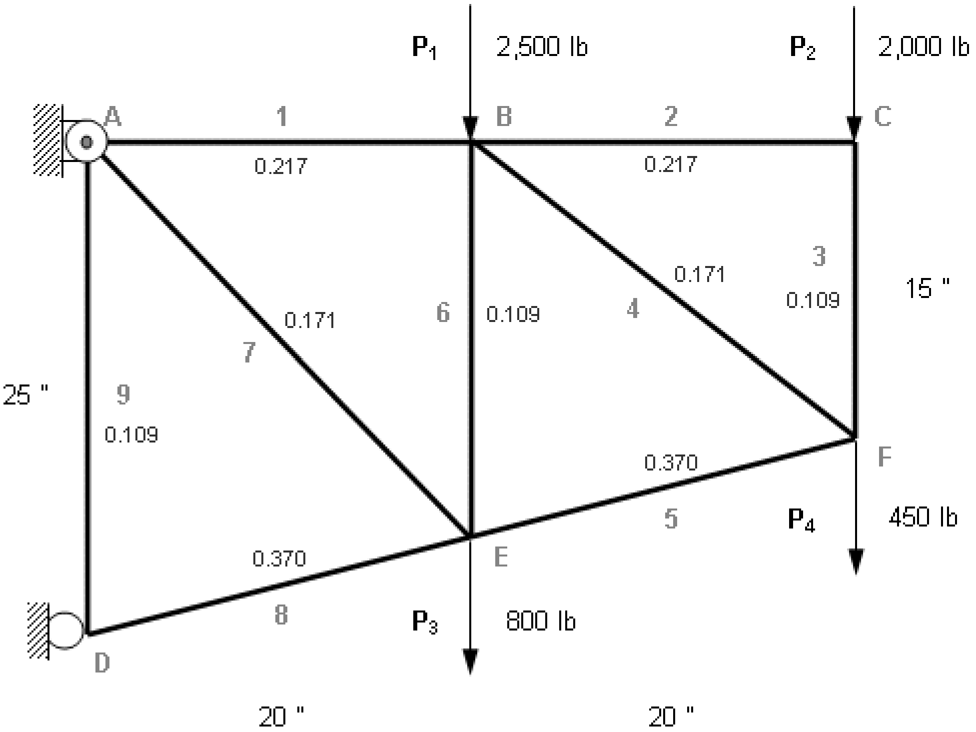
Page A7.24 Influence Coefficients - Truss with Pinned Joints

Deflections at points B, C, E and F given:

A_1 = 0.217 in^2 A_2 = 0.217 in^2 A_3 = 0.109 in^2 A_4 =0.171 in^2 A_5 = 0.370 in^2

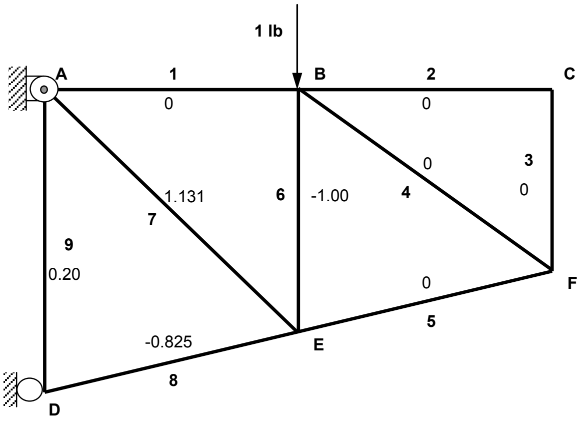
A_6 = 0.109 in^2 A_7 = 0.171 in^2 A_8 = 0.370 in^2 A_9 = 0.109 in^2 E = 10 E6 psi

Graphic

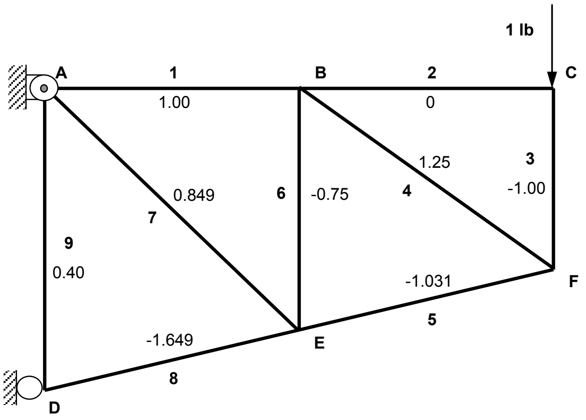


Unit Load Distribution

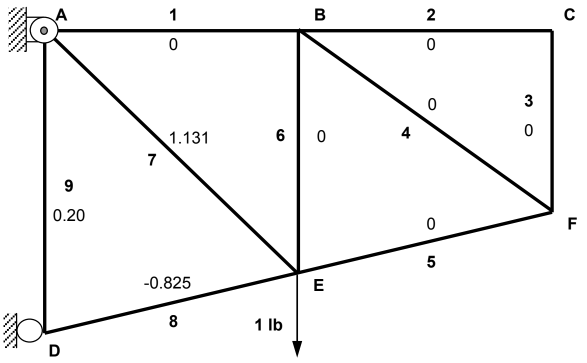
Column 1



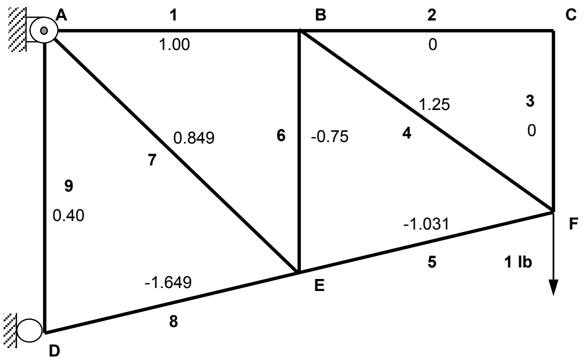
Column 2



Column 3



Column 4



$$[G_{im}] = \begin{bmatrix} 0 & 1.00 & 0 & 1.00 \\ 0 & 0 & 0 & 0 \\ 0 & -1.00 & 0 & 0 \\ 0 & 1.25 & 0 & 1.25 \\ 0 & -1.031 & 0 & -1.031 \\ -1.000 & -0.750 & 0 & -0.750 \\ 1.131 & 0.849 & 1.131 & 0.849 \\ -0.825 & -1.649 & -0.825 & -1.649 \\ 0.20 & 0.40 & 0.20 & 0.40 \end{bmatrix}$$

Transpose

$$[G_{im}]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1.131 & -0.82 & 0.20 \\ 1 & 0 & -1 & 1.25 & -1.031 & -0.75 & 0.85 & -1.65 & 0.40 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.131 & -0.82 & 0.20 \\ 1 & 0 & 0 & 1.25 & -1.031 & -0.75 & 0.85 & -1.65 & 0.40 \end{bmatrix}$$

Flexibility Coefficients

$$[\alpha_{ij}] = 1/E \begin{bmatrix} 92.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 92.17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 137.61 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 146.20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 55.72 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 183.49 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 165.41 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 55.72 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 229.36 \end{bmatrix}$$

Multiply

$$[\alpha_{ij}]\,[G_{im}]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -183.49 & 187.13 & -45.95 & 45.87 \\ 92.17 & 0 & -137.61 & 182.75 & -57.43 & -137.61 & 140.35 & -91.89 & 91.74 \\ 0 & 0 & 0 & 0 & 0 & 0 & 187.13 & -45.95 & 45.87 \\ 92.17 & 0 & 0 & 182.75 & -57.43 & -137.61 & 140.35 & -91.89 & 91.74 \end{bmatrix}$$

Matrix Triple Product

$$[G_{im}]\,[\alpha_{ij}]\,[G_{im}]^T = 1/E \begin{bmatrix} 442.3 & 390.5 & 258.8 & 390.5 \\ 390.5 & 928.0 & 252.9 & 790.4 \\ 258.8 & 252.9 & 258.8 & 252.9 \\ 390.5 & 790.4 & 252.9 & 790.4 \end{bmatrix}$$

Deflections

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = 1/E \begin{bmatrix} 442.3 & 390.5 & 258.8 & 390.5 \\ 390.5 & 928.0 & 252.9 & 790.4 \\ 258.8 & 252.9 & 258.8 & 252.9 \\ 390.5 & 790.4 & 252.9 & 790.4 \end{bmatrix} \begin{Bmatrix} 2{,}500 \\ 2{,}000 \\ 800 \\ 450 \end{Bmatrix} = \begin{Bmatrix} 0.227 \\ 0.339 \\ 0.147 \\ 0.312 \end{Bmatrix} \text{ in}$$

Three Load Cases

$$\begin{Bmatrix} \delta_B \\ \delta_C \\ \delta_E \\ \delta_F \end{Bmatrix} = 1/E \begin{bmatrix} 442.3 & 390.5 & 258.8 & 390.5 \\ 390.5 & 928.0 & 252.9 & 790.4 \\ 258.8 & 252.9 & 258.8 & 252.9 \\ 390.5 & 790.4 & 252.9 & 790.4 \end{bmatrix} \begin{Bmatrix} \mathbf{2{,}500} & -1{,}200 & 1{,}800 \\ \mathbf{2{,}000} & -800 & 1{,}470 \\ \mathbf{800} & -2{,}100 & -1{,}200 \\ \mathbf{450} & -1{,}750 & -1{,}100 \end{Bmatrix} = 1/E \begin{bmatrix} 2{,}269{,}486 & -2{,}070{,}007 & 630{,}039 \\ 3{,}390{,}247 & -3{,}125{,}245 & 894{,}179 \\ 1{,}473{,}615 & -1{,}498{,}906 & 248{,}846 \\ 3{,}115{,}017 & -3{,}015{,}154 & 691{,}885 \end{bmatrix}$$

$$\begin{Bmatrix} \delta_B \\ \delta_C \\ \delta_E \\ \delta_F \end{Bmatrix} = 1/E \begin{bmatrix} 442.3 & 390.5 & 258.8 & 390.5 \\ 390.5 & 928.0 & 252.9 & 790.4 \\ 258.8 & 252.9 & 258.8 & 252.9 \\ 390.5 & 790.4 & 252.9 & 790.4 \end{bmatrix} \begin{array}{ccc} 1 & 2 & 3 \\ \left\{\begin{matrix} \mathbf{2{,}500} & -1{,}200 & 1{,}800 \\ \mathbf{2{,}000} & -800 & 1{,}470 \\ \mathbf{800} & -2{,}100 & -1{,}200 \\ \mathbf{450} & -1{,}750 & -1{,}100 \end{matrix}\right\} & & \end{array} = \begin{Bmatrix} 0.227 & -0.207 & 0.063 \\ 0.339 & -0.313 & 0.089 \\ 0.147 & -0.150 & 0.025 \\ 0.312 & -0.302 & 0.069 \end{Bmatrix} \text{ in}$$

Page A7.25 Influence Coefficients - Landing Gear Unit

Flexibility Coefficients

$\alpha_{44} = L^3 / 3EI = 15{,}552 / EI$ *rounded up* $15{,}600 / EI$ for $E = 10\,E6$ psi and $G = 3{,}846{,}154$ psi

Polar Moment of Inertia $J = I_x + I_y = 2I$ $EI / GJ = (10 * I) / (3.846154 * 2I) = 1.30$

$GJ = EI / 1.30$

$$\alpha_{22} = \frac{L_{AB}}{GJ} = \frac{1.30\left(3\text{ in}\right)}{EI} = \frac{3.90}{EI} \qquad \alpha_{77} = \frac{L_{BC}}{GJ} = \frac{1.30\left(36\text{ in}\right)}{EI} = \frac{46.80}{EI}$$

$$[\alpha_{ij}] = 1/EI \begin{bmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.900 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15{,}552 & 648 & 0 & 0 & 0 \\ 0 & 0 & 0 & 648 & 36 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 36 & 0 & 648 \\ 0 & 0 & 0 & 0 & 0 & 0 & 46.800 & 0 \\ 0 & 0 & 0 & 0 & 0 & 648 & 0 & 15{,}552 \end{bmatrix}$$

Unit Load Distribution

$[G_{73}] = 3\cos 20^\circ = 2.819$ *instead of* 2.810 $[G_{62}] = -\cos 20^\circ = -0.940$ *instead of* -0.937

$$[G_{im}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.342 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & -0.940 & 1.026 \\ 0 & 0.342 & 2.819 \\ 0 & 0 & 1 \end{bmatrix}$$

Transpose

$$[G_{im}]^T = \begin{bmatrix} 1 & 0 & 0 & 0.3420 & 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -0.9397 & 0.3420 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1.0261 & 2.8191 & 1 \end{bmatrix}$$

Multiply

$$[\alpha_{ij}]\,[G_{im}]^T = 1/EI \begin{bmatrix} 9 & 0 & 0 & 7{,}263 & 329.6 & 0 & 0 & 0 \\ 0 & 3.896 & 0 & 0 & 0 & -33.829 & 15.991 & -608.921 \\ 0 & 0 & 9 & 0 & 0 & 684.9 & 131.8 & 16{,}216.9 \end{bmatrix}$$

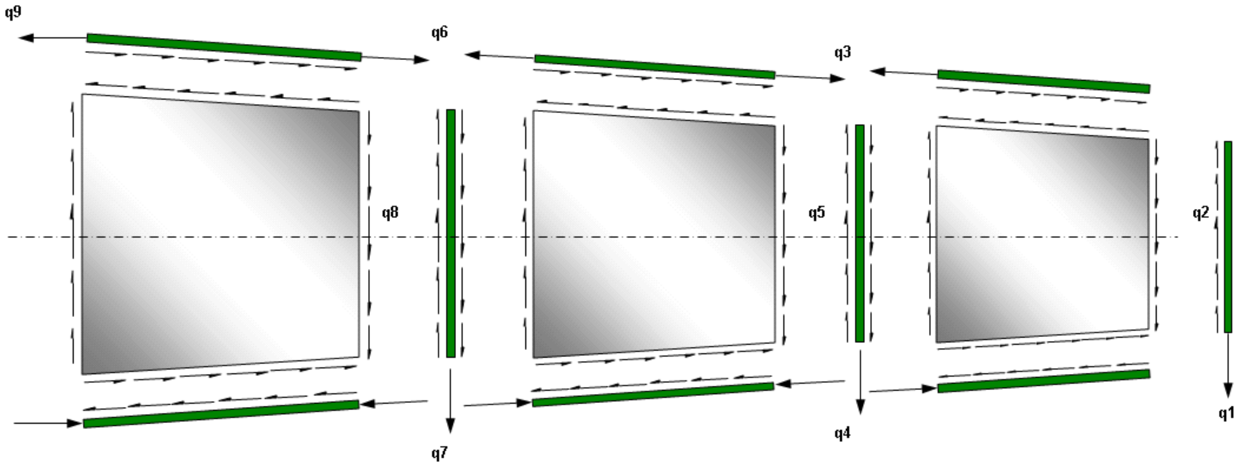
Matrix Triple Product

$$[G_{im}]\,[\alpha_{ij}]\,[G_{im}]^T = 1/EI \begin{bmatrix} 9 & 0 & 0 \\ 0 & 3.900 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Page A7.26 Influence Coefficients – Tapered Shear Beam

Thin Web Aluminum Beam Determine the deflection at Point G (See Bruhn page A7.26)

Free Body Diagram



Flexibility Coefficients

Spar Webs

Panel	h_1 (in)	h_2 (in)	Length L (in)	Area S (in^2)	Thickness t (in)	α_{ii} S / G t	α_{jj} $(h_1 / h_2)^2 \alpha_{ii}$
Bay 1 Spar Web	18.33	20.00	20	383.3 in^2	0.032 in	31,146 / E	**26,171 / E**
Bay 2 Spar Web	16.67	18.33	20	350.0 in^2	0.032 in	28,437 / E	**23,502 / E**
Bay 3 Spar Web	15.00	16.67	20	316.7 in^2	0.032 in	25,729 / E	**20,841 / E**

Spar Caps and Stiffeners

Spar Caps

	AB	**BC**	**CD**	
L	20.017	20.017	20.017	in
A_i	0.150	0.150	0.150	in^2
A_j	0.150	0.150	0.150	in^2
α_{ii}	44.48	44.48	44.48	/ E
α_{ij}	22.24	22.24	22.24	/ E
α_{jj}	44.48	44.48	44.48	/ E

$\alpha_{ii} = L / 3AE$

$\alpha_{ij} = L / 6AE$

Stiffeners

	BF	**CG**	**HI**	
L	18.333	16.667	15.00	in
A_i	0.080	0.080	0.050	in^2
A_j	0.080	0.080	0.050	in^2
α_{ii}	76.39	69.44	100	/ E
α_{ij}	38.19	34.72	50	/ E
α_{jj}	76.39	69.44	100	/ E

Matrix Form

$$[\,\alpha_{ij}\,] = 10 \;/\; E \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2{,}084.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 17.79 & 0 & 0 & 4.45 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.94 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2{,}350.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.45 & 0 & 0 & 17.79 & 0 & 0 & 4.45 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7.64 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}617.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.45 & 0 & 0 & 8.90 \end{bmatrix}$$

Unit Load Distribution

$$[\,G_{im}\,] = \begin{bmatrix} 1 & 0 & 0 \\ 0.0667 & 0 & 0 \\ 1.2010 & 0 & 0 \\ 0 & 1 & 0 \\ 0.0540 & 0.0600 & 0 \\ 2.1837 & 1.0919 & 0 \\ 0 & 0 & 1 \\ 0.0446 & 0.0496 & 0.0545 \\ 3 & 2 & 1 \end{bmatrix}$$

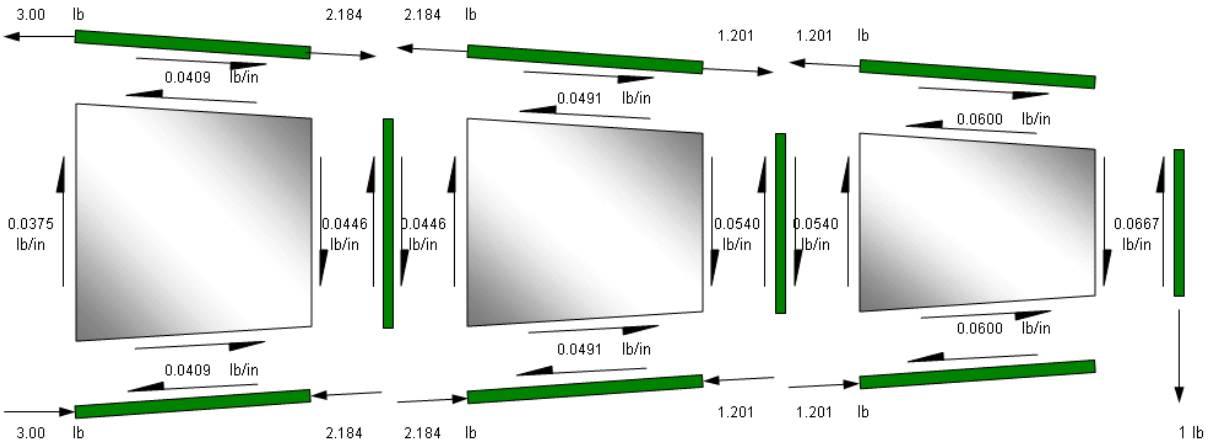
Transpose

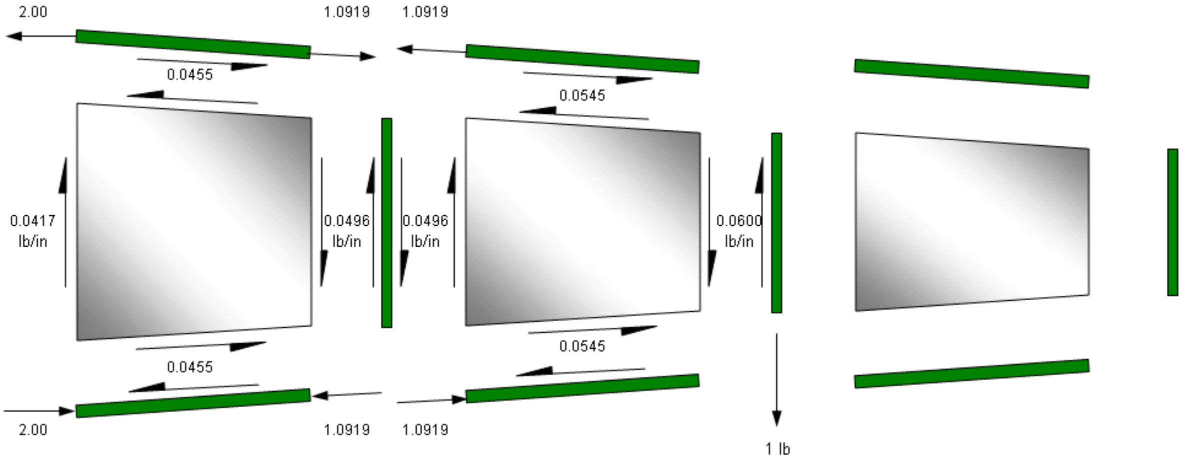
$$[\,G_{im}\,]^T = \begin{bmatrix} 1 & 0.067 & 1.201 & 0 & 0.054 & 2.1837113 & 0 & 0.045 & 3 \\ 0 & 0 & 0 & 1 & 0.060 & 1.092 & 0 & 0.050 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.055 & 1 \end{bmatrix}$$

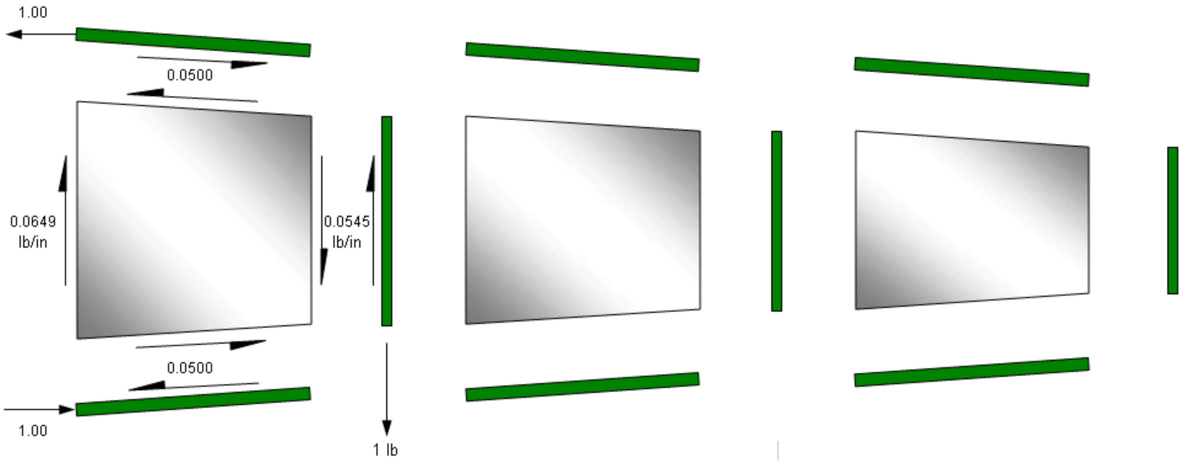
Multiply

$$[\,\alpha_{ij}\,]\;[\,G_{im}\,]^T = \begin{bmatrix} 10 & 138.94 & 31.08 & 0 & 126.91 & 57.54 & 0 & 116.80 & 36.40 \\ 0 & 0 & 4.86 & 6.94 & 141.01 & 28.32 & 0 & 129.77 & 22.65 \\ 0 & 0 & 0 & 0 & 0 & 4.45 & 7.64 & 142.75 & 8.90 \end{bmatrix}$$

Unit Load Distribution







Matrix Triple Product

$$[G_{im}]\ [\alpha_{ij}]\ [G_{im}]^T \quad = \quad 10\ /\ E \begin{bmatrix} 303.5 & 149.0 & 42.8 \\ 149.0 & 98.1 & 29.7 \\ 42.8 & 29.7 & 24.3 \end{bmatrix}$$

Deflections

$$\begin{Bmatrix} \delta_H \\ \delta_G \\ \delta_F \end{Bmatrix} = 10 \;/\; E \begin{bmatrix} 303.5 & 149.0 & 42.8 \\ 149.0 & 98.1 & 29.7 \\ 42.8 & 29.7 & 24.3 \end{bmatrix} \begin{Bmatrix} 1{,}800 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \delta_H \\ \delta_G \\ \delta_F \end{Bmatrix} = 10 \;/\; E \begin{Bmatrix} 546{,}352 \\ 268{,}275 \\ 76{,}994 \end{Bmatrix} = \begin{Bmatrix} 0.546 \\ \mathbf{0.268} \\ 0.077 \end{Bmatrix} \text{ in}$$

Deflection at Point G

δ_G = 0.268 in

See *Analysis and Design of Flight Vehicle Structures*, pages A7.22, A7.26 and A15.27.

Page A7.28 Method of Elastic Weights - Mohr's Method

Column 1

"For a unit load at point b, Fig. d …"	*should be*	"For a unit load at point B, Fig. d …"
"For deflection of point c, draw …"	*should be*	"For deflection of point C, draw …"

Column 2

"… at points b and c due …"	*should be*	"… at points B and C due …"
"… acting at point a …"	*should be*	"… acting at point A …"

Likewise in paragraph 2 change each instance of "b" to "B" and "c" to "C". Thanks to Dr. Howard W. Smith.

Page A7.29 Method of Elastic Weights – Mohr's Method

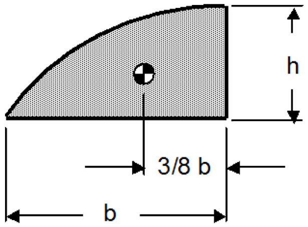
Column 1, Figure A7.42a Add $M_{\max} = \dfrac{w\,L^2}{8}$ to the moment diagram.

Slope at Supports **Slope at Center**

$$\alpha_{supports} = \frac{wL^3}{24EI}$$

$$\alpha_{center} = \left(\frac{wL^3}{24} - \frac{wL^3}{24}\right)\frac{1}{EI} = 0$$

Area of Each Half of the Moment Curve



$$Area = \frac{2}{3}\,b\,h = \frac{2}{3}\left(\frac{L}{2}\right)\frac{w\,L^2}{8} = \frac{w\,L^3}{24}$$

Deflection at Center of Beam

$$\delta_{center} = \left[\frac{w\,L^3}{24}\left(\frac{L}{2}\right) - \frac{w\,L^3}{24}\left(\frac{3\,L}{16}\right)\right]\frac{1}{E\,I} = \frac{5}{384}\left(\frac{w\,L^4}{E\,I}\right)$$

$$\delta_{center} = \frac{5}{384}\left(\frac{w\,L^4}{E\,I}\right)$$

Slope at Supports **Slope at Center**

$$\alpha_{supports} = \frac{wL^3}{24EI} \qquad \alpha_{center} = \left(\frac{wL^3}{24} - \frac{wL^3}{24}\right)\frac{1}{EI} = 0$$

Page A7.29 Mohr’s Method – Cantilever Wing Example

Column 2, Figure below Table A7.6 … I believe the values of the reactions have been swapped. I calculate the left reaction at 299,679 lb (vs. 299,090) and the right reaction at 225,919 lb (vs. 226,570).

Figure A7.43

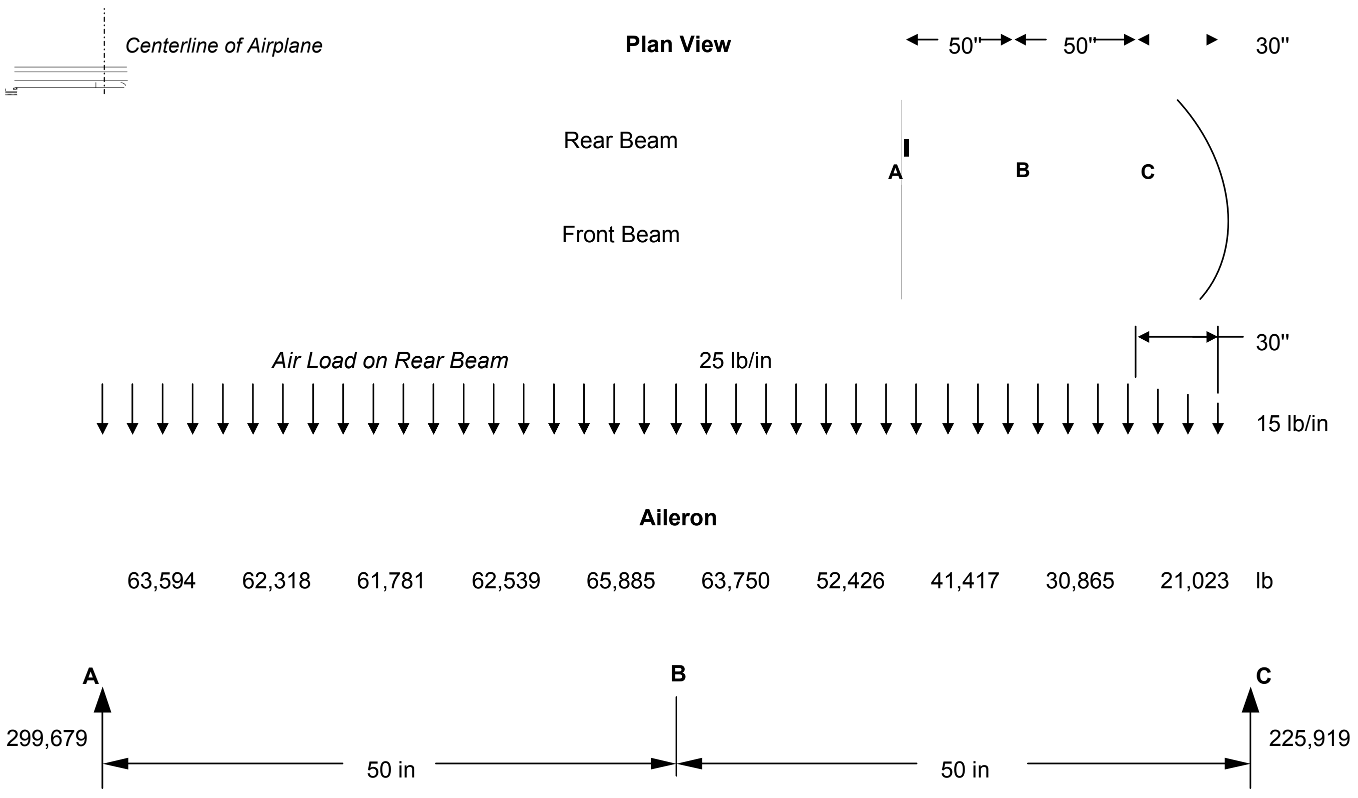


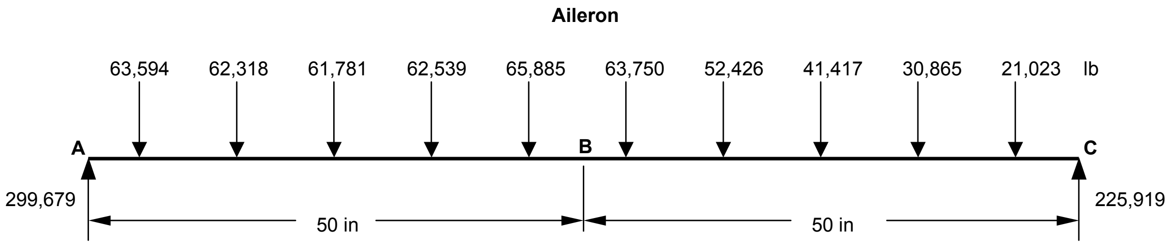
Table A7.6

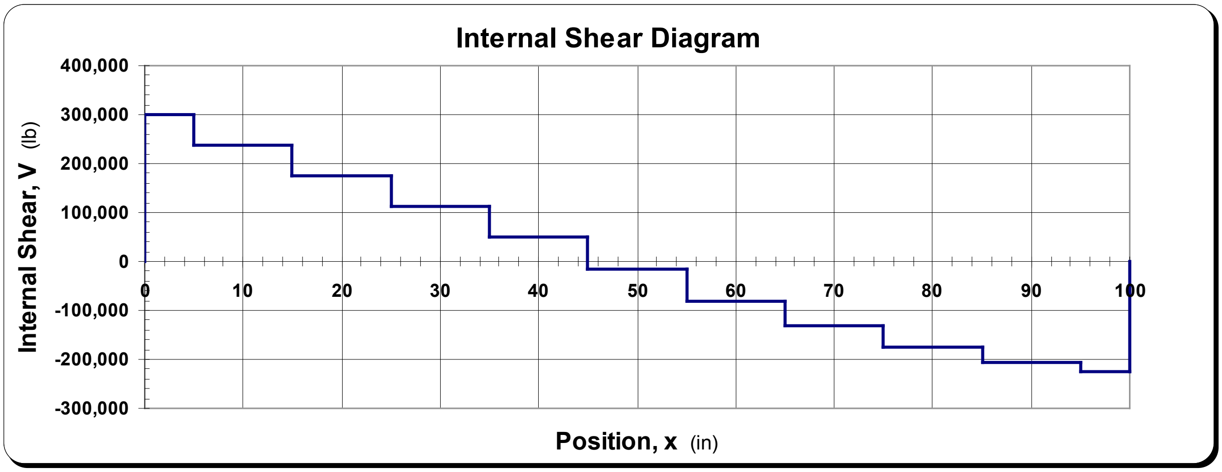
	Table A7.6					
	Strip No.	ds (in)	Midpoint (in)	Moment (midpoint)	Inertia (in^4)	Elastic Load *
C	1	10	5	11,563	5.5	21,023
	2	10	15	20,063	6.5	30,865
	3	10	25	31,063	7.5	41,417
	4	10	35	44,563	8.5	52,426
B	5	10	45	60,563	9.5	63,750
	6	10	55	79,063	12	65,885
	7	10	65	100,063	16	62,539
	8	10	75	123,563	20	61,781
	9	10	85	149,563	24	62,318
A	10	10	95	178,063	28	63,594

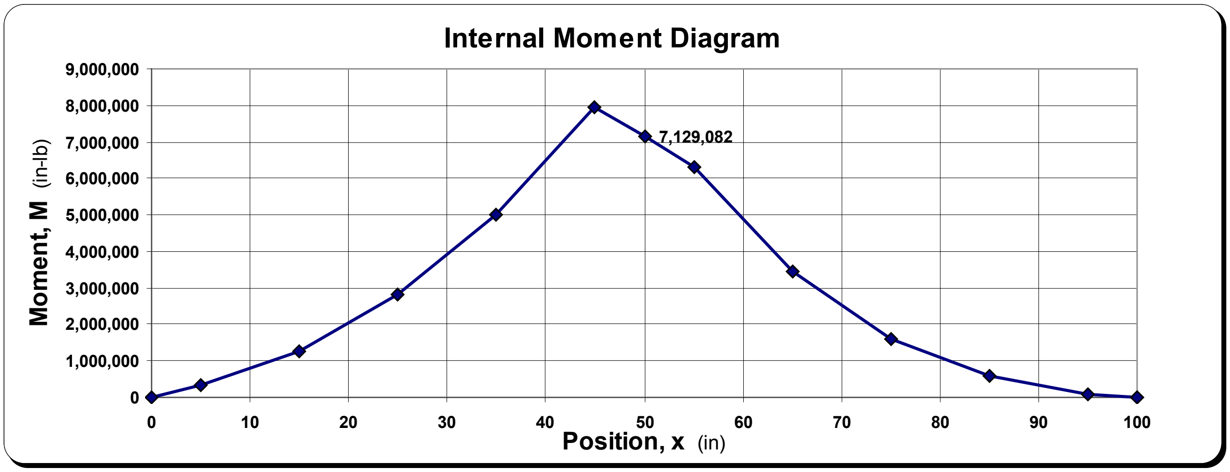
** Elastic Load = M ds / I*

Page A7.30 Mohr's Method – Seaplane Cantilever Wing Example

Column 1, Example Problem 34, should divide each δ_c by 1,000. Using 20.02 in lieu of 20, I get a deflection of C (divided by 1,000) equal to 517,867 / E I. Using 266.67 (267), 576.47 (576) yields 433.33 instead of 433 and 123.53 instead of 124. This gives me a tip deflection (divided by 1,000) of **102,310,667 / E I** in lieu of 102,200,000 / E I.







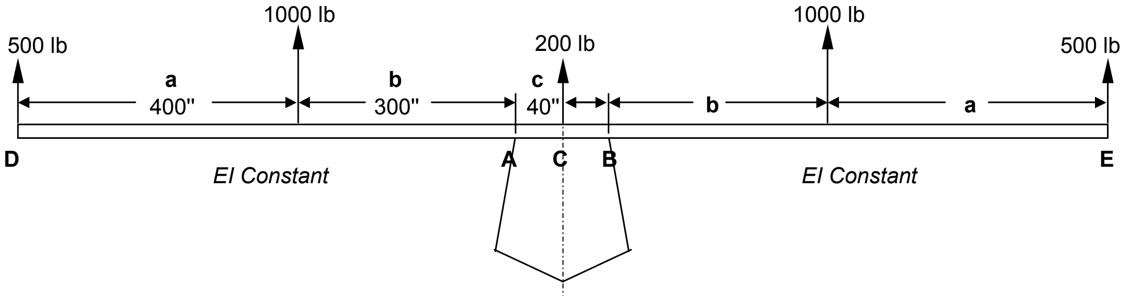
Page A7.31 Moment Area Method – Seaplane Cantilever Wing Example

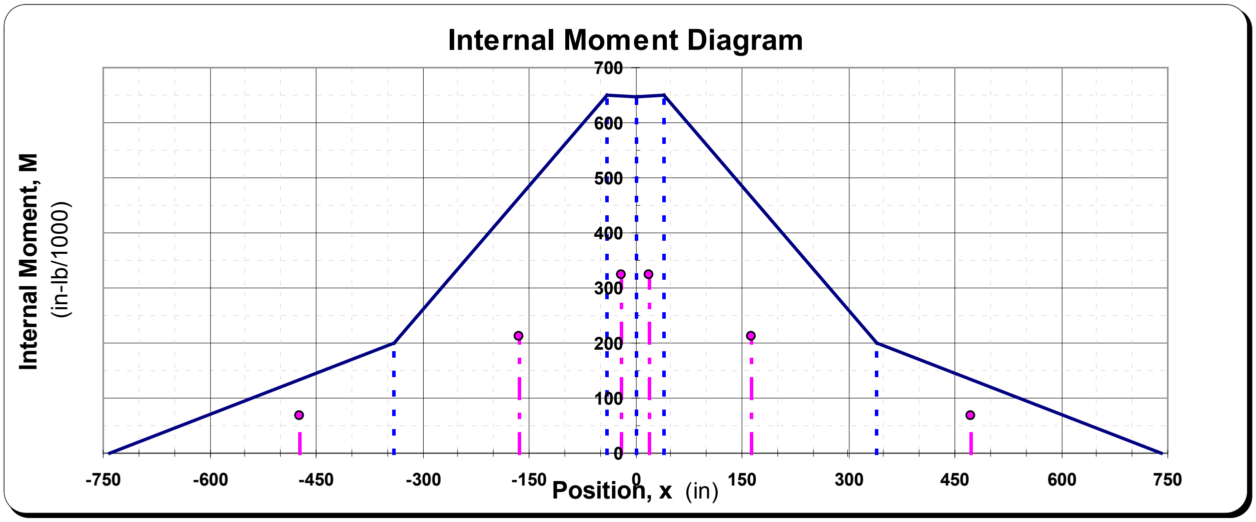
Column 2, Example Problem 36, Last Paragraph.

> "… and subtract deflection of a with respect to tangent at C ." *should be*
>
> "… and subtract deflection of A with respect to tangent at C."

Using values in the previous problem, 19.98 for 20 and 719.98 for 720 … gives me a tip deflection (divided by 1,000) of **102,310,667 / E I** in lieu of 102,180,000 / E I.

Figure A7.46





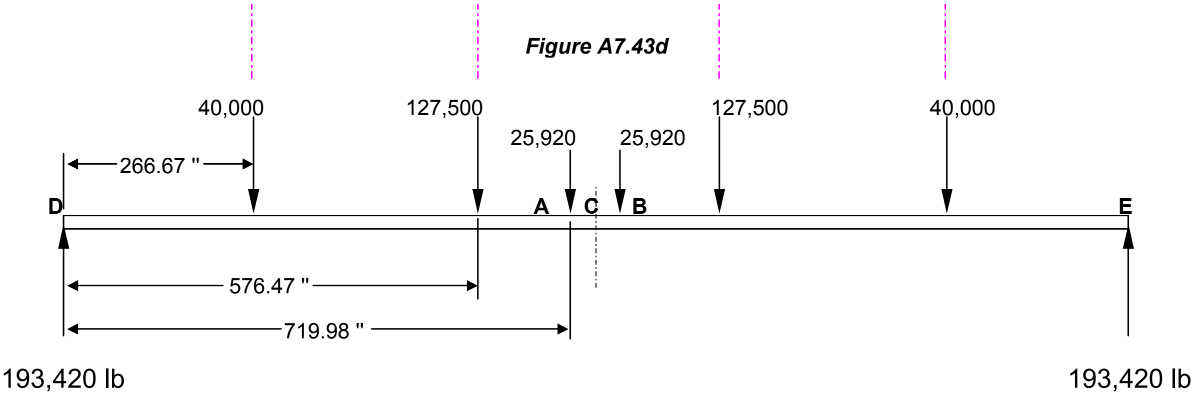


Figure A7.43d

Wing Deflection by the Moment Area Method

Deflection of D (with respect to C)

$$\frac{650 + 646}{2} \left(40'' \right) 19.98''$$

δ / 1,000	**517,867 / E I**

Deflection of A (with respect to tangent at C)

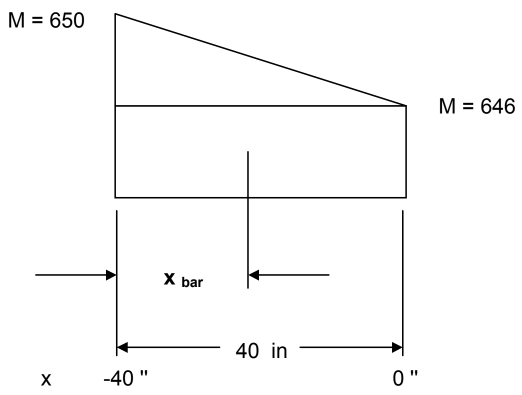
40,000 266.67 " + 127,500 576.47 " + 25,920 719.98 "

δ / 1,000	**102,828,533 / E I**

Calculate Centroid of Trapezoids

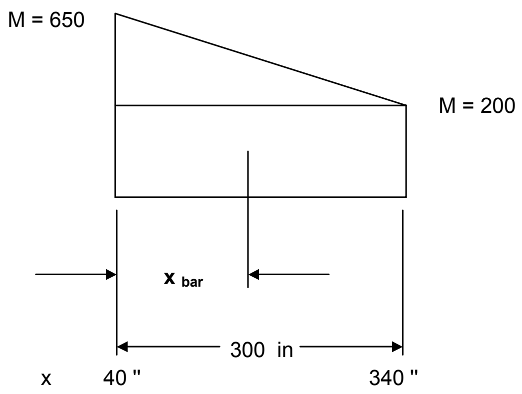
	x	**A**	**x A**
Rectangle	20	25,840	516,800
Triangle	13.33	80	1,066.7
	Σ	25,920	517,866.7

$$x_{bar} = \frac{\Sigma\, x\, A}{\Sigma\, A} = \mathbf{19.98} \text{ in}$$



	x	**A**	**x A**
Rectangle	150	60,000	9,000,000
Triangle	100	67,500	6,750,000
	Σ	127,500	15,750,000

$$x_{bar} = \frac{\Sigma\, x\, A}{\Sigma\, A} = \mathbf{123.53} \text{ in}$$



	40,000	127,500	25,920	25,920	127,500	40,000	
	266.67 "	576.47 "	719.98 "	760.02 "	903.53 "	1213.33 "	
ΣM_D	10,666,667	73,500,000	18,661,867	19,699,733	1.152E+08	48,533,333	**193,420**
	1213.33	903.53	760.02	719.98	576.47	266.67	
193,420	48,533,333	1.152E+08	19,699,733	18,661,867	73,500,000	10,666,667	ΣM_E

OUTPUT

Deflection of A (with respect to C)	δ / 1,000	**517,867**	/ E I
Deflection of D (with respect to tangent at C)	δ / 1,000	**102,828,533**	/ E I
Deflection of Tip	δ / 1,000	**102,310,667**	/ E I

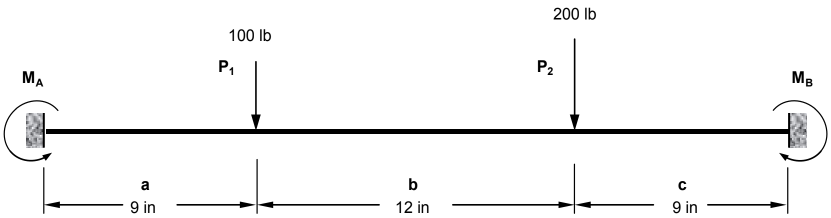
Page A7.32 Moment Area Method – Fixed Beam

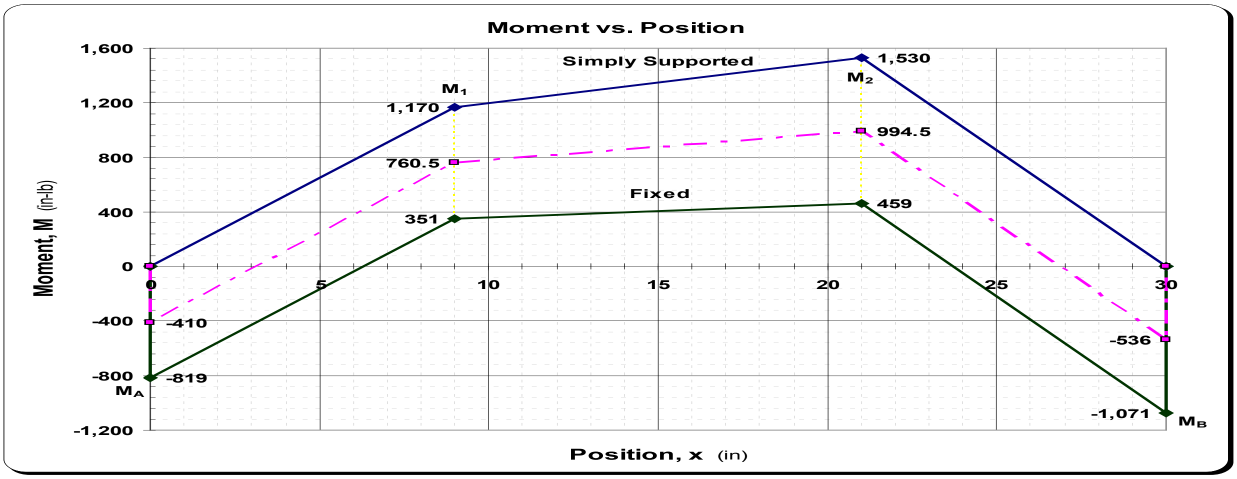
Figure A7.47

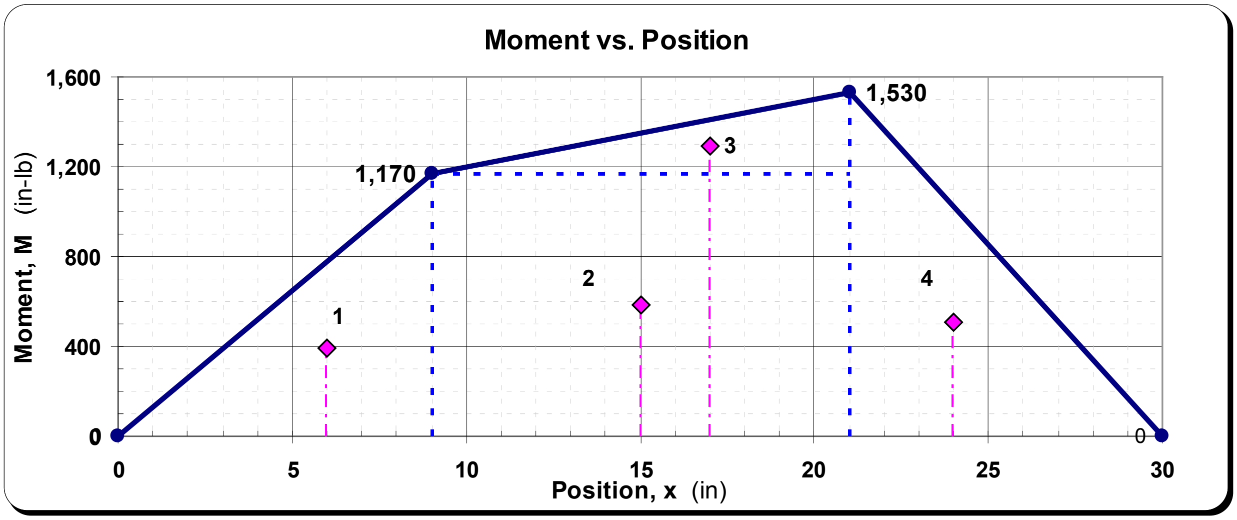
$P\,a\,(L-a)$ *should be* $$\frac{P\,a\,(L-a)}{L}$$

L

Example Problem 37





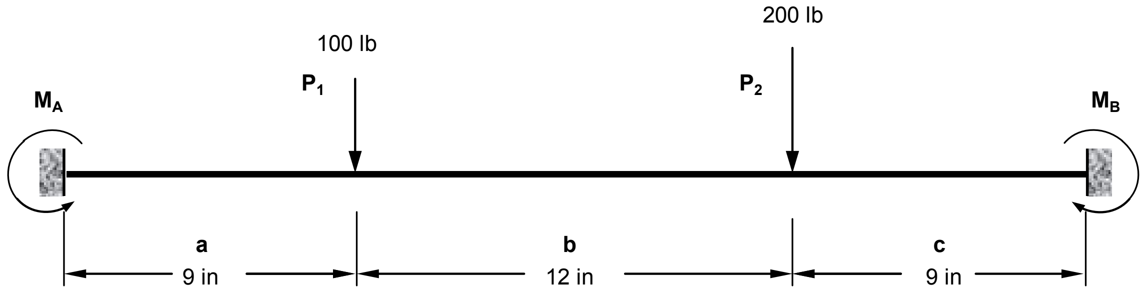


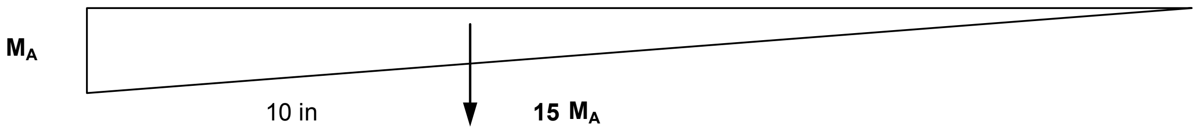
Simply Supported

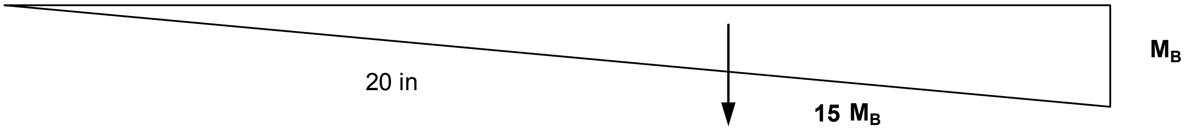
	M_1	M_2
Due to P_1	630	270
Due to P_2	540	1,260
Σ M	**1,170**	**1,530**

	Area 1	**Area 2**	**Area 3**	**Area 4**	**M_A**	**M_B**	
	585	1,170	180	765	0.5	0.5	
	9	12	12	9	30	30	
1	**5,265**	**14,040**	**2,160**	**6,885**	**15 Ma**	**15 Mb**	=0
x bar	6.00 in	15 in	17.00 in	24.00 in	10.00 in	20.00 in	
2	**31,590**	**210,600**	**36,720**	**165,240**	**150 Ma**	**300 Mb**	=0

Fixed







Two Equations, Two Unknowns

1	**15 Ma + 15 Mb + 28,350 = 0**
2	**150 Ma + 300 Mb + 444,150 = 0**

$$\left|\begin{matrix} \mathbf{15} & \mathbf{15} \\ \mathbf{150} & \mathbf{300} \end{matrix}\right| \begin{matrix} M_A = -28,350 \\ M_B = -444,150 \end{matrix}$$

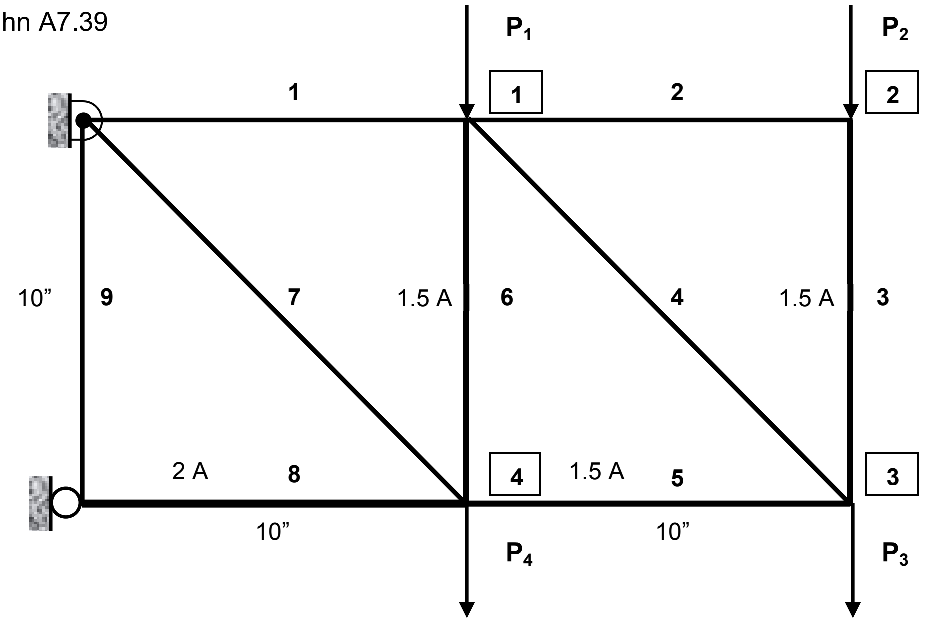
$$\begin{matrix} M_A \\ M_B \end{matrix} = \begin{bmatrix} 0.1333 & -0.0067 \\ -0.0667 & 0.0067 \end{bmatrix} \begin{matrix} -28,350 \\ -444,150 \end{matrix} = \begin{matrix} \mathbf{-819} & \text{in-lb} \\ \mathbf{-1,071} & \text{in-lb} \end{matrix}$$

M_A = -819 in-lb *versus* M_A = -816 in-lb

M_B = -1,071 in-lb *versus* M_B = -1,074 in-lb

Page A7.39 Influence Coefficient Matrix - Truss

Bruhn A7.39



Load Case 1

P_1 = 1,000 lb
P_2 = 500 lb
P_3 = 800 lb
P_4 = 400 lb

Load Case 2

P_1 = 300 lb
P_2 = 700 lb
P_3 = 400 lb
P_4 = 600 lb

Influence Coefficients

$$[\alpha_{ij}] = 1/E \begin{bmatrix} 10.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6.67 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 14.14 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.67 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 14.14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.00 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.00 \end{bmatrix}$$

Unit Load Distribution

$$[G_{im}] = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1.414 & 0 & 1.414 \\ 0 & -1 & 0 & -1 \\ -1 & -1 & 0 & -1 \\ 1.414 & 1.414 & 1.414 & 1.414 \\ -1 & -2 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Transpose

$$[G_{im}]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1.414 & -1 & 0 \\ 1 & 0 & -1 & 1.414 & -1 & -1 & 1.414 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.414 & -1 & 0 \\ 1 & 0 & 0 & 1.414 & -1 & -1 & 1.414 & -2 & 0 \end{bmatrix}$$

Multiply

$$[\alpha_{ij}]\,[G_{im}]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -6.667 & 20 & -5 & 0 \\ 10 & 0 & -6.667 & 20 & -6.667 & -6.667 & 20 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & -5 & 0 \\ 10 & 0 & 0 & 20 & -6.667 & -6.667 & 20 & -10 & 0 \end{bmatrix}$$

Matrix Triple Product

$$[G_{im}]\,[\alpha_{ij}]\,[G_{im}]^T = 1/E \begin{bmatrix} 39.95 & 44.95 & 33.28 & 44.95 \\ 44.95 & 106.57 & 38.28 & 99.90 \\ 33.28 & 38.28 & 33.28 & 38.28 \\ 44.95 & 99.90 & 38.28 & 99.90 \end{bmatrix}$$

Deflections

$$\left\{ \delta_{mk} \right\} = \frac{1}{E} \begin{bmatrix} 39.95 & 44.95 & 33.28 & 44.95 \\ 44.95 & 106.57 & 33.28 & 99.90 \\ 33.28 & 38.28 & 33.28 & 38.28 \\ 44.95 & 99.90 & 38.28 & 99.90 \end{bmatrix} \begin{Bmatrix} 1{,}000 & 300 \\ 500 & 700 \\ 800 & 400 \\ 400 & 600 \end{Bmatrix}$$

Greatest Deflection of Point 4

For E = 10.0 x 10^6 psi

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = \frac{1}{10\left(10^6\right)} \begin{bmatrix} 39.95 & 44.95 & 33.28 & 44.95 \\ 44.95 & 106.57 & 33.28 & 99.90 \\ 33.28 & 38.28 & 33.28 & 38.28 \\ 44.95 & 99.90 & 38.28 & 99.90 \end{bmatrix} \begin{Bmatrix} 1{,}000 & 300 \\ 500 & 700 \\ 800 & 400 \\ 400 & 600 \end{Bmatrix} = \begin{Bmatrix} 0.011 & 0.008 \\ 0.017 & 0.016 \\ 0.009 & 0.007 \\ 0.017 & 0.016 \end{Bmatrix}$$

Load Case 1

$$\delta_1 = 0.011 \;\; in \qquad \delta_2 = 0.017 \;\; in \qquad \delta_3 = 0.009 \;\; in \qquad \underline{\underline{\delta_4 = 0.017 \;\; in}} \longleftarrow$$

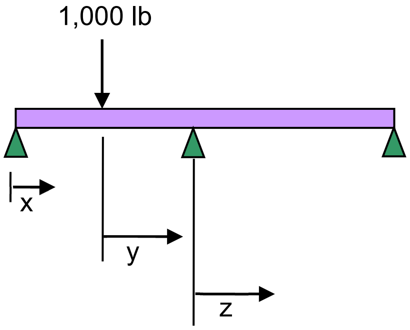
Load Case 2

$$\delta_1 = 0.008 \;\; in \qquad \delta_2 = 0.016 \;\; in \qquad \delta_3 = 0.007 \;\; in \qquad \underline{\underline{\delta_4 = 0.016 \;\; in}} \longleftarrow$$

Page A8.2 Theorem of Least Work

Figure A8.2

The x, y and z dimensions are measured as shown in this sketch:



The moment equation in the right column is actually written as:

$M = (500 + R_x)(L/2 + y) - 1{,}000\, y$

but it simplifies to the Bruhn equation shown.

Thanks to Jim Baldwin.

Page A8.3 Method of Least Work – Fixed Beam

Column 1,

For M … $0 < y < 2L$

M_r	*should be*	M_R

Then …

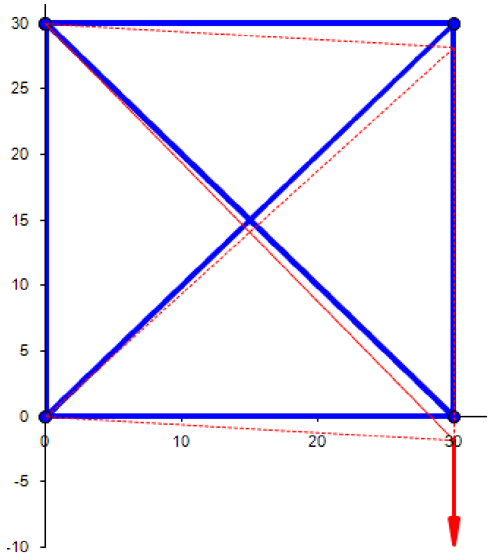
P_L	*should be*	$P\,L$

Last equation

$2\,P_L$	*should be*	$2\,P\,L$
P_L	*should be*	$P\,L$

Page A8.7 Method of Dummy Unit Loads

Elmer F. Bruhn *Analysis and Design of Flight Vehicles Structures* page A8.7



S Loads

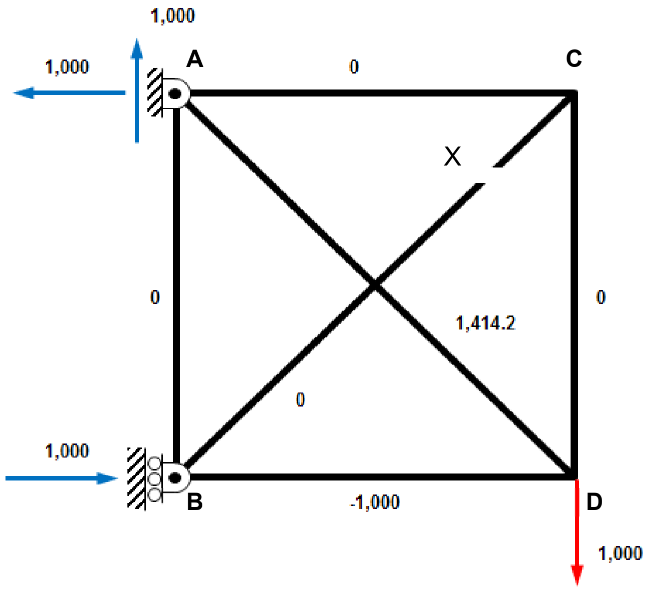
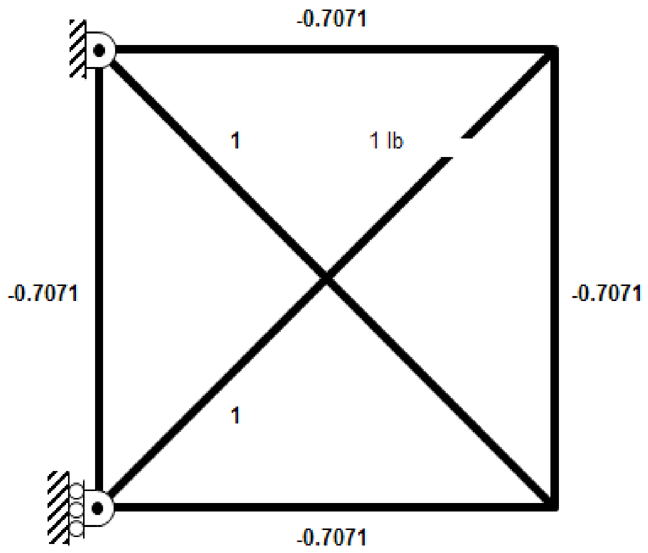


Table A8.1

Member	Length, L (inch)	Area, A (in ^{2})	Load, S (lb)	Unit Load, u	S u L / A	u ^{2} L / A	True Load S + X u (lb)
AB	30.00	1.0	0	-0.7071	0	15.00	395.3
BC	30.00	1.0	-1,000	-0.7071	21,213.2	15.00	-604.7
CD	30.00	1.0	0	-0.7071	0	15.00	395.3
DA	30.00	1.0	0	-0.7071	0	15.00	395.3
AC	42.426	2.0	0	1.000	0	21.21	-559.0
BD	42.426	1.5	1,414.2	1.000	40,000	28.28	855.2
				Σ	61,213.2	109.50	

Unit Loads



$$X = -\frac{S\,u\,L\,/\,A}{u^2\,L\,/\,A} = -\frac{(\,61{,}213.2\,)}{109.50} = -559.0\ lb$$

True Loads

P_{AB} = S + X u_x = 0 lb - 559.0 (- 0.7071) = 395.3

P_{BC} = S + X u_x = -1,000 lb - 559.0 (- 0.7071) = -604.7

P_{CD} = S + X u_x = 0 lb - 559.0 (- 0.7071) = 395.3

P_{BD} = S + X u_x = 0 lb - 559.0 (- 0.7071) = 395.3

P_{CE} = S + X u_x = 0 lb - 559.0 (1.00) = 559.0

P_{CE} = S + X u_x = 1,414.2 lb - 559.0 (1.00) = 855.2

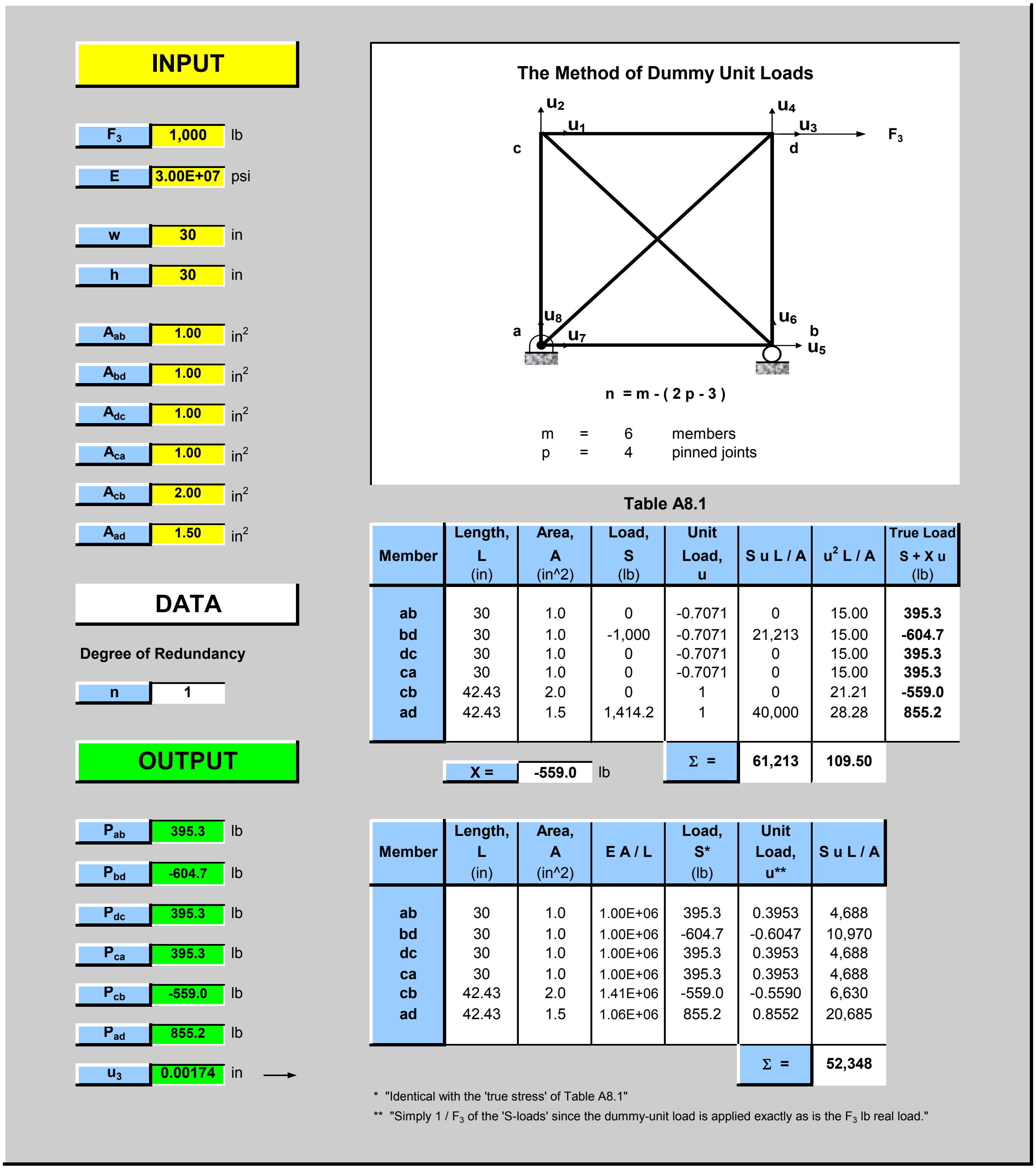
Degree of Redundancy

n = m - (2 p - 3)

m	6 Members
p	4 Pinned Joints
n	n = m - (2 p - 3) = 6 - [2 (4) - 3] = 1

Spreadsheet

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* page A8.7



INPUT

F_3	1,000	lb
E	3.00E+07	psi
w	30	in
h	30	in
A_{ab}	1.00	in^2
A_{bd}	1.00	in^2
A_{dc}	1.00	in^2
A_{ca}	1.00	in^2
A_{cb}	2.00	in^2
A_{ad}	1.50	in^2

DATA

Degree of Redundancy

n	1

OUTPUT

P_{ab}	395.3	lb
P_{bd}	-604.7	lb
P_{dc}	395.3	lb
P_{ca}	395.3	lb
P_{cb}	-559.0	lb
P_{ad}	855.2	lb
u_3	0.00174	in →

The Method of Dummy Unit Loads

n = m - (2 p - 3)

m = 6 members
p = 4 pinned joints

Table A8.1

Member	Length, L (in)	Area, A (in^2)	Load, S (lb)	Unit Load, u	S u L / A	u^2 L / A	True Load S + X u (lb)
ab	30	1.0	0	-0.7071	0	15.00	**395.3**
bd	30	1.0	-1,000	-0.7071	21,213	15.00	**-604.7**
dc	30	1.0	0	-0.7071	0	15.00	**395.3**
ca	30	1.0	0	-0.7071	0	15.00	**395.3**
cb	42.43	2.0	0	1	0	21.21	**-559.0**
ad	42.43	1.5	1,414.2	1	40,000	28.28	**855.2**
				Σ =	**61,213**	**109.50**	

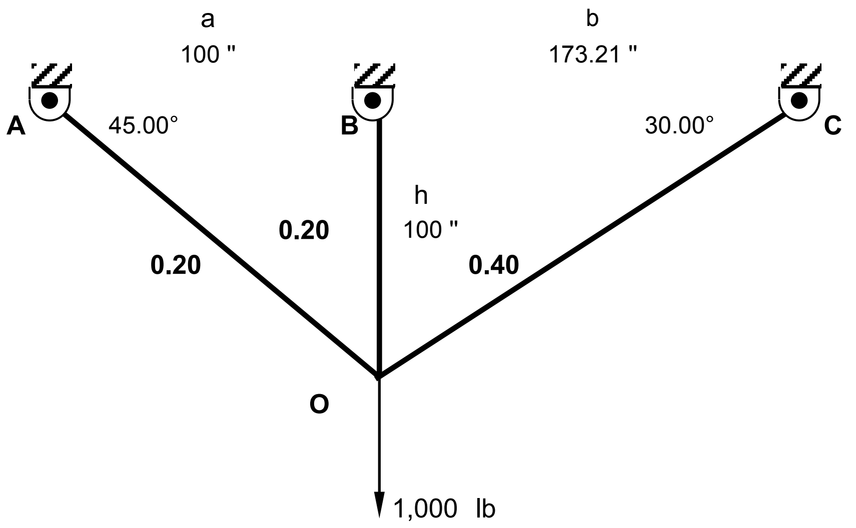
X = -559.0 lb

Member	Length, L (in)	Area, A (in^2)	E A / L	Load, S* (lb)	Unit Load, u**	S u L / A
ab	30	1.0	1.00E+06	395.3	0.3953	4,688
bd	30	1.0	1.00E+06	-604.7	-0.6047	10,970
dc	30	1.0	1.00E+06	395.3	0.3953	4,688
ca	30	1.0	1.00E+06	395.3	0.3953	4,688
cb	42.43	2.0	1.41E+06	-559.0	-0.5590	6,630
ad	42.43	1.5	1.06E+06	855.2	0.8552	20,685
					Σ =	**52,348**

* "Identical with the 'true stress' of Table A8.1"

** "Simply 1 / F_3 of the 'S-loads' since the dummy-unit load is applied exactly as is the F_3 lb real load."

Page A8.8 Method of Dummy Unit Loads, Singly Redundant Truss



Column 1, second table u = 1.224 should be 1.225 or 1.2247

Member	Length, L (in)	Area, A (in^2)	Load, S (lb)	Unit Load, u_x	$S\, u_x\, L / A$	$u_x^2\, L / A$	True Load S + X u (lb)
AO	**141.42**	**0.20**	**0**	**1.2247**	0	1,060.7	**335.5**
BO	**100**	**0.20**	**1,000**	**-1.3660**	-6.8E+05	933.0	**625.8**
CO	**200**	**0.40**	**0**	**1**	0	500.0	**273.9**
				Σ =	**-6.8E+05**	**2,493.67**	

Dummy Unit Load Method – Unequal Areas

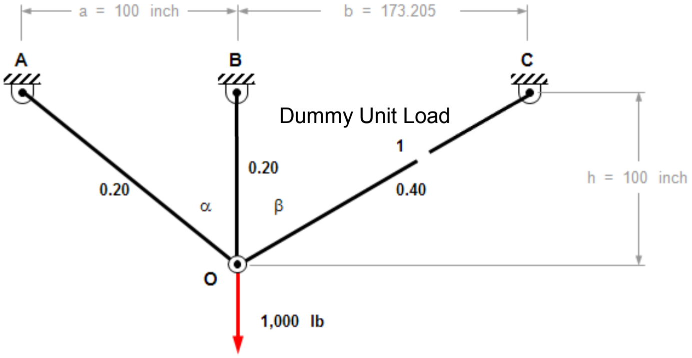
Elmer F. Bruhn *Analysis and Design of Flight Vehicles Structures* page A8.8

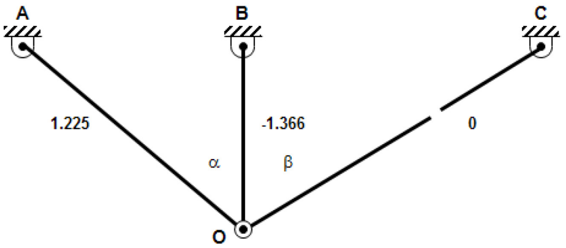
Given

P = 1,000 lb AO = 141.421 inch BO = 100 inch CO = 200 inch

Cross Sectional Areas $A_1 = 0.200\ \text{in}^2$ $A_2 = 0.200\ \text{in}^2$ $A_3 = 0.400\ \text{in}^2$

$\alpha = 45^\circ$ $\beta = 60^\circ$ E = 10.0 E+06 psi





S Loads

$\Sigma F_x = 0$ $- AO \sin\alpha + CO \sin\beta = 0$

$- AO\ (0.7071) + \mathbf{1}\ (0.8660) = 0$ AO = 1.22474

$\Sigma F_y = 0$ $AO \cos\alpha + BO + CO \cos\beta = 0$

$1.22474\ (0.7071) + BO + \mathbf{1}\ (0.500) = 0$ BO = -1.36603

Member Forces

Member	**Length L** (inch)	**Area A** (in^2)	**Load S** (lb)	**Unit Load u**	$\mathbf{S\,u_x\,L/A}$	$\mathbf{u_x^{\,2}\,L/A}$	**True Load S + X u** (lb)
AO	141.421	0.200	0	1.22474	0	1,060.66	**335.5**
BO	100	0.200	1,000	-1.36603	-683,012.70	933.01	**625.8**
CO	200	0.400	0	**1**	0	500	**273.9**
				Σ	**-683,012.70**	**2,493.67**	

$$X = -\frac{S\,u\,L\,/\,A}{u^2\ L\ /\ A} = -\frac{(-683{,}012.70)}{2{,}493.67} = 273.9\ lb$$

$R_1 = S + X\,u = 0 + 273.9\ (1.2247) = 335.5$ lb

$R_2 = S + X\,u = 1{,}000 + 273.9\ (-1.3360) = 625.8$ lb

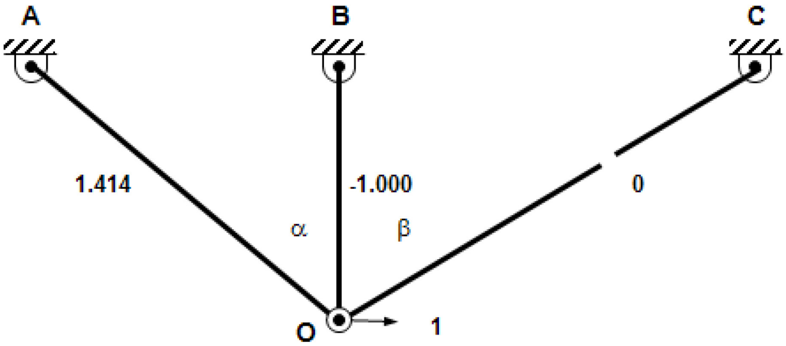
$R_3 = S + X\,u = 0 + 273.9\ (\mathbf{1}) = 273.9$ lb

Displacements

Vertical Displacements

$$\delta_y = \frac{S\,u\,L\,/\,A}{E} = \frac{(-683{,}012.70)}{10.0E06\;psi} = -0.0683\;inch \quad \downarrow$$

Horizontal Displacements



$\Sigma F_x = 0$ $\qquad - AO \sin\alpha + CO \sin\beta + 1 = 0$

$- AO\;(0.7071) + 0 + \mathbf{1} = 0 \qquad AO = 1.4142$

$\Sigma F_y = 0$ $\qquad AO \cos\alpha + BO + CO \cos\beta = 0$

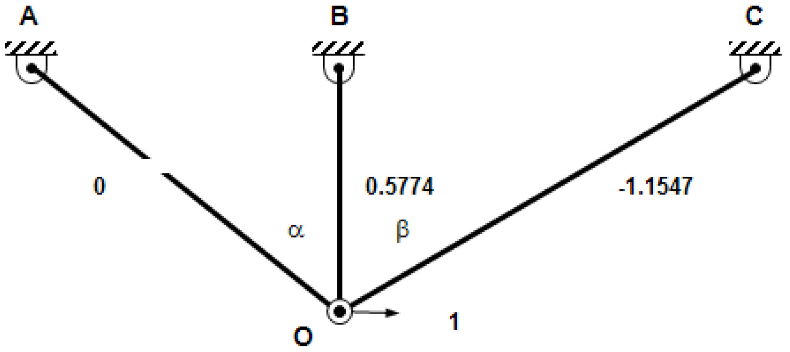
$1.4142\;(0.7071) + BO + 0\;(0.500) = 0 \qquad BO = -1.000$

Member	Length, L (inch)	Area, A (in ^{2})	Load, S * (lb)	Unit Load, u	S u L / A
AO	141.42	0.20	335.5	1.4142	335,456
BO	100	0.20	625.8	-1.000	-312,924
CO	200	0.40	273.9	0	0
				Σ =	22,531.5

* True loads from example above.

$$\delta_x = \frac{S\,u\,L\,/\,A}{E} = \frac{22{,}531.5}{10.0E06\;psi} = 0.0023\;inch \quad \longrightarrow$$

Horizontal Displacements - Another Way



$\Sigma F_x = 0$ $\quad - AO \sin\alpha + CO \sin\beta + 1 = 0$

$0 + CO\ (0.86603) + \mathbf{1} = 0 \qquad CO = -1.1547$

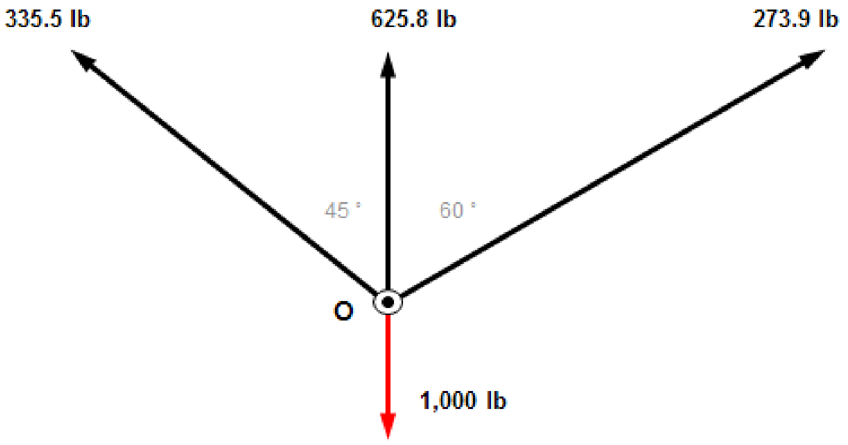
$\Sigma F_y = 0$ $\quad AO \cos\alpha + BO + CO \cos\beta = 0$

$0\ (0.7071) + BO - 1.1547\ (0.500) = 0 \qquad BO = 0.5774$

Member	Length, L (inch)	Area, A (in ^{2})	Load, S * (lb)	Unit Load, u	S u L / A
AO BO CO	141.42 100 200	0.20 0.20 0.40	335.5 625.8 273.9	0 0.5774 -1.1547	0 180,667 -158,135
				Σ =	22,531.5

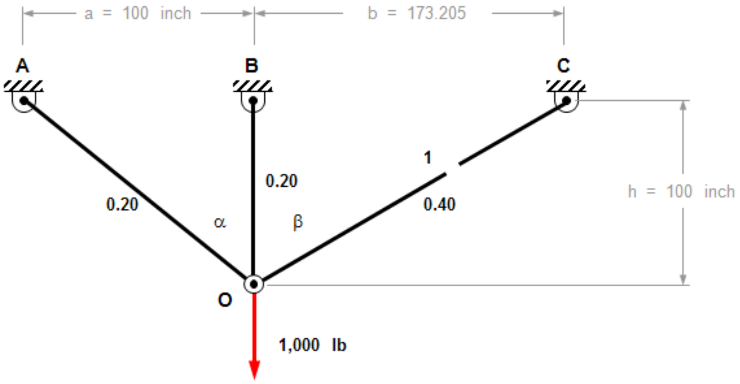
* True loads from example above.

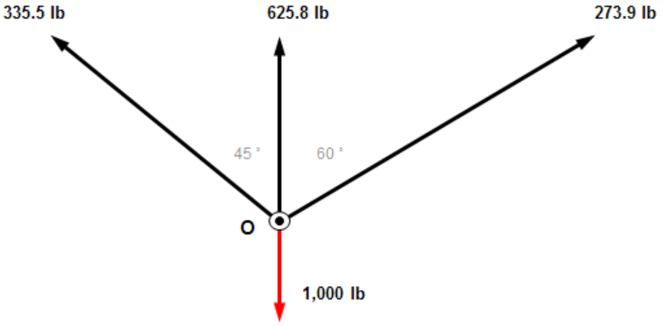
$$\delta_x = \frac{S\,u\,L\,/\,A}{E} = \frac{22{,}531.5}{10.0E06\ psi} = 0.0023\ inch \longrightarrow$$



Spreadsheet Page 1

Load	P	1,000	lb
Young's Modulus	E	1.00E+07	psi
Geometry	h	100	inch
	a	100	inch
	b	173.205	inch
Area	A_{AO}	0.20	in^2
	A_{BO}	0.20	in^2
	A_{CO}	0.40	in^2
Lengths	AO	141.421	inch
	BO	100	inch
	CO	200	inch
Angles	α	45	degrees
	β	60	degrees





Degree of Redundancy

n	1	Three Members, Two Static Equations

Member Forces

Member	Length, L (inch)	Area, A (in^2)	Load, S (lb)	Unit Load, u	S u L / A	u^2 L / A	True Load S + X u (lb)
AO	141.421	0.20	0	1.225	0	1,060.66	**335.5**
BO	100	0.20	1,000	-1.366	-683,013	933.01	**625.8**
CO	200	0.40	0	**1**	0	500.00	**273.9**
				Σ =	**-683,013**	**2,493.67**	

$$X = - \frac{-683,013}{2,493.67} = 273.9 \text{ lb}$$

$$\delta_y = \frac{-683,013}{1.00E+07} = -0.0683 \text{ inch}$$

True Loads

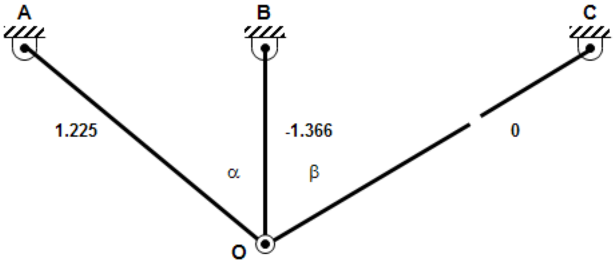
AO	335.5	lb
BO	625.8	lb
CO	273.9	lb

Displacements

δ_y	-0.0683	inch	↓
δ_x	0.0023	inch	→
δ_{total}	0.0683	inch	

Spreadsheet Page 2

Vertical Displacements



$\Sigma\ x = 0 \quad -AO \ \sin\alpha + CO \ \sin\beta = 0$

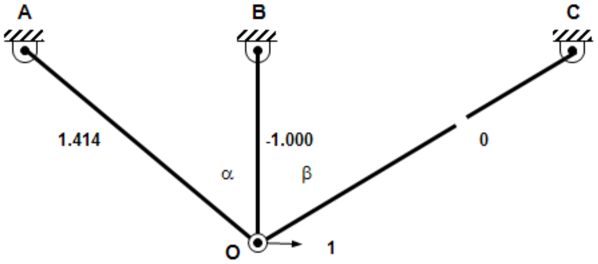
$-AO \left(0.70711\right) + \mathbf{1} \left(0.86603\right) = 0 \qquad AO = 1.2247$

$\Sigma\ y = 0 \quad AO \ \cos\alpha + BC + CO \ \cos\beta = 0$

$1.225 \left(0.70711\right) + BO + \mathbf{1} \left(0.500\right) = 0 \qquad BO = -1.3660$

Member	Length, L (inch)	Area, A (in ^{2})	Load, S (lb)	Unit Load, u	S u L / A	u ^{2} L / A	True Load S + X u (lb)
AO	141.421	0.20	0	1.225	0	1,060.66	**335.5**
BO	100	0.20	1,000	-1.366	-683,013	933.01	**625.8**
CO	200	0.40	0	**1**	0	500.00	**273.9**
				Σ =	**-683,013**	**2,493.67**	

Horizontal Displacements



$\Sigma\ x = 0 \quad -AO \ \sin\alpha + 1 = 0$

$-AO \left(0.70711\right) + 1 = 0 \qquad AO = 1.4142$

$\Sigma\ y = 0 \quad AO \ \cos\alpha + BO + CO \ \cos\beta = 0$

$1.414 \left(0.70711\right) + BO + \mathbf{0} \left(0.500\right) = 0 \qquad BO = -1.000$

Member	Length, L (inch)	Area, A (in ^{2})	Load, S * (lb)	Unit Load, u	S u L / A
AO	**141.42**	**0.20**	**335.5**	**1.4142**	335,456
BO	**100**	**0.20**	**625.8**	**-1.000**	-312,924
CO	**200**	**0.40**	**273.9**	**0**	0
				Σ =	**22,531.5**

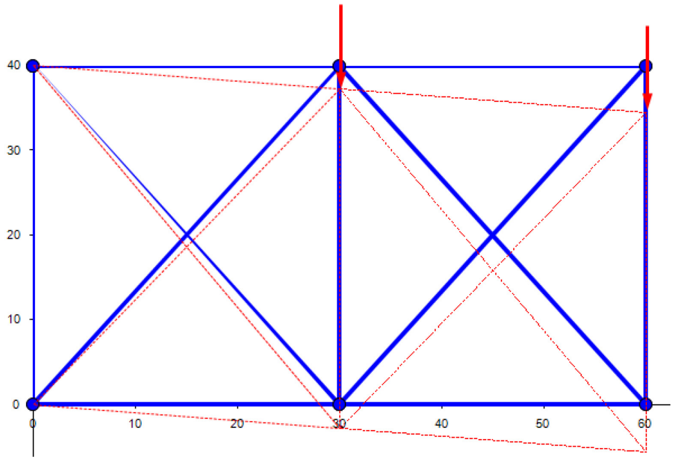
* True loads from example above.

E	**1.00E+07**

δ_x	**0.0023**	inch →

Page A8.10 Trusses with Double Redundancy

Elmer F. Bruhn *Analysis and Design of Flight Vehicles Structures* page A8.10



S Loads

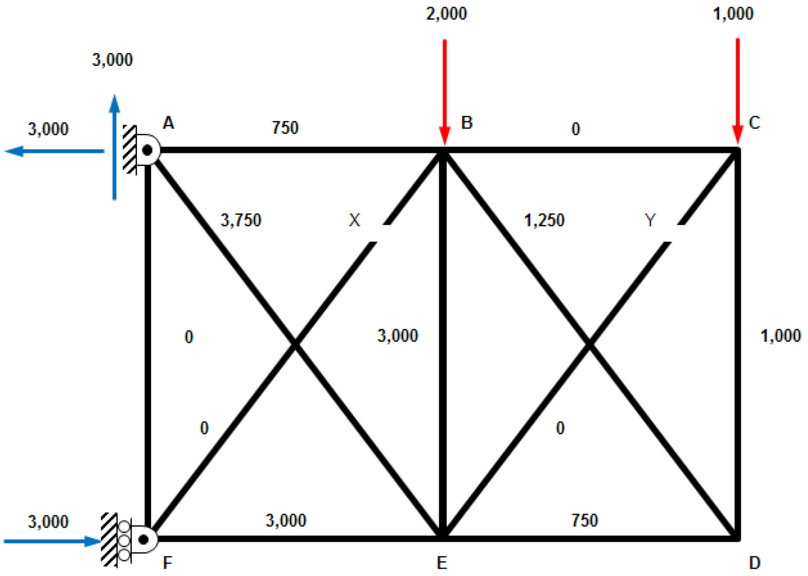
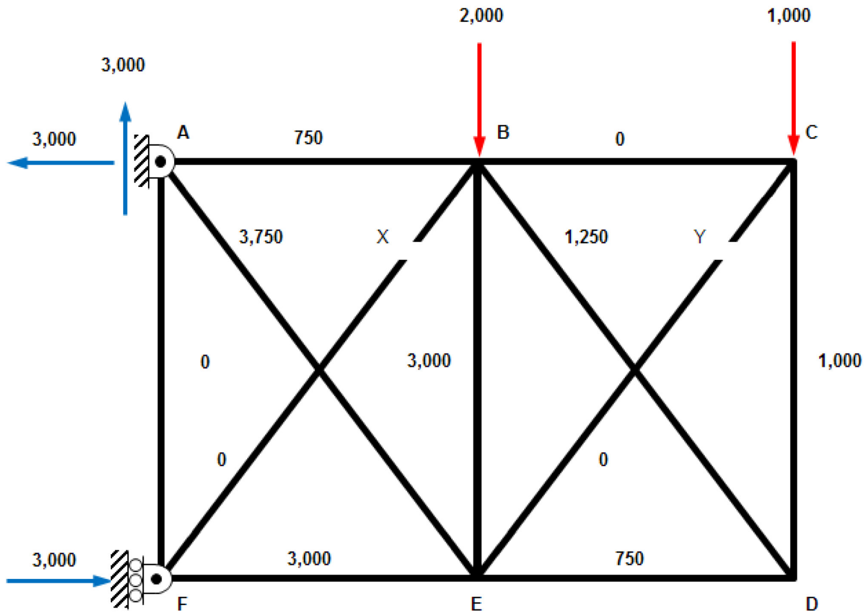


Table A8.2

Member	Length, L (inch)	Area, A (in ^{2})	Load, S (lb)	Unit Load, u_x	Unit Load, u_y	$S u_x L / A$	$S u_y L / A$	$u_x^2 L / A$	$u_y^2 L / A$	$u_x u_y L/A$	True Load $S + X u$ (lb)
AB	30	0.50	750	-0.600	0	-27,000	0	21.60	0	0	1,686.4
BC	30	0.25	0	0	-0.600	0	0	0	43.20	0	558.2
CD	40	0.50	-1,000	0	-0.800	0	64,000	0	51.20	0	-255.7
BD	50	0.25	1,250	0	1	0	250,000	0	200	0	319.6
CE	50	0.50	0	0	1	0	0	0	100	0	-930.4
BE	40	0.70	-3,000	-0.800	-0.800	137,143	137,143	36.57	36.57	36.57	-1,007.2
ED	30	0.50	-750	0	-0.600	0	27,000	0	21.60	0	-191.8
BF	50	0.80	0	1.000	0	0	0	62.50	0	0	-1,560.6
AE	50	0.50	3,750	1.000	0	375,000	0	100	0	0	2,189.4
EF	30	0.70	-3,000	-0.600	0	77,143	0	15.43	0	0	-2,063.6
AF	40	0.25	0	-0.800	0	0	0	102.40	0	0	1,248.5
					Σ	562,286	478,143	338.50	452.57	36.57	

S Loads Expanded



$\alpha = \text{atan}\ (\ 40\ \text{inch}\ /\ 30\ \text{inch}\ ) = 53.13\ \text{degrees}$ $\sin\alpha = 0.80$ $\cos\alpha = 0.60$

Joint C

$\Sigma F_x = 0$ $BC - CE\cos\alpha = 0$ $BC - 0 = 0$ $BC = 0\ \text{lb}$

$\Sigma F_y = 0$ $CD - CE - P_2 = 0$ $CD - 0 - 1{,}000 = 0$ $CD = 1{,}000\ \text{lb}$

Joint D

$\Sigma F_y = 0$ $CD - BD\sin\alpha = 0$ $1{,}000\ \text{lb} - BD\ (\ 0.80\ ) = 0$ $BD = 1{,}250\ \text{lb}$

$\Sigma F_x = 0$ $DE - BD\cos\alpha = 0$ $DE - 1{,}250\ \text{lb}\ (\ 0.60\ ) = 0$ $DE = 750\ \text{lb}$

Joint F

$\Sigma F_x = 0$ $3{,}000\ \text{lb} - EF - BF\cos\alpha = 0$ $3{,}000\ \text{lb} - EF - 0 = 0$ $EF = -3{,}000\ \text{lb}$

$\Sigma F_y = 0$ $AF - BF\sin\alpha = 0$ $AF - 0\ (\ 0.80\ ) = 0$ $AF = 0\ \text{lb}$

Joint E

$\Sigma F_x = 0$ $DE - EF - AE\cos\alpha + CE\cos\alpha = 0$

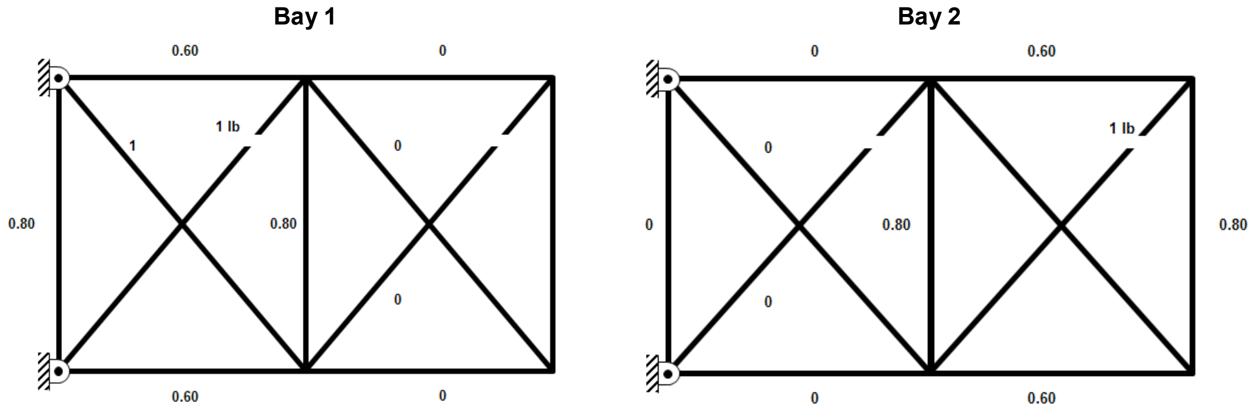
$750 - 3{,}000 - AE\ (\ 0.60\ ) + 0\ (\ 0.60\ ) = 0$ $AE = 3{,}750\ \text{lb}$

$\Sigma F_y = 0$ $BE + AE\sin\alpha + CE\sin\alpha = 0$ $BE + 3{,}750\ (\ 0.80\ ) - 0 = 0$ $BE = -3{,}000\ \text{lb}$

Joint A

$\Sigma F_x = 0$ $-3{,}000\ \text{lb} + AB + AE\cos\alpha = 0$ $-3{,}000 + AB + 3{,}750\ (\ 0.60\ ) = 0$ $AB = 750\ \text{lb}$

Figure



Unit X Loads

Joint B

$\Sigma F_x = 0$	$- AB - BF \cos\alpha = 0$	$- AB - 1 \text{ lb } (0.60) = 0$	$AB = - 0.60$ lb
$\Sigma F_y = 0$	$- BE - BF \sin\alpha = 0$	$- BE - 1 \text{ lb } (0.80) = 0$	$CD = - 0.80$ lb

Joint A

$\Sigma F_x = 0$	$AB - AE \cos\alpha = 0$	$0.60 \text{ lb } - AE (0.60) = 0$	$AE = 1.00$ lb
$\Sigma F_y = 0$	$- AF - AE \sin\alpha = 0$	$- AF - 1.00 (0.80) = 0$	$AF = - 0.80$ lb

Joint F

$\Sigma F_x = 0$	$EF + BF \cos\alpha = 0$	$EF + 1 \text{ lb } (0.60) = 0$	$EF = - 0.60$ lb

Unit Y Loads

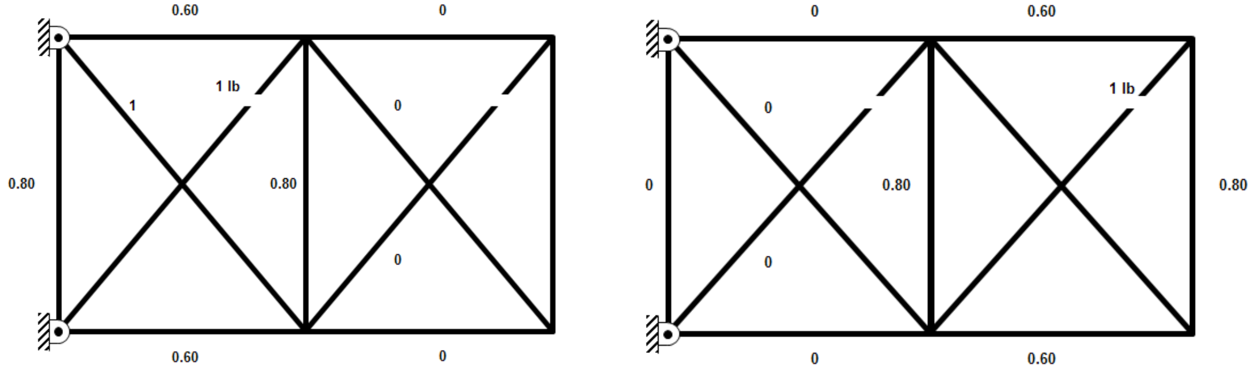
Joint C

$\Sigma F_x = 0$	$- BC - CE \cos\alpha = 0$	$- AB - 1 \text{ lb } (0.60) = 0$	$BC = - 0.60$ lb
$\Sigma F_y = 0$	$- CD - CE \sin\alpha = 0$	$- CD - 1 \text{ lb } (0.80) = 0$	$CD = - 0.80$ lb

Joint D

$\Sigma F_y = 0$	$- CD - BD \sin\alpha = 0$	$- (- 0.80) - BD (0.80) = 0$	$BD = 1.00$ lb
$\Sigma F_x = 0$	$- DE - BD \cos\alpha = 0$	$- DE - 1 \text{ lb } (0.60) = 0$	$DE = - 0.60$ lb

Unit Loads


Bay 1

Bay 2

Two Equations, Two Unknowns

$$X \sum \frac{u_x^{\,2}\, L}{A} + Y \sum \frac{u_x\, u_y\, L}{A} = -\sum \frac{S\, u_x\, L}{A} \qquad 338.5\ X + 36.6\ Y = -562{,}285.7$$

$$X \sum \frac{u_x\, u_y\, L}{A} + Y \sum \frac{u_y^{\,2}\, L}{A} = -\sum \frac{S\, u_y\, L}{A} \qquad 36.6\ X + 452.6\ Y = -478{,}142.9$$

$$\begin{bmatrix} 338.5 & 36.6 \\ 36.6 & 452.6 \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{Bmatrix} -562{,}285.7 \\ -478{,}142.9 \end{Bmatrix}$$

$$\begin{bmatrix} 338.5 & 36.6 \\ 36.6 & 452.6 \end{bmatrix}^{-1} = \begin{bmatrix} 0.00298 & -0.00024 \\ -0.00024 & 0.00223 \end{bmatrix}$$

$$\begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{bmatrix} 0.00298 & -0.00024 \\ -0.00024 & 0.00223 \end{bmatrix} \begin{Bmatrix} -562{,}285.7 \\ -478{,}142.9 \end{Bmatrix} = \begin{Bmatrix} -1{,}560.59 \\ -930.39 \end{Bmatrix}$$

$X = -1{,}560.6 \quad Y = -930.4$

True Loads

$P_{AB} = S + X\, u_x + Y\, u_y = 750 \text{ lb} - 1{,}560.6\ (-0.600) - 930.4\ (0) = 1{,}686.4$

$P_{BC} = S + X\, u_x + Y\, u_y = 0 \text{ lb} - 1{,}560.6\ (0) - 930.4\ (-0.600) = 558.2$

$P_{CD} = S + X\, u_x + Y\, u_y = -1{,}000 \text{ lb} - 1{,}560.6\ (0) - 930.4\ (-0.800) = -255.7$

$P_{BD} = S + X\, u_x + Y\, u_y = 1{,}250 \text{ lb} - 1{,}560.6\ (0) - 930.4\ (1) = 319.6$

$P_{CE} = S + X\, u_x + Y\, u_y = 0 \text{ lb} - 1{,}560.6\ (0) - 930.4\ (1) = -930.4$

$P_{BE} = S + X\, u_x + Y\, u_y = -3{,}000 \text{ lb} - 1{,}560.6\ (-0.800) - 930.4\ (-0.800) = -1{,}007.2$

$P_{ED} = S + X\, u_x + Y\, u_y = -750 \text{ lb} - 1{,}560.6\ (0) - 930.4\ (-0.600) = -191.8$

$P_{BF} = S + X\, u_x + Y\, u_y = 0 \text{ lb} - 1{,}560.6\ (1) - 930.4\ (0) = -1{,}560.8$

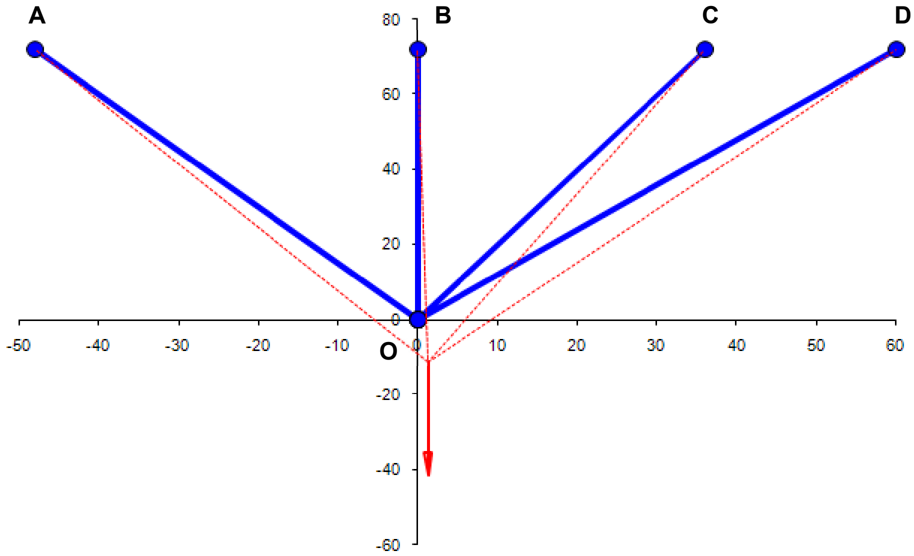
$P_{AE} = S + X\, u_x + Y\, u_y = -3{,}750 \text{ lb} - 1{,}560.6\ (1) - 930.4\ (0) = 2.189.4$

$P_{EF} = S + X\, u_x + Y\, u_y = -3{,}000 \text{ lb} - 1{,}560.6\ (-0.600) - 930.4\ (0) = -2{,}063.6$

$P_{AF} = S + X\, u_x + Y\, u_y = 0 \text{ lb} - 1{,}560.6\ (-1) - 930.4\ (0) = 1{,}248.5$

Page A8.11 Dummy Unit Load Method – Truss with Double Redundancy

Elmer F. Bruhn *Analysis and Design of Flight Vehicles Structures* page A8.11



Solution

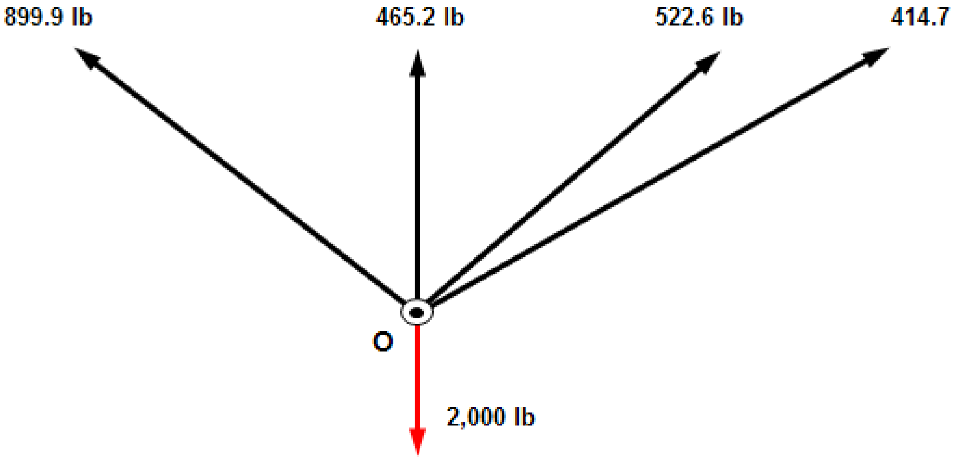
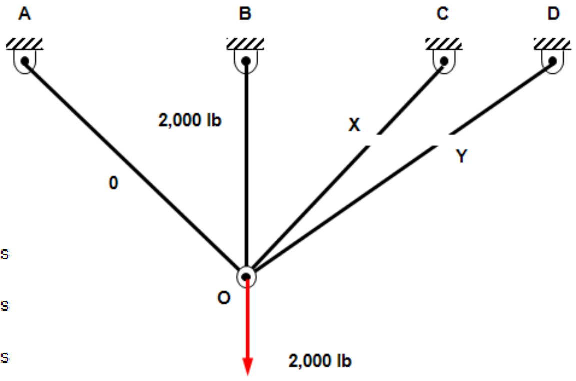


Table A8.3

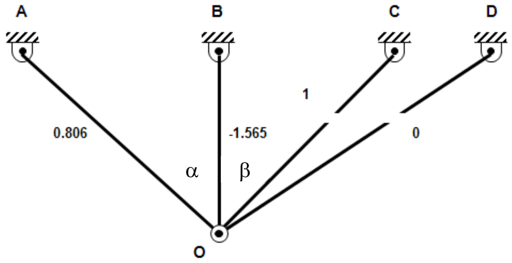
Member	Length, L (inch)	Area, A (in ^{2})	Load, S (lb)	Unit Load, u_x	Unit Load, u_y	$S\ u_x\ L / A$	$S\ u_y\ L / A$	$u_x^2\ L / A$	$u_y^2\ L / A$	$u_x\ u_y\ L/A$	True Load (lb)
AO	86.533	0.200	0	0.8062	1.1541	0	0	281.23	576.30	402.58	**899.9**
BO	72.000	0.100	2,000	-1.5652	-1.7285	-2,253,957	-2,489,037	1,764.00	2,151.15	1,947.98	**465.2**
CO	80.498	0.200	0	1	0	0	0	402.49	0	0	**522.6**
DO	93.723	0.300	0	0	1	0	0	0	312.41	0	**414.7**
					Σ	-2,253,957	-2,489,037	2,447.73	3,039.85	**2,350.56**	

S Loads

α	33.69	degrees
β	26.57	degrees
γ	39.81	degrees



Unit Loads u_x



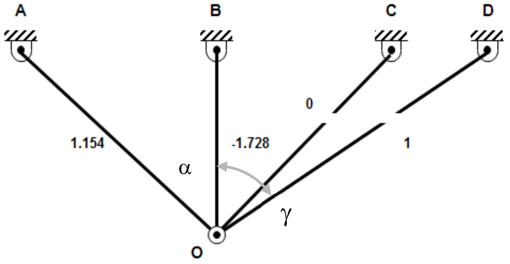
$\Sigma F_x = 0$ $\quad - AO \sin\alpha + CO \sin\beta + DO \sin\gamma = 0$

$-AO \sin(33.69°) + \mathbf{1} \sin(25.57°) + 0 = 0$ $\quad AO = 0.8062$

$\Sigma F_y = 0$ $\quad AO \cos\alpha + BO + CO \cos\beta + DO \cos\gamma = 0$

$0.8062 \cos(33.69°) + BO + \mathbf{1} \cos(25.57°) + 0 = 0$ $\quad BO = -1.5652$

Unit Loads u_y



$\Sigma F_x = 0$ $\quad - AO \sin\alpha + CO \sin\beta + DO \sin\gamma = 0$

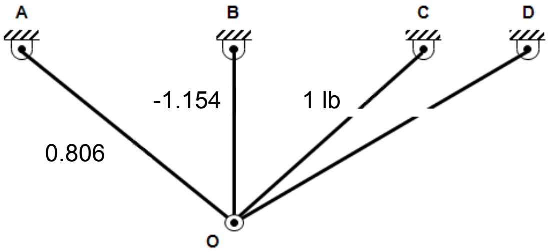
$-AO \sin(33.69°) + 0 + \mathbf{1} \sin(39.81°) = 0$ $\quad AO = 1.1541$

$\Sigma F_y = 0$ $\quad AO \cos\alpha + BO + CO \cos\beta + DO \cos\gamma = 0$

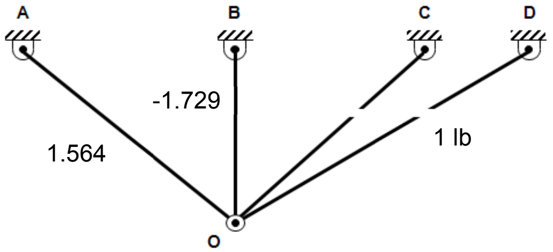
$1.1541 \cos(33.69°) + BO + 0 + \mathbf{1} \cos(39.81°) = 0$ $\quad BO = -1.7285$

Unit Loads

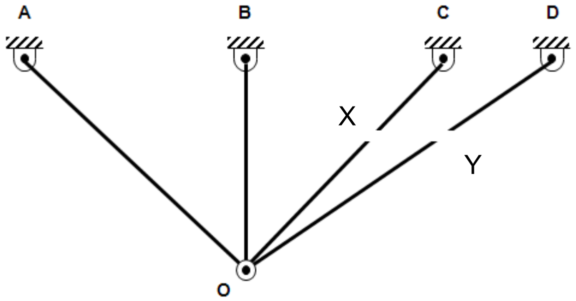
Member 3



Member 4



Two Equations, Two Unknowns



$$X \sum \frac{u_x^{\,2}\ L}{A} + Y \sum \frac{u_x\ u_y\ L}{A} = -\sum \frac{S\ u_x\ L}{A}$$

$$2{,}444.8\ X + 2{,}349.4\ Y = 2{,}252{,}160$$

$$X \sum \frac{u_x\ u_y\ L}{A} + Y \sum \frac{u_y^{\,2}\ L}{A} = -\sum \frac{S\ u_y\ L}{A}$$

$$2{,}349.4\ X + 3{,}041.0\ Y = 2{,}439{,}760$$

Matrix Form

$$\begin{bmatrix} 2{,}444.8 & 2{,}349.4 \\ 2{,}349.4 & 3{,}041.0 \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{Bmatrix} 2{,}252{,}160 \\ 2{,}439{,}760 \end{Bmatrix}$$

$$\begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{bmatrix} 0.00159 & -0.00123 \\ -0.00123 & 0.00128 \end{bmatrix} \begin{Bmatrix} 2{,}252{,}160 \\ 2{,}439{,}760 \end{Bmatrix} = \begin{Bmatrix} 521.9 \\ 415.5 \end{Bmatrix}$$

$X = 521.9$ $\quad$ $Y = 415.5$

True Loads

$P_{AO} = S + X\ u_x + Y\ u_y = 0$ lb $+ 521.9\ (0.806) + 415.5\ (1.154) = 899.9$

$P_{BO} = S + X\ u_x + Y\ u_y = 2{,}000$ lb $+ 521.9\ (-1.564) + 415.5\ (-1.729) = 465.2$

$P_{CO} = S + X\ u_x + Y\ u_y = 0$ lb $+ 521.9\ (1.00) + 415.5\ (0) = 522.6$

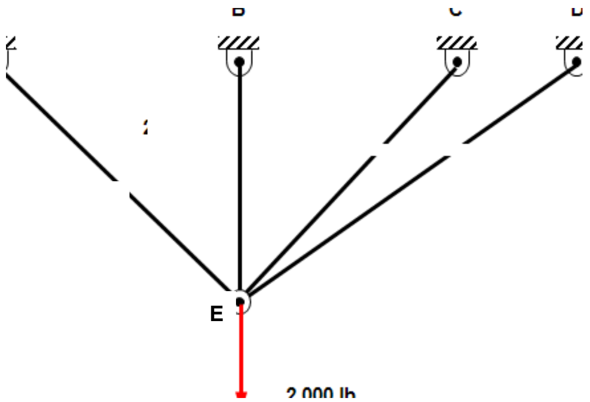
$P_{DO} = S + X\ u_x + Y\ u_y = 0$ lb $+ 521.9\ (0) + 415.5\ (1.00) = 414.7$

Degree of Redundancy

Degree of Redundancy = 2 Two Equations Four Members = Four Unknowns

Flexibility Coefficients

Elmer F. Bruhn *Analysis and Design of Flight Vehicles Structures* pages A8.31 and A8.32



Member	Length, L (in)	Area, A (in^2)	True Load S + X u (lb)
ae	**86.53**	**0.20**	**899.95**
be	**72.00**	**0.10**	**465.19**
ce	**80.50**	**0.20**	**522.59**
de	**93.72**	**0.30**	**414.71**

$$[\alpha_{ij}] = 1 / E \begin{bmatrix} 432.67 & 0 & 0 & 0 \\ 0 & 720 & 0 & 0 \\ 0 & 0 & 402.49 & 0 \\ 0 & 0 & 0 & 312.41 \end{bmatrix}$$

$$[g_{ir}] = \begin{bmatrix} 0.8062 & 1.1541 \\ -1.5652 & -1.7285 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[g_{ri}] = \begin{bmatrix} 0.8062 & -1.5652 & 1 & 0 \\ 1.1541 & -1.7285 & 0 & 1 \end{bmatrix}$$

$$[\alpha_{ij}]\{g_{ir}\} = 1 / E \left\{ \begin{matrix} 348.83 & -1,126.98 & 402.49 & 0 \\ 499.34 & -1,244.52 & 0 & 312.41 \end{matrix} \right\}$$

$$\begin{array}{cccccc} & \textbf{First} & \textbf{Second} & \textbf{Third} & \textbf{\%} & \textbf{Problem 3} \\ & \text{(Cut)} & \text{(Guess)} & \text{(True)} & \text{(True)} & \text{(page A8.11)} \\ \{g_{im}\} = & \left\{\begin{matrix} 0 \\ 1 \\ 0 \\ 0 \end{matrix}\right\} & \left\{\begin{matrix} 0.400 \\ 0.258 \\ 0.400 \\ 0.0672 \end{matrix}\right\} & \left\{\begin{matrix} 0.4500 \\ 0.2326 \\ 0.2613 \\ \mathbf{0.2074} \end{matrix}\right\} & \left\{\begin{matrix} 0.391 \\ 0.202 \\ 0.227 \\ 0.180 \end{matrix}\right\} & \left\{\begin{matrix} 899.95 \\ 465.19 \\ 522.59 \\ 414.71 \end{matrix}\right\} \text{lb} \\ & & & & & 2,302.44 \text{ lb} \\ [\alpha_{rn}] = 1 / E & \left\{\begin{matrix} -1,127.0 \\ -1,244.5 \end{matrix}\right\} & \left\{\begin{matrix} 9.77 \\ -100.35 \end{matrix}\right\} & \left\{\begin{matrix} 0.00 \\ 0.00 \end{matrix}\right\} & \left\{\begin{matrix} 0.00 \\ 0.00 \end{matrix}\right\} & \end{array}$$

Page A8.15 Example Problem 9

Column 2, Figure A8.24

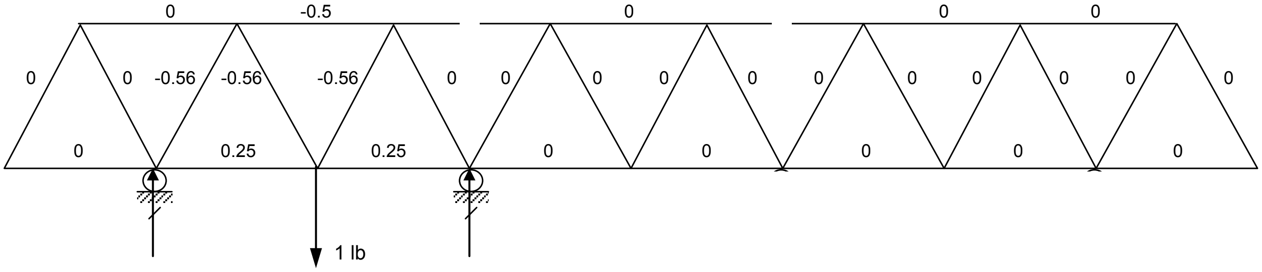
Add "L = 15 inches"

Thanks to Dr. Howard W. Smith.

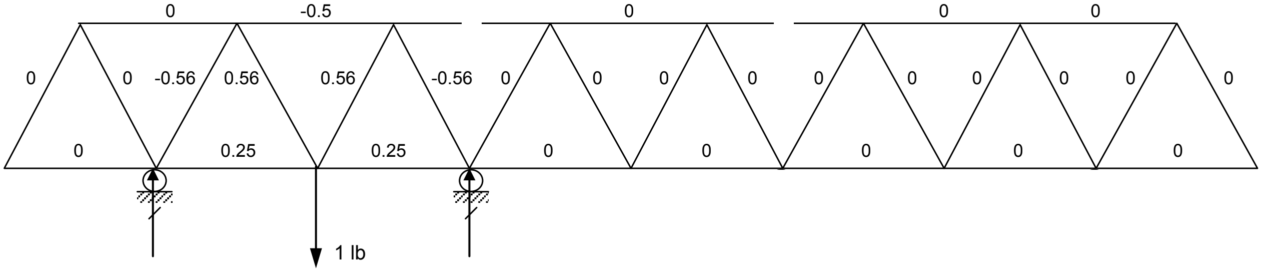
Page A8.21 Continuous Truss

Column 2

Loading for column { g_{i2} } of the matrix [g_{im}]



should be



Member Flexibility Coefficient

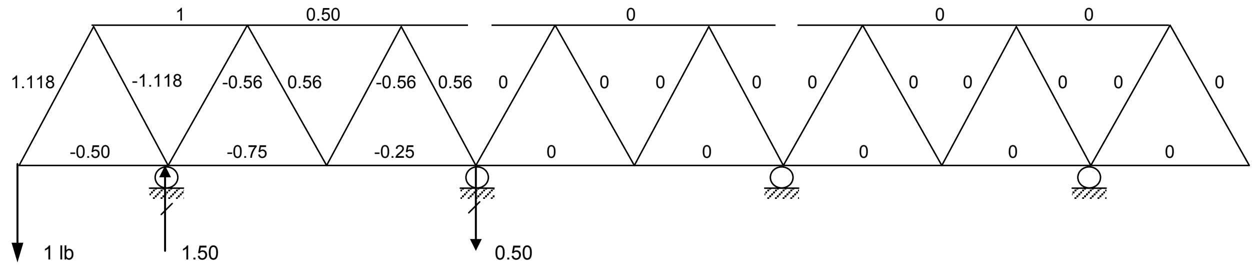
$L_{1\text{-}7}$ / $A_{1\text{-}7}$ = 20 / 1.00 = 20 $L_{8\text{-}15}$ / $A_{8\text{-}15}$ = 20 / 0.50 = 40 $L_{16\text{-}31}$ / $A_{16\text{-}31}$ = 22.361 / 1.00 = 22.361

α_i = 1 / E

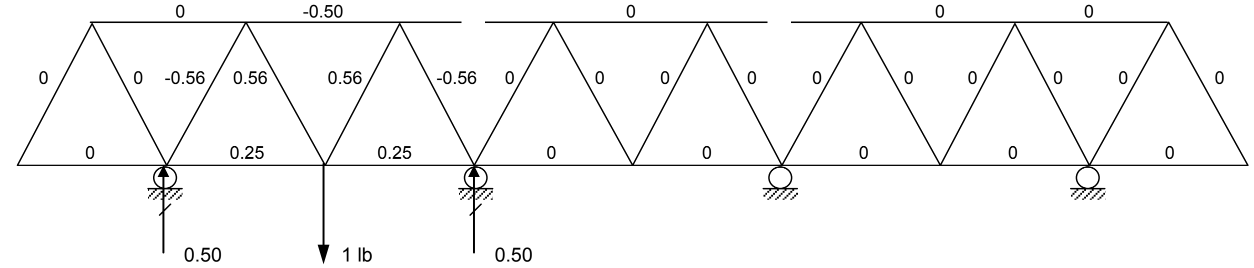
$$\left\{\left(\begin{array}{ccccccccccccccccccccccccccccccc}
20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36
\end{array}\right)\right\}$$

Matrix [g $_{im}$]

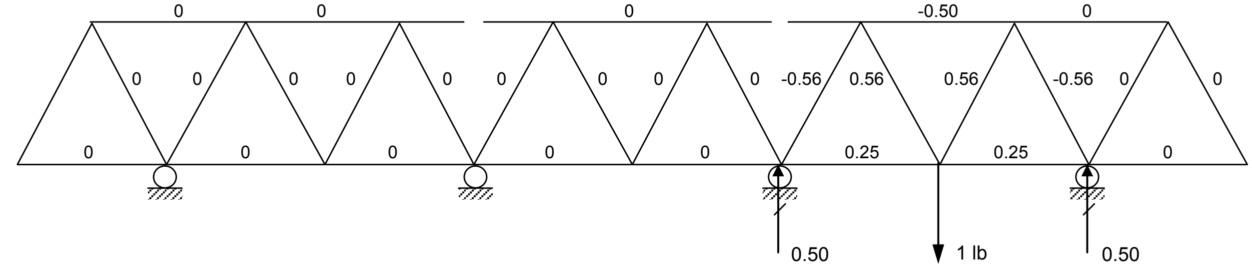
[g $_{i1}$]



[g $_{i2}$]



[g $_{i3}$]



[g $_{i4}$]

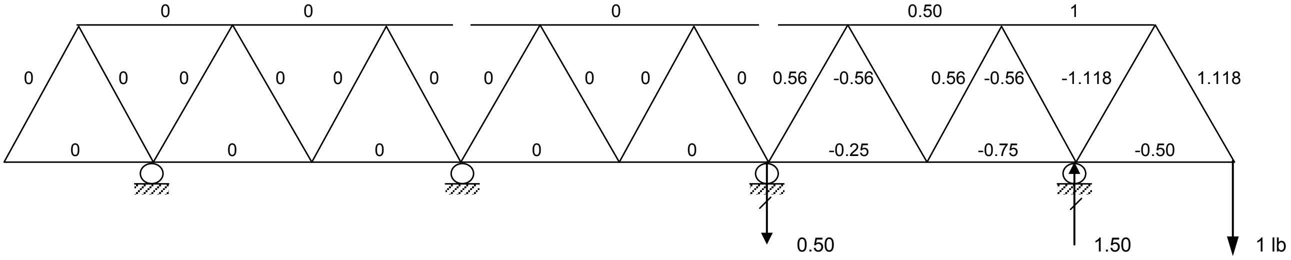


Figure A8.30a

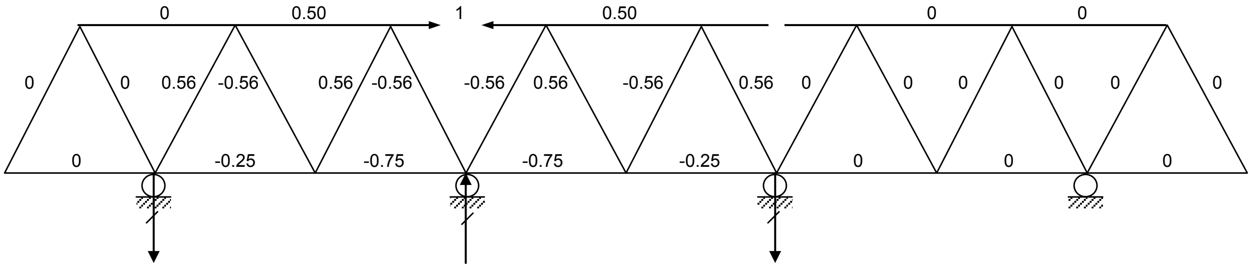
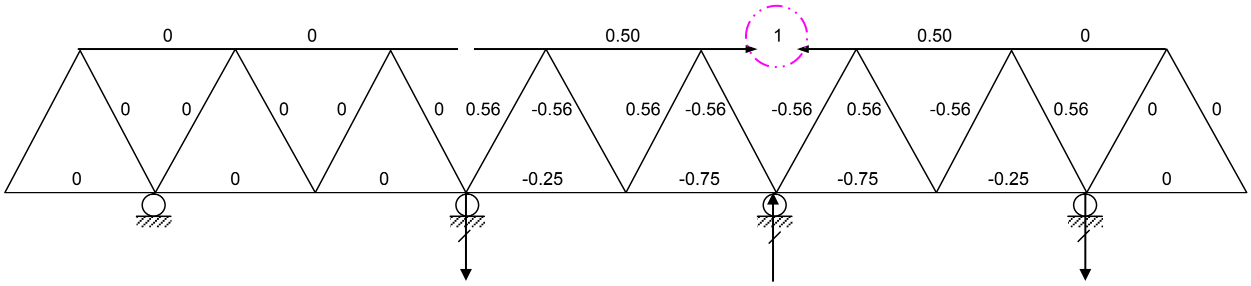


Figure A8.30b

Missing the “1#” label



Calculations

Bruhn, page A8.22

$$[\alpha_{rs}] = [g_{ri}]\ [\alpha_{ij}]\ [g_{js}] = \frac{1}{E}\begin{bmatrix}135.9 & -7.951\\ -7.951 & 135.9\end{bmatrix}$$

$$[\alpha_{rn}] = [g_{ri}]\ [\alpha_{ij}]\ [g_{jn}] = \frac{1}{E}\begin{bmatrix}-7.951 & -15 & 0 & 0\\ 0 & 0 & -15 & -7.951\end{bmatrix}$$

$$[\alpha_{rs}]^{-1} = E\begin{bmatrix}0.00738 & 0.00043\\ 0.00043 & 0.00738\end{bmatrix} = \frac{E}{18{,}432}\begin{bmatrix}136.1 & 7.962\\ 7.962 & 136.1\end{bmatrix}$$

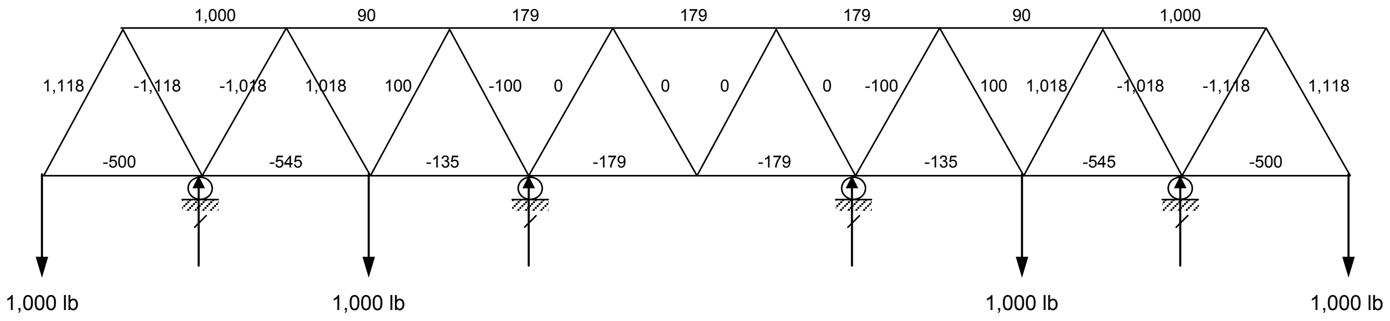
$$[\alpha_{rs}]^{-1}\ [\alpha_{rn}] = E\begin{bmatrix}0.0074 & 0.0004\\ 0.0004 & 0.0074\end{bmatrix} \times \frac{1}{E}\begin{bmatrix}-7.951 & -15 & 0 & 0\\ 0 & 0 & -15 & -7.951\end{bmatrix} = \begin{bmatrix}-0.0587 & -0.1108 & -0.0065 & -0.0034\\ -0.0034 & -0.0065 & -0.1108 & -0.0587\end{bmatrix}$$

$$-\ [\alpha_{rs}]^{-1}\ [\alpha_{rn}] = \begin{bmatrix}0.0587 & 0.1108 & 0.0065 & 0.0034\\ 0.0034 & 0.0065 & 0.1108 & 0.0587\end{bmatrix}$$

$$\begin{array}{cccccc} & [g_{im}] & & -\,[\alpha_{rs}]^{-1}\,[\alpha_{rn}]\,[g_{ir}] & & \\ [G_{im}] = & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.50 & -0.50 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.50 & 0.50 \\ 0 & 0 & 0 & 1 \\ -0.5 & 0 & 0 & 0 \\ -0.75 & 0.25 & 0 & 0 \\ -0.25 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & -0.25 \\ 0 & 0 & 0.25 & -0.75 \\ 0 & 0 & 0 & -0.50 \\ 1.118 & 0 & 0 & 0 \\ -1.118 & 0 & 0 & 0 \\ -0.559 & -0.559 & 0 & 0 \\ 0.559 & 0.559 & 0 & 0 \\ -0.559 & 0.559 & 0 & 0 \\ 0.559 & -0.559 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.559 & 0.559 \\ 0 & 0 & 0.559 & -0.559 \\ 0 & 0 & 0.559 & 0.559 \\ 0 & 0 & -0.559 & -0.559 \\ 0 & 0 & 0 & -1.118 \\ 0 & 0 & 0 & 1.118 \end{bmatrix} & + & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.029 & 0.055 & 0.003 & 0.002 \\ 0.059 & 0.111 & 0.006 & 0.003 \\ 0.031 & 0.059 & 0.059 & 0.031 \\ 0.003 & 0.006 & 0.111 & 0.059 \\ 0.002 & 0.003 & 0.055 & 0.029 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.015 & -0.028 & -0.002 & -0.001 \\ -0.044 & -0.083 & -0.005 & -0.003 \\ -0.045 & -0.085 & -0.033 & -0.017 \\ -0.017 & -0.033 & -0.085 & -0.045 \\ -0.003 & -0.005 & -0.083 & -0.044 \\ -0.001 & -0.002 & -0.028 & -0.015 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.033 & 0.062 & 0.004 & 0.002 \\ -0.033 & -0.062 & -0.004 & -0.002 \\ 0.033 & 0.062 & 0.004 & 0.002 \\ -0.033 & -0.062 & -0.004 & -0.002 \\ -0.031 & -0.058 & 0.058 & 0.031 \\ 0.031 & 0.058 & -0.058 & -0.031 \\ -0.031 & -0.058 & 0.058 & 0.031 \\ 0.031 & 0.058 & -0.058 & -0.031 \\ -0.002 & -0.004 & -0.062 & -0.033 \\ 0.002 & 0.004 & 0.062 & 0.033 \\ -0.002 & -0.004 & -0.062 & -0.033 \\ 0.002 & 0.004 & 0.062 & 0.033 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & = & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.529 & -0.445 & 0.003 & 0.002 \\ 0.059 & 0.111 & 0.006 & 0.003 \\ 0.031 & 0.059 & 0.059 & 0.031 \\ 0.003 & 0.006 & 0.111 & 0.059 \\ 0.002 & 0.003 & -0.445 & 0.529 \\ 0 & 0 & 0 & 1 \\ -0.50 & 0 & 0 & 0 \\ -0.765 & 0.222 & -0.002 & 0 \\ -0.294 & 0.167 & -0.005 & -0.003 \\ -0.045 & -0.085 & -0.033 & -0.017 \\ -0.017 & -0.033 & -0.085 & -0.045 \\ -0.003 & -0.005 & 0.167 & -0.294 \\ 0 & -0.002 & 0.222 & -0.765 \\ 0 & 0 & 0 & -0.50 \\ 1.118 & 0 & 0 & 0 \\ -1.118 & 0 & 0 & 0 \\ -0.526 & -0.497 & 0.004 & 0.002 \\ 0.526 & 0.497 & -0.004 & -0.002 \\ -0.526 & 0.621 & 0.004 & 0.002 \\ 0.526 & -0.621 & -0.004 & -0.002 \\ -0.031 & -0.058 & 0.058 & 0.031 \\ 0.031 & 0.058 & -0.058 & -0.031 \\ -0.031 & -0.058 & 0.058 & 0.031 \\ 0.031 & 0.058 & -0.058 & -0.031 \\ -0.002 & -0.004 & -0.621 & 0.526 \\ 0.002 & 0.004 & 0.621 & -0.526 \\ -0.002 & -0.004 & 0.497 & 0.526 \\ 0.002 & 0.004 & -0.497 & -0.526 \\ 0 & 0 & 0 & -1.118 \\ 0 & 0 & 0 & 1.118 \end{bmatrix} \end{array}$$

Example

For $P_1 = P_2 = P_3 = P_4 = 1{,}000$ lb



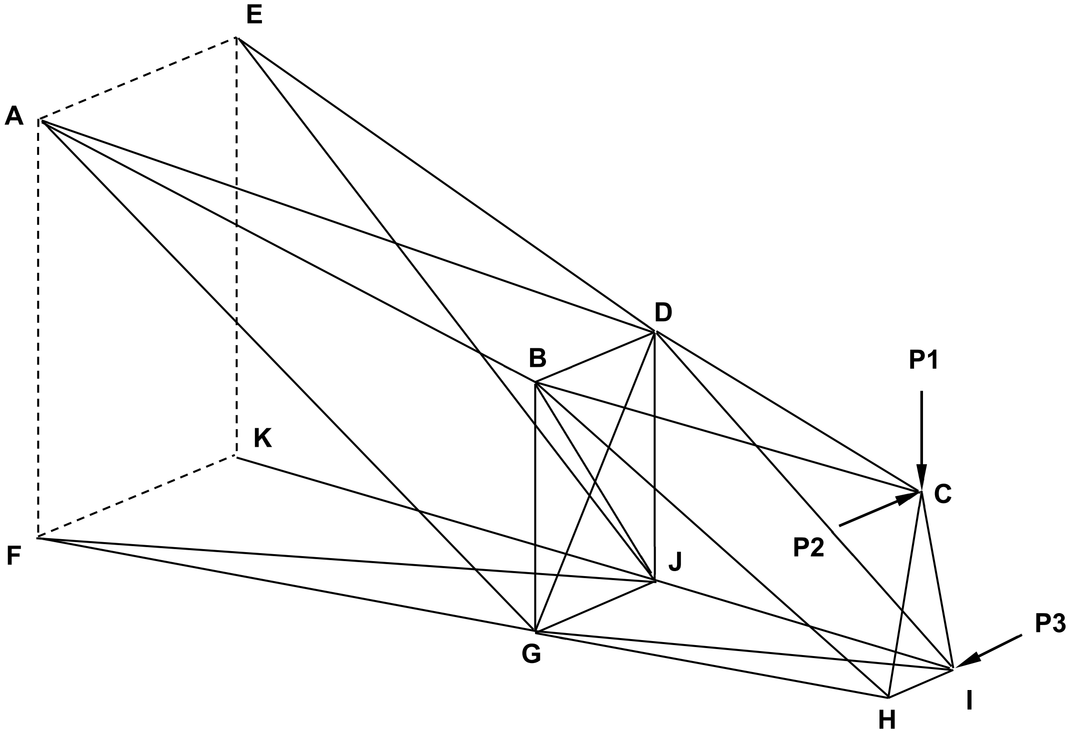
Member Forces for Unit Applied External Loads

Elmer F. Bruhn, *Analysis and Design of Flight Vehicle Structures*, page A8.22.

Check the signs on the member forces with a white background:

$$
[\,G_{i\,m}\,] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0.529 & -0.445 & 0.003 & 0.002 \\
0.059 & 0.111 & 0.006 & 0.003 \\
0.031 & 0.059 & 0.059 & 0.031 \\
0.003 & 0.006 & 0.111 & 0.059 \\
0.002 & 0.003 & -0.445 & 0.529 \\
0 & 0 & 0 & 1 \\
-0.500 & 0 & 0 & 0 \\
-0.765 & 0.222 & -0.002 & 0 \\
-0.294 & 0.167 & -0.005 & -0.003 \\
-0.045 & -0.085 & -0.033 & -0.017 \\
-0.017 & -0.033 & -0.085 & -0.045 \\
-0.003 & -0.005 & 0.167 & -0.294 \\
0 & -0.002 & 0.222 & -0.765 \\
0 & 0 & 0 & -0.500 \\
1.118 & 0 & 0 & 0 \\
-1.118 & 0 & 0 & 0 \\
-0.526 & -0.497 & 0.004 & 0.002 \\
0.526 & 0.497 & -0.004 & -0.002 \\
-0.526 & 0.621 & 0.004 & 0.002 \\
0.526 & -0.621 & -0.004 & -0.002 \\
-0.031 & -0.058 & 0.058 & 0.031 \\
0.031 & 0.058 & -0.058 & -0.031 \\
-0.031 & -0.058 & 0.058 & 0.031 \\
0.031 & 0.058 & -0.058 & -0.031 \\
-0.002 & -0.004 & -0.621 & 0.526 \\
0.002 & 0.004 & 0.621 & -0.526 \\
-0.002 & -0.004 & 0.497 & 0.526 \\
0.002 & 0.004 & -0.497 & -0.526 \\
0 & 0 & 0 & -1.118 \\
0 & 0 & 0 & 1.118
\end{bmatrix}
$$

Page A8.23 Tubular Tail Fuselage Truss



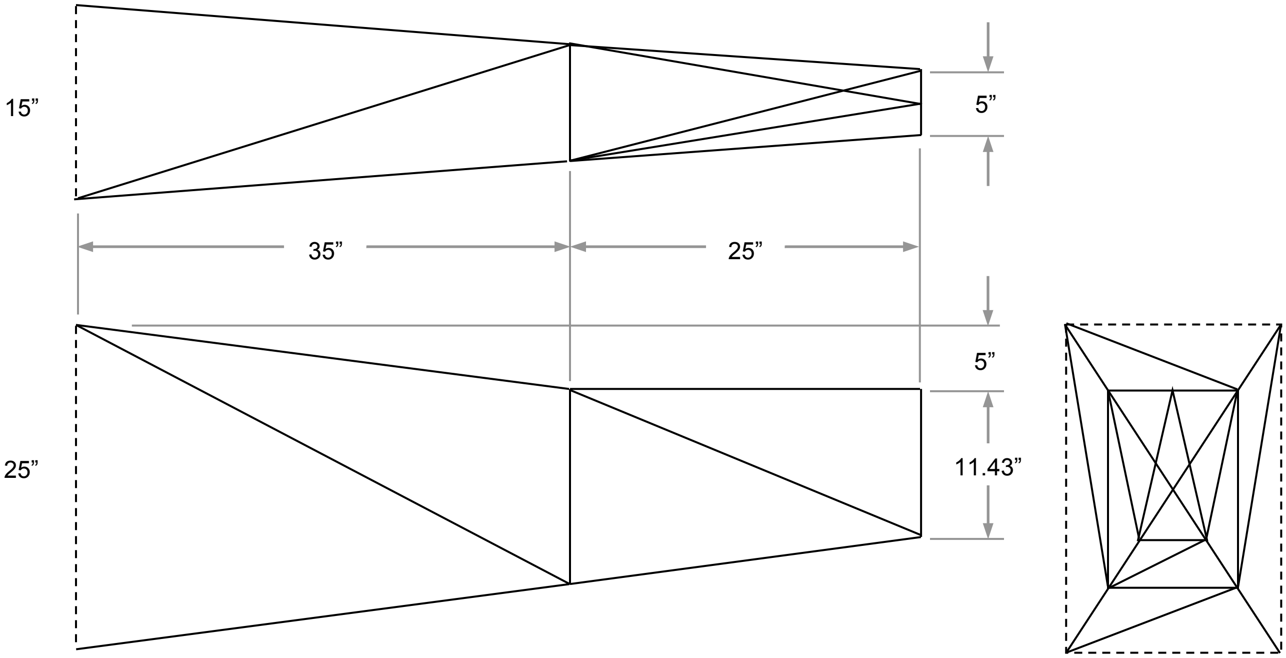


Figure A8.31

Member	Number	Length L (in)	Area A (in^2)	L / A (in^{-1})
AB	1	35.474	0.565	62.79
BC	2	25.420	0.499	50.94
BD	3	9.200	0.165	55.76
CD	4	25.420	0.499	50.94
DE	5	35.474	0.565	62.79
FG	6	35.474	0.565	62.79
GH	7	25.341	0.565	44.85
HI	8	5.000	0.165	30.30
IJ	9	25.341	0.565	44.85
GJ	10	9.200	0.165	55.76
JK	11	35.474	0.565	62.79
AG	12	40.415	0.630	64.15
BG	13	15.000	0.165	90.91
BH	14	27.569	0.500	55.14
HC	15	11.700	0.395	29.62
IC	16	11.700	0.395	29.62
DI	17	27.569	0.500	55.14
DJ	18	15.000	0.165	90.91
EJ	19	40.415	0.630	64.15
AD	20	37.369	0.565	66.14
FJ	21	37.369	0.565	66.14
GI	22	26.233	0.500	52.47
DG	23	17.597	0.165	106.65
BJ	24	17.597	0.165	106.65

Joint Coordinates

Joint	x	y	z
A	0	25.00	0
B	2.90	20.00	35.00
C	7.50	20.00	60.00
D	12.10	20.00	35.00
E	15.00	25.00	0
F	**0**	**0**	**0**
G	2.90	5.00	35.00
H	5.00	8.57	60.00
I	10.00	8.57	60.00
J	12.10	5.00	35.00
K	15.00	0	0

Statics

Page A8.23, Column 2

Joint C

	x	y	z
C	7.5	20	60
B	2.9	20	35
BC	i	j	k
# 2	-4.6	0	-25
	25.41968 in		
	α	β	γ
cos	**-0.180962**	0	**-0.98349**
	100.426°	90°	169.574°

	x	y	z
C	7.5	20	60
D	12.1	20	35
CD	i	j	k
# 4	4.6	0	-25
	25.41968 in		
	α	β	γ
cos	**0.180962**	0	**-0.9834901**
	79.57419°	90°	169.5742°

	x	y	z
C	7.5	20	60
H	5	8.57	60
HC	i	j	k
# 15	-2.5	-11.43	0
	11.70021 in		
	α	β	γ
cos	**-0.213671**	**-0.976906**	0
	102.338°	167.662°	90°

	x	y	z
C	7.5	20	60
I	10	8.57	60
IC	i	j	k
# 16	2.5	-11.43	0
	11.70021 in		
	α	β	γ
cos	**0.213671**	**-0.976906**	0
	77.66241°	167.6624°	90°

$\Sigma F_x = 0$ $\quad 0.21367\ q_{16} - 0.21367\ q_{15} + 0.180962\ q_4 - 0.180962\ q_2 + P_2 = 0$

$$\mathbf{0.21367\ q_{16} - 0.21367\ q_{15} + 0.18096\ q_4 - 0.18096\ q_2 = -P_2}$$

$\Sigma F_y = 0$ $\quad -0.97691\ q_{15} - 0.97691\ q_{16} + P_1 = 0$

$$0.97691\ q_{15} + 0.97691\ q_{16} = -P_1$$

$$\mathbf{q_{15} + q_{16} = -1.02364\ P_1}$$

$\Sigma F_z = 0$ $\quad -0.98349\ q_2 - 0.98349\ q_4 = 0$

$$\mathbf{q_2 + q_4 = 0}$$

Joint B

	x	y	z		x	y	z
B	2.9	20	35	B	2.9	20	35
A	0	25	0	C	7.5	20	60

AB	i	j	k	BC	i	j	k
# 1	-2.9	5	-35	# 2	4.6	0	25
	35.47408 in				25.41968 in		

	α	β	γ		α	β	γ
cos	-0.08175	0.140948	-0.986636	cos	0.180962	0	0.9834901
	94.6892°	81.8973°	170.622°		79.57419°	90°	10.42581°

	x	y	z		x	y	z
B	2.9	20	35	B	2.9	20	35
D	12.1	20	35	G	2.9	5	35

BD	i	j	k	BG	i	j	k
# 3	9.2	0	0	# 13	0	-15	0
	9.2 in				15 in		

	α	β	γ		α	β	γ
cos	1	0	0	cos	0	-1	0
	0°	90°	90°		90°	180°	90°

	x	y	z		x	y	z
B	2.9	20	35	B	2.9	20	35
H	5	8.57	60	J	12.1	5	35

BH	i	j	k	BJ	i	j	k
# 14	2.1	-11.43	25	# 24	9.2	-15	0
	27.56909 in				17.59659 in		

	α	β	γ		α	β	γ
cos	0.076172	-0.414595	0.906813	cos	0.522829	-0.852438	0
	85.6314°	114.494°	24.9315°		58.47782°	148.4778°	90°

$\Sigma F_x = 0$ $\qquad -0.08175\, q_1 + 0.18096\, q_2 + q_3 + 0\, q_{13} + 0.07617\, q_{14} + 0.52283\, q_{24} = 0$

$$\mathbf{0.08175\, q_1 - 0.18096\, q_2 - q_3 - 0.07617\, q_{14} = 0.52283\, q_{24}}$$

$\Sigma F_y = 0$ $\qquad 0.14095\, q_1 + 0\, q_2 + 0\, q_3 - q_{13} - 0.41460\, q_{14} - 0.85244\, q_{24} = 0$

$$\mathbf{-\, 0.14095\, q_1 + q_{13} + 0.41460\, q_{14} = -\, 0.85244\, q_{24}}$$

$\Sigma F_z = 0$ $\qquad -0.98664\, q_1 + 0.98349\, q_2 + 0\, q_3 + 0\, q_{13} + 0.90681\, q_{14} + 0\, q_{24} = 0$

$$0.98664\, q_1 - 0.98349\, q_2 - 0.90681\, q_{14} = 0$$

$$\mathbf{q_1 - 0.99681\, q_2 - 0.91910\, q_{14} = 0}$$ and so on …

Joints

			q 2	q 4	q 15	q 16			P_1	P_2	P_3
C	1	ΣFx = 0	-0.18096	0.18096	-0.21367	0.21367				-1	
		ΣFy = 0	0	0	-0.97691	-0.97691			1		
	2	ΣFy = 0	0	0	1	1			-1.02364		
	3	ΣFz = 0	-0.98349	-0.98349	0	0					
			q 7	q 8	q 14	q 15					
H	4	ΣFx = 0	-0.08287	1	-0.07617	0.21367					
	5	ΣFy = 0	-0.14088	0	0.41459	0.97691					
	6	ΣFz = 0	-0.98655	0	-0.9068126	0					
			q 8	q 9	q 16	q 17	q 22		P_1	P_2	P_3
I	7	ΣFx = 0	-1	0.08287	-0.21367	0.07617	-0.27065				-1
	8	ΣFy = 0	0	-0.14088	0.97691	0.41459	-0.13609				
	9	ΣFz = 0	0	-0.98655	0.00000	-0.90681	-0.95301				
			q1	q2	q3	q13	q14		*q24*		
B	10	ΣFx = 0	-0.08175	0.18096	1	0	0.07617		*0.52283*		
	11	ΣFy = 0	0.14095	0	0	-1	-0.41459		*-0.85244*		
		ΣFz = 0	-0.98664	0.98349	0	0	0.90681		*0*		
	12	ΣFz = 0	1	-0.996811			-0.919095		*0*		
			q3	q4	q5	q17	q18	q20			
D	13	ΣFx = 0	-1	-0.18096	0.08175	-0.07617	0	-0.32380			
	14	ΣFy = 0	0	0	0.14095	-0.41459	-1	0.13380			
	15	ΣFz = 0	0	0.98349	-0.98664	0.90681	0	-0.93662			
			q6	q7	q10	q12	q13		q22	*q23*	
G	16	ΣFx = 0	-0.08175	0.08287	1	-0.07175	0		0.27065	*0.52283*	
	17	ΣFy = 0	-0.14095	0.14088	0	0.49486	1		0.13609	*0.85244*	
	18	ΣFz = 0	-0.98664	0.98655	0	-0.86601	0		0.95301	*0*	
			q9	q10	q11	q18	q19	q21	*q24*		
J	19	ΣFx = 0	-0.08287	-1	0.08175	0	0.071755	-0.3238015	*-0.52283*		
	20	ΣFy = 0	0.14088	0	-0.14095	1	0.49486	-0.1338023	*0.85244*		
	21	ΣFz = 0	0.98655	0	-0.98664	0	-0.866005	-0.936616	*0*		
						q22	q23	q24	*q22*	*q23*	*q24*
	22	GI	q22 = q22			1			*1*		
	23	DG	q23 = q23				1			*1*	
	24	BJ	q24 = q24					1			*1*

Matrix [C ij]

$$
\begin{bmatrix}
0 & 0.181 & 0 & 0.181 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.214 & -0.214 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.983 & 0 & -0.983 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.083 & 1 & 0 & 0 & 0 & 0 & 0 & 0.076 & 0.214 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.141 & 0 & 0 & 0 & 0 & 0 & 0 & -0.415 & 0.977 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.987 & 0 & 0 & 0 & 0 & 0 & 0 & 0.907 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.083 & 0 & 0 & 0 & 0 & 0 & 0 & -0.214 & -0.076 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.141 & 0 & 0 & 0 & 0 & 0 & 0 & 0.977 & -0.415 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.907 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.082 & 0.181 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.076 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.141 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -0.415 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0.997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.919 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0.181 & 0.082 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.076 & 0 & 0 & 0.324 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.141 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.415 & -1 & 0 & -0.134 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.983 & -0.987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.907 & 0 & 0 & 0.937 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.082 & 0.083 & 0 & 0 & 1 & 0.072 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.141 & 0.141 & 0 & 0 & 0 & -0.495 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.987 & 0.987 & 0 & 0 & 0 & 0.866 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.083 & 1 & 0.082 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.072 & 0 & 0.324 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.141 & 0 & -0.141 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -0.495 & 0 & 0.134 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.987 & 0 & -0.987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.866 & 0 & 0.937 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Inverse of [C_{ij}] Matrix

-2.754	-0.825	-0.507	-1.107	1.6892	-0.148	-1.107	-2E-16	-0.093	5E-16	0	1	-5E-16	1E-48	-7E-17	0	0	0	0	8E-17	0	0	0	0
2.763	2E-16	0.5084	-2.763	0	0.2321	-2.763	-6E-16	-0.232	-5E-16	0	0	5E-16	-1E-48	1E-16	0	0	0	0	2E-16	0	0	0	0
-0.275	-9E-04	-0.051	0.2705	0.0019	-0.023	0.2705	6E-17	0.0227	1	0	-0.082	-4E-17	1E-49	-1E-17	0	0	0	0	-2E-17	0	0	0	0
2.763	2E-16	-0.508	-2.763	0	0.2321	-2.763	-6E-16	-0.232	-5E-16	0	0	5E-16	-1E-48	1E-16	0	0	0	0	2E-16	0	0	0	0
-2.754	0.8295	0.746	-1.107	-0.005	0.0937	-1.107	-1.694	0.1488	-2.365	0	0.1933	2.3649	-7E-33	-0.818	0	0	0	0	8E-17	0	0	0	0
-0.483	1.2344	0.089	2.4475	-1.693	-0.98	2.4475	-0.834	-0.567	0	0	0	0	-1.245	-0.178	2.0758	0	0.8416	-2.076	1.2455	0.5397	0	0	0
3E-16	-0.825	-3E-17	-3.862	1.6894	1.0968	-3.862	-8E-16	-0.324	0	0	0	0	0	7E-17	0	0	0	0	3E-16	0	0	0	0
4E-17	-0.107	-4E-18	0.5	0	-0.042	-0.5	-1E-16	-0.042	0	0	0	0	0	9E-18	0	0	0	0	3E-17	0	0	0	0
3E-16	0.8252	-3E-17	-3.862	0	0.3244	-3.862	-1.689	-1.097	0	0	0	0	0	7E-17	0	0	0	0	3E-16	0	0	0	0
-3E-19	0.0009	4E-20	0.0044	-0.002	-0.001	0.0044	8E-19	0.0004	0	0	0	0	0	-8E-20	1	0	-0.083	0	-3E-19	0	0	0	0
0.5508	-0.466	-0.101	1.611	0.0045	-0.133	1.611	0.9502	1.0158	0	0	0	0	1.419	0.2027	-2.365	0	0.196	2.3649	-1.419	-0.615	0	0	0
-0.341	0.2885	0.0627	-0.996	-0.003	0.0824	-0.996	-0.588	-0.628	0	0	0	0	-0.878	-0.125	1.4629	-1	0.0216	-1.463	0.8777	0.3804	0	0	0
0.3882	-0.256	0.0714	-1.586	0.5239	0.0584	-1.586	-3E-16	-0.133	0	-1	-0.141	0	0	3E-17	0	0	0	0	1E-16	0	0	0	0
-3E-16	0.8977	4E-17	4.2015	-1.838	-0.09	4.2015	8E-16	0.3529	0	0	0	0	0	-7E-17	0	0	0	0	-3E-16	0	0	0	0
-2E-16	0.5	2E-17	2.34	0	-0.197	2.34	5E-16	0.1966	0	0	0	0	0	-4E-17	0	0	0	0	-2E-16	0	0	0	0
2E-16	0.5	-2E-17	-2.34	0	0.1966	-2.34	-5E-16	-0.197	0	0	0	0	0	4E-17	0	0	0	0	2E-16	0	0	0	0
3E-16	0.8977	-4E-17	-4.202	0	0.3529	-4.202	-1.838	-0.09	0	0	0	0	0	7E-17	0	0	0	0	3E-16	0	0	0	0
-0.388	-0.256	0.0714	1.5858	0	-0.133	1.5858	0.5239	0.0584	0	0	0	0	-1	-0.143	0	0	0	0	-1E-16	0	0	0	0
0.6276	0.4138	-0.115	-2.564	0	0.2154	-2.564	-0.847	-0.095	0	0	0	0	1.6166	0.2309	0	0	0	0	-1.617	0.2309	0	0	0
-1E-16	0.0046	0.252	1E-16	-0.005	0.0007	1E-16	-0.005	0.0007	-2.491	0	0.2037	2.4912	-8E-33	0.2064	0	0	0	0	0	0	0	0	0
-5E-34	-0.005	0	2E-17	0.0047	0.0022	2E-17	0.0047	0.0022	0	0	0	0	0	0	-2.491	0	0.2064	2.4912	0	0.2064	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Unit Stress Distribution– Determinate Structure [g_{im} ¦ g_{ir}]

[P1 P2 P3 ¦ q22 q23 q24]

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x

P1	P2	P3	q22	q23	q24
0	**-1**	0	0	0	0
1.024	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	**-1**	**0.271**	0	0
0	0	0	**0.136**	0	0
0	0	0	**0.953**	0	0
0	0	0	0	0	**0.523**
0	0	0	0	0	**-0.852**
0	0	0	0	0	**0**
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	**0.271**	**-0.523**	0
0	0	0	**0.136**	**-0.852**	0
0	0	0	**0.953**	**0**	0
0	0	0	0	0	**0.523**
0	0	0	0	0	**-0.852**
0	0	0	0	0	**0**
0	0	0	**1**	0	0
0	0	0	0	**1**	0
0	0	0	0	0	**1**

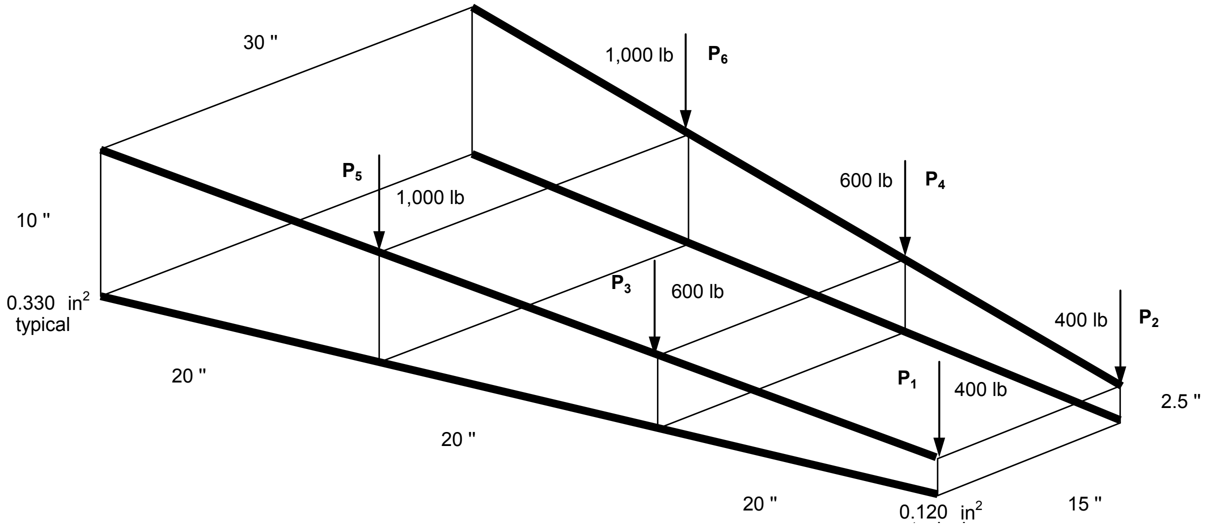
=

External Load			Redundant Load		
-0.845	2.75	1.11	-0.39	0	0
0	-2.76	2.76	-0.97	0	0
-0.001	0.27	-0.27	0.09	0	0.523
0	-2.76	2.76	-0.97	0	0
0.849	2.75	1.11	-0.39	0	-1.236
1.264	0.48	-2.45	1.37	-1.085	-2.147
-0.845	-3E-16	3.862	-1.354	0	-2E-16
-0.109	-4E-17	0.5	-0.175	0	-3E-17
0.845	-3E-16	3.862	-2.320	0	-2E-16
0.001	3E-19	-0.004	0.193	-0.523	3E-19
-0.477	-0.551	-1.611	1.080	1.236	2.446
0.295	0.341	0.996	-0.668	0.088	-1.513
-0.262	-0.388	1.586	-0.556	0	0.852
0.919	3E-16	-4.202	1.474	0	3E-16
0.512	2E-16	-2.340	0.821	0	1E-16
0.512	-2E-16	2.340	-0.821	0	-1E-16
0.919	-3E-16	4.202	-1.474	0	-3E-16
-0.262	0.388	-1.586	0.556	0	9E-17
0.424	-0.628	2.564	-0.899	0	1.378
0	0.000	0.000	0.000	0	-1.302
0	5E-34	0.000	-0.475	1.302	1.302
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

and so on …

Page A8.25 Idealized Box Beam

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures*



Flexibility Coefficients

For E = 10 E6 psi G = 3.85 E6 psi

Panel	h_1	h_2	Length L	Area S	Thickness t	α_{ii} S / G t	α_{ii} $(h_1/h_2)^2 a_{ii}$	Upr & Lwr Skin $2\alpha_{jj}$
	(in)	(in)	(in)	(in^2)	(in)			
Bay 1 Skin	25	30	20	550 in^2	0.032	44,643 / E	31,002 / E	**62,004** / E
Bay 2 Skin	20	25	20	450 in^2	0.025	46,753 / E	29,922 / E	**59,844** / E
Bay 3 Skin	15	20	20	350 in^2	0.020	45,455 / E	25,568 / E	**51,136** / E
Bay 1 Spar Web	7.5	10.0	20	175 in^2	0.065	6,993 / E	**3,934** / E	
Bay 2 Spar Web	5.0	7.5	20	125 in^2	0.050	6,494 / E	**2,886** / E	
Bay 3 Spar Web	2.5	5.0	20	75 in^2	0.040	4,870 / E	**1,218** / E	

Matrix

α_{ij} = 1 / E

$$\begin{bmatrix}
1{,}218 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 51{,}136 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1{,}218 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{134.5} & 0 & 0 & 0 & 0 & \mathbf{24.64} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{134.5} & 0 & 0 & 0 & 0 & \mathbf{24.64} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2{,}886 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 59{,}844 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}886 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{24.64} & 0 & 0 & 0 & 0 & \mathbf{80.7} & 0 & 0 & 0 & 0 & \mathbf{14.69} & 0 \\
0 & 0 & 0 & 0 & \mathbf{24.64} & 0 & 0 & 0 & 0 & \mathbf{80.7} & 0 & 0 & 0 & 0 & \mathbf{14.69} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3{,}934 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 62{,}004 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3{,}934 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{14.69} & 0 & 0 & 0 & 0 & 27.62 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{14.69} & 0 & 0 & 0 & 0 & 27.62
\end{bmatrix}$$

Tapered Bar

$L = 60$ inches $\quad A_i = 0.120\ \text{in}^2 \quad A_j = 0.330\ \text{in}^2 \quad A_i / A_j = 0.364$

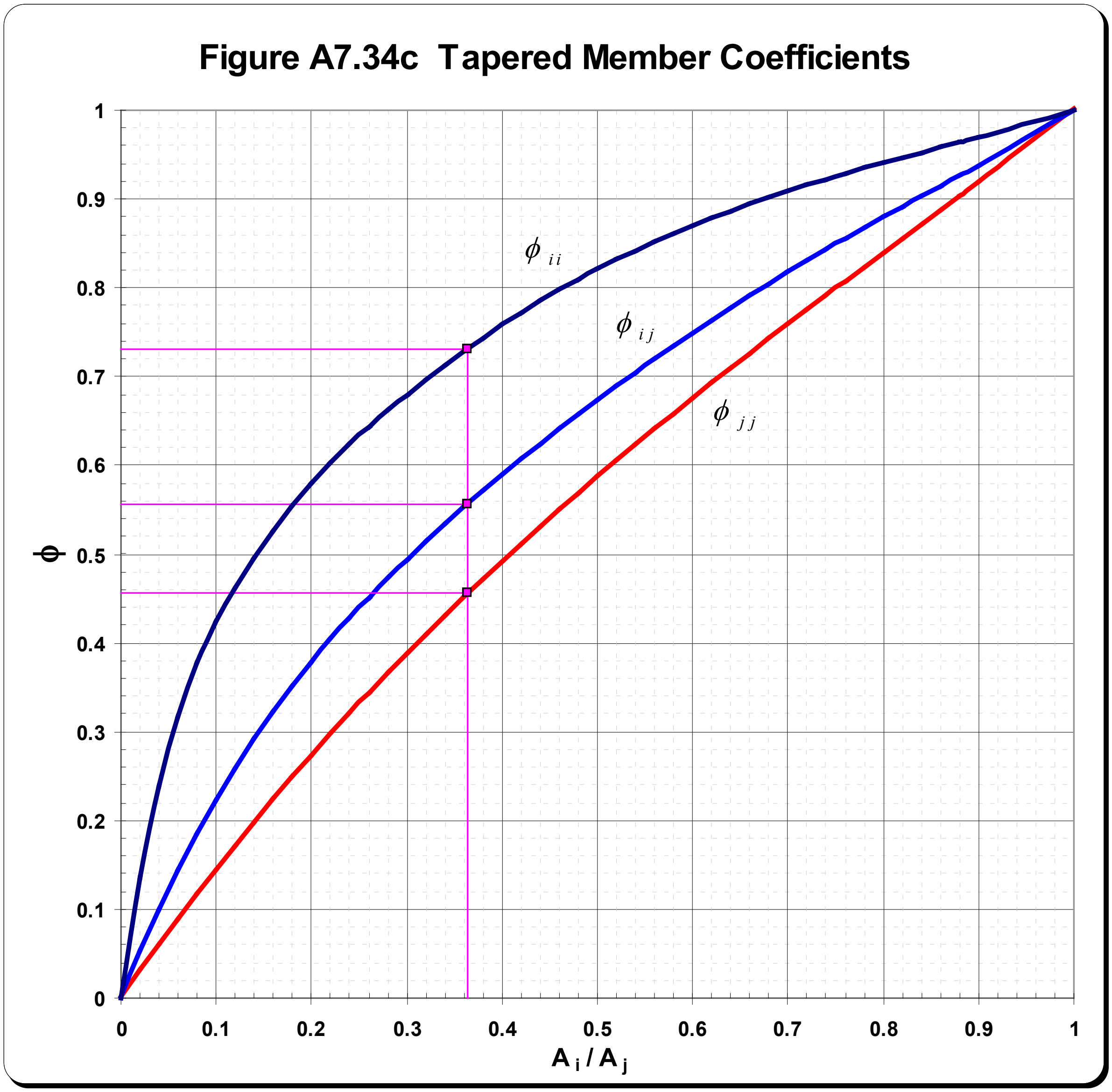
$\phi_{ii} = 0.7310 \quad \phi_{ij} = 0.5569 \quad \phi_{jj} = 0.4558$

$\alpha_{ii} = [\ L\ /\ (\ 3\ A_i\ E\ )\ ]\ \phi_{ii} = 121.84\ /\ E$

$\alpha_{ij} = [\ L\ /\ (\ 6\ A_i\ E\ )\ ]\ \phi_{ij} = 46.41\ /\ E$

$\alpha_{jj} = [\ L\ /\ (\ 3\ A_j\ E\ )\ ]\ \phi_{jj} = 27.62\ /\ E \qquad \boldsymbol{\alpha_{jj} = 27.62\ /\ E}$

See Bruhn page A7.23



My Results (Bruhn, page A8.26)

$$[\alpha_{rs}] = [g_{ri}]\ [\alpha_{ij}]\ [g_{js}] \qquad [g_{js}] = [g_{ir}]$$

$$[\alpha_{rs}] = 1{,}000{,}000\ /\ E \begin{bmatrix} 0.3873 & 0.1935 & 0.0513 \\ 0.1935 & 0.2895 & 0.0722 \\ 0.0513 & 0.0722 & 0.1254 \end{bmatrix}$$

$$[\alpha_{rn}] = [g_{ri}]\ [\alpha_{ij}]\ [g_{jn}] \qquad [g_{jn}] = [g_{im}]$$

$$[\alpha_{rn}] = 1{,}000{,}000\ /\ E \begin{bmatrix} -0.0348 & 0.0349 & -0.0031 & 0.0100 & -0.0016 & 0.0016 \\ -0.0251 & 0.0248 & -0.0052 & 0.0126 & -0.0022 & 0.0022 \\ -0.0081 & 0.0081 & -0.0039 & 0.0048 & -0.0023 & 0.0023 \end{bmatrix}$$

$$[\alpha_{rs}]^{-1} = E\ /\ 1{,}000{,}000 \begin{bmatrix} 3.8781 & -2.5649 & -0.1097 \\ -2.5649 & 5.7284 & -2.2472 \\ -0.1097 & -2.2472 & 9.3106 \end{bmatrix}$$

$$[G_{sn}] = -[\alpha_{rs}]^{-1}\ [\alpha_{rn}]$$

$$[G_{sn}] = \begin{bmatrix} 0.0697 & -0.0708 & -0.0017 & -0.0060 & 0.0002 & -0.00017 \\ 0.0364 & -0.0345 & 0.0130 & -0.0358 & 0.0036 & -0.0036 \\ 0.0153 & -0.0159 & 0.0240 & -0.0148 & 0.0160 & -0.0160 \end{bmatrix}$$

True Stresses

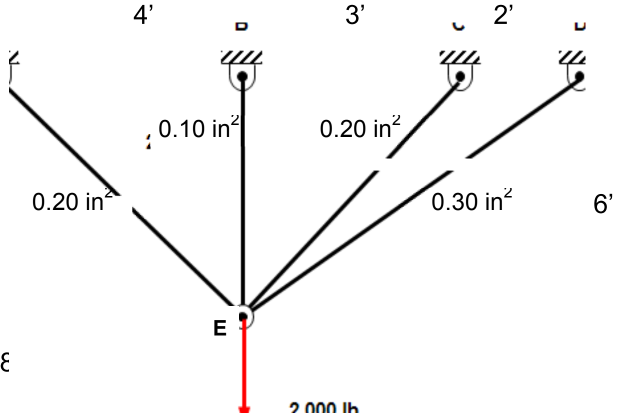
$$[G_{im}] = [g_{im}] - [g_{ir}]\ [G_{rm}] \qquad [G_{rm}] = [G_{sn}]$$

$$[G_{im}] = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0.3303 & 0.0708 & 0.0017 & 0.0060 & -0.0002 & 0.0002 \\ 2 & 0.0697 & -0.0708 & -0.0017 & -0.0060 & 0.0002 & -0.0002 \\ 3 & 0.0697 & 0.3292 & -0.0017 & -0.0060 & 0.0002 & -0.0002 \\ 4 & 2.260 & 1.773 & 0.0433 & 0.1497 & -0.0044 & 0.0044 \\ 5 & 1.746 & 2.233 & -0.0433 & -0.1497 & 0.0044 & -0.0044 \\ 6 & 0.0854 & 0.0124 & 0.1864 & 0.0340 & -0.0035 & 0.0035 \\ 7 & 0.0364 & -0.0345 & 0.0130 & -0.0358 & 0.0036 & -0.0036 \\ 8 & 0.0146 & -0.0124 & 0.0136 & 0.1660 & 0.0035 & -0.0035 \\ 9 & 2.817 & 2.492 & -0.346 & 1.178 & -0.109 & 0.109 \\ 10 & 2.524 & 2.849 & 0.346 & 1.493 & 0.109 & -0.109 \\ 11 & 0.0459 & -0.0006 & 0.0672 & 0.0070 & 0.1180 & 0.0153 \\ 12 & 0.0153 & -0.0159 & 0.0240 & -0.0148 & 0.0160 & -0.0160 \\ 13 & -0.0015 & 0.0451 & 0.0217 & -0.0070 & 0.0153 & 0.1180 \\ 14 & 3.473 & 2.749 & 2.934 & 1.530 & 1.397 & 0.606 \\ 15 & 2.758 & 3.482 & 1.072 & 2.476 & 0.606 & 1.397 \\ \hline \end{array}$$

Page A8.32 Influence Coefficient Matrix – Redundant Truss

Influence Coefficients

$$\left[\alpha_{ij}\right] = \frac{1}{E}\begin{bmatrix} 432 & 0 & 0 & 0 \\ 0 & 720 & 0 & 0 \\ 0 & 0 & 402 & 0 \\ 0 & 0 & 0 & 312 \end{bmatrix}$$



Using my values for L / A from Problem 3 on page A8

$$\left[\alpha_{ij}\right] = \begin{bmatrix} 432.67 & 0 & 0 & 0 \\ 0 & 720 & 0 & 0 \\ 0 & 0 & 402.49 & 0 \\ 0 & 0 & 0 & 312.41 \end{bmatrix}$$

Member	Length, L (in)	Area, A (in^2)	True Load S + X u (lb)
ae	86.53	0.20	900.16
be	72.00	0.10	465.31
ce	80.50	0.20	521.91
de	93.72	0.30	415.51

Unit Load Distribution

$$\left[g_{ir}\right] = \begin{bmatrix} 0.8062 & 1.1541 \\ -1.5652 & -1.7285 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Transpose

$$\left[g_{ri}\right] = \begin{bmatrix} 0.8062 & -1.5652 & 1 & 0 \\ 1.1541 & -1.7285 & 0 & 1 \end{bmatrix}$$

Multiply

$$\left[\alpha_{ij}\right]\left\{g_{ri}\right\} = 1 / E \left\{\begin{matrix} 348.83 & -1{,}126.98 & 402.49 & 0 \\ 499.34 & -1{,}244.52 & 0 & 312.41 \end{matrix}\right\}$$

Determinate Stress Distribution

$[g_{41}] = 0.280$ *should be* $[g_{41}] = 0.208$ or 0.207 depending on significant digits of $[g_{ir}]$ … etc.

$$[g_{im}]_{TRUE} = \begin{Bmatrix} 0.450 \\ 0.232 \\ 0.260 \\ 0.280 \end{Bmatrix} \quad \textit{should be} \quad [g_{im}]_{TRUE} = \begin{Bmatrix} 0.450 \\ 0.233 \\ 0.261 \\ 0.207 \end{Bmatrix}$$

Dividing the forces from Example Problem 3 on page A8.11 by 2,000 yields the distribution above. Using a percentage will give you the same distribution.

Redundant Forces

	First (Cut)	**Second** (Guess)	**Third** (True)	**%** (True)	**Problem 3** (page A8.11)
$\{g_{im}\}$ =	0 1 0 0	0.400 0.258 0.400 0.0672	0.4500 0.2326 0.2613 **0.2074**	0.391 0.202 0.227 0.180	899.95 465.19 522.59 414.71 lb 2,302.44 lb
$[\alpha_{rn}]$ = 1 / E	-1,127.0 -1,244.5	9.77 -100.35	0.00 0.00	0.00 0.00	

Note:

$$[\alpha_{rn}]_{CUT} = [g_{ri}]\,[\alpha_{ij}]\,\{g_{jn}\}_{CUT} = \begin{Bmatrix} -1126 \\ -1245 \end{Bmatrix} \frac{1}{E\,I}$$

should be …

$$[\alpha_{rn}]_{CUT} = [g_{ri}]\,[\alpha_{ij}]\,\{g_{jn}\}_{CUT} = \begin{Bmatrix} -1126 \\ -1245 \end{Bmatrix} \frac{1}{E}$$

Page A9.13 Calculation of Frame Elastic Properties

Table A9.5, Column 3

w = ds *should be* $w = \dfrac{ds}{I}$

I

Thanks to Dr. Howard W. Smith.

Page A11.9 Moment Distribution Method – Fixed Beam with Support Deflections

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* page A11.9

Column 1, Figure A11.18

The “d” dimension is missing an arrow.

d and L *should be* d / 2 L / 2
2 2

Below Figure A11.18, to avoid any possible confusion

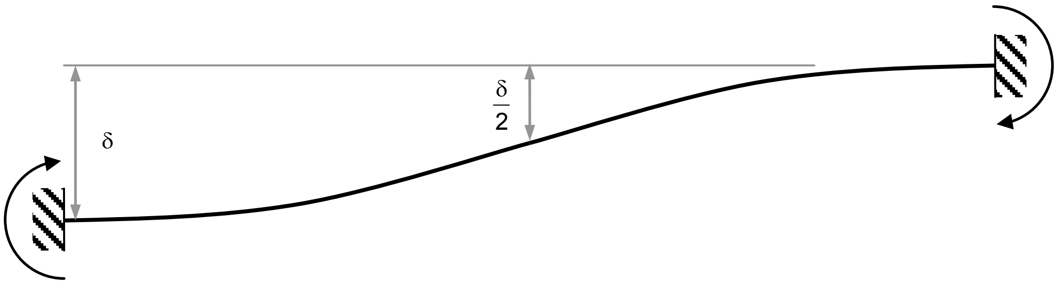
$$-\delta = \frac{ML}{4EI}\cdot\frac{L}{6}\,\frac{-ML}{4EI}\cdot\frac{5}{6}\,L$$

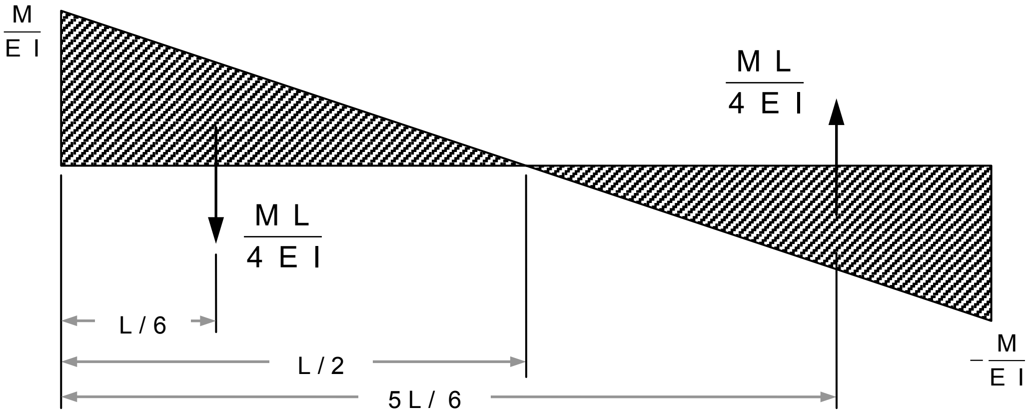
should look something like this:

$$-\delta = \left[\frac{M\,L}{4\,E\,I}\cdot\left(\frac{L}{6}\right)\right]-\left[\frac{M\,L}{4\,E\,I}\cdot\left(\frac{5}{6}\right)L\right] = -\frac{4\,M\,L^2}{24\,E\,I}$$

where the area of each moment curve $A = \frac{1}{2}\left(\frac{L}{2}\right)\frac{M}{EI} = \frac{M\,L}{4\,E\,I}$

Therefore $\delta = \frac{M\,L^2}{6\,E\,I}$





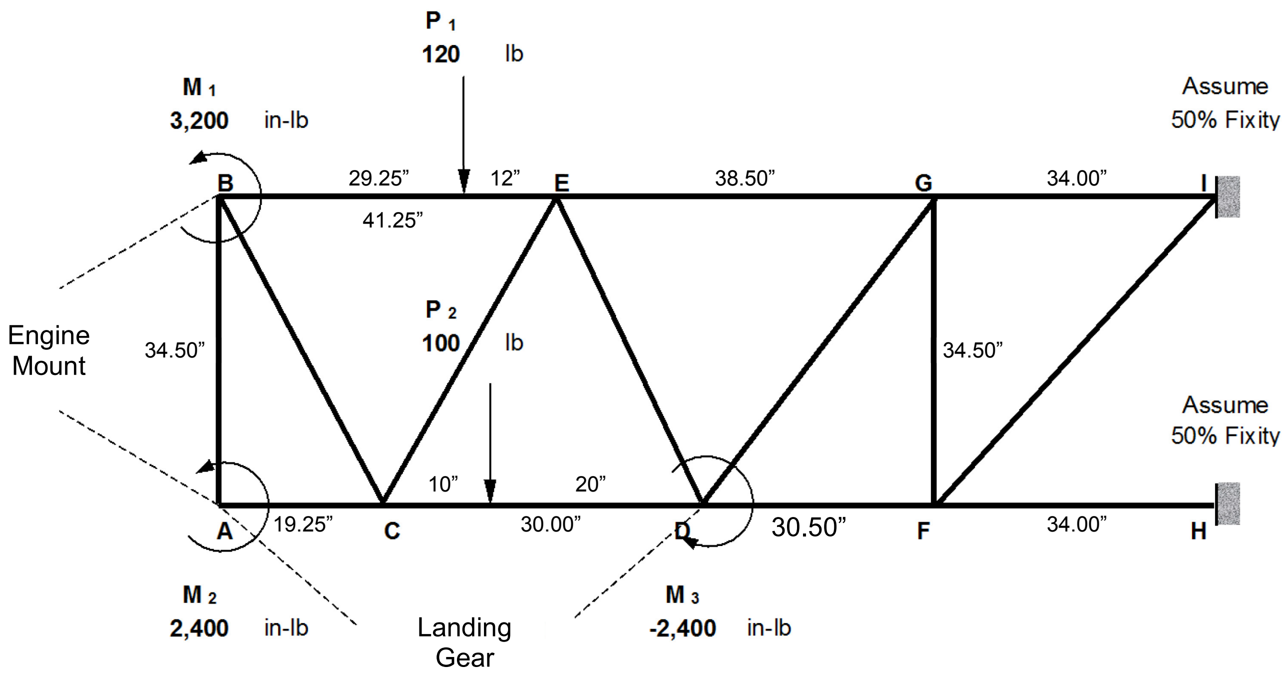
Page A11.9 Moment Distribution Method – Continuous Beam with Deflected Supports

Column 2, "The fixed-end moments for a trapezoidal loading from Table A11.4 …"

Table A11.4 *should be* Table A11.1 (page A11.3)

Page A11.13 Fuselage Side Truss

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Example Problem #8



Example Problem #8, Table A11.2 Second instance of member "CD" in Column 1 should be "CE".

Upper tubes are longer than the lower tubes yet AB and FG are both 34.50 inches long. Using DF = 30.50 inches (instead of 30 inches) and EG = 38.50 inches (instead of 38 inches) I get the following lengths:

ED = 35.42 inch instead of 35.40 inch
BC = 39.51 inch instead of 39.50 inch
CE = 40.92 inch instead of 41.00 inch

DG = 46.05 inch instead of 46.00 inch
FI = 48.44 inch instead of 48.50 inch

Moments (in-lb)

M_{ab} = -1,614.5

M_{ba} = -1,863.1

M_{bc} = -652.5

M_{cb} = -159.9

M_{cd} = -212.7

M_{dc} = 1,098.3

M_{de} = 207.3

M_{ed} = -111.7

M_{eg} = -201.2

M_{ge} = -117.4

M_{fg} = -67.0

M_{gf} = -51.3

M_{gi} = -22.1

M_{ig} = -6.3

M_{ac} = -785.3

M_{ca} = -51.3

M_{be} = -684.2

M_{eb} = 397.5

M_{ce} = -1.5

M_{ec} = -100.2

M_{df} = 641.7

M_{fd} = 206.3

M_{dg} = 452.7

M_{gd} = 192.0

M_{fi} = -47.8

M_{if} = -13.7

M_{hf} = -25.4

M_{fh} = -88.7

Page A11.15 Moment Distribution Method – Fixed Beam with Variable Inertia

Table A11.3

	Static M / I Curve			Trial M_A / I Curve			Trial M_B / I Curve		
Beam Portion	**Avg Ord.** y	**Mom. Arm** x	**Moment** y x	**Avg Ord.** y	**Mom. Arm** x	**Moment** y x	**Avg Ord.** y	**Mom. Arm** x	**Moment** y x
1	106.7	2.67	284	110	2.06	226.7	6.7	2.67	17.8
2	397.6	6.31	2,508	132.7	6.06	804.8	24.8	6.31	156.8
3	770.9	10.16	7,835	142.7	10.01	1,429.1	48.2	10.16	489.7
4	1,120.0	14.10	15,787	130.0	14.05	1,826.7	70.0	14.10	986.7
5	1,440.0	18.07	26,027	110.0	18.06	1,986.7	90.0	18.07	1,626.7
6	1,760.0	22.06	38,827	90.0	22.07	1,986.7	110.0	22.06	2,426.7
7	1,614.5	26.13	42,182	67.3	26.13	1,757.6	123.6	26.02	3,217.0
8	974.5	30.23	29,459	40.6	30.23	1,227.5	117.0	30.06	3,516.0
9	420.0	34.35	14,427	17.5	34.35	601.1	90.8	34.12	3,098.9
10	100.0	38.67	3,867	4.2	38.67	161.1	62.5	38.13	2,383.3
	8,704.2		**181,202**	**845.0**		**12,007.9**	**743.6**		**17,919.4**

It appears that the orientations of the trapezoids from the moment diagram were not considered when calculating x_{bar} for some areas. I calculate x_{bar} from the left of each area using $x_{bar} = 2/3\ (x_2-x_1)$ for the triangular portion of areas with the point on left side and $x_{bar} = 1/3\ (x_2-x_1)$ for the triangular portion of areas with the point on the right side. My guess is that $x_{bar} = 2/3\ (x_2-x_1)$ was used throughout the analysis regardless of the orientation of the trapezoid.

Static M / I Curve: The sum of the average ordinate, y is wrong in Bruhn. Adding up the values in his table yields 8,704 instead of 8,504. This gives a value for x_{bar} = 179,809 / 8,704 = 20.66 inch instead of 21.15 inch. The last four values in my column for Moment Arm, x take into account the x_{bar} discussion above. x_{bar} = 181,202 / 8,704.2 = 20.82 inch.

Trial M_A / I Curve: The last eight values in the column for Moment Arm, x take into account the x_{bar} discussion above. x_{bar} = 12,008 / 845.0 = 14.21 in

Trial M_B / I Curve: The last three values in the column for Moment Arm, x take into account the x_{bar} discussion above. x_{bar} = 17,919.4 / 743.6 = 24.10 in

Fixed End Moments My numbers yield M_A = -341.7 in-lb and M_B = 782.3 in-lb

Carry Over Factors My numbers yield COF_{BA} = 0.543 and COF_{AB} = 0.670

Constant, c c_{BA} = 0.721 and c_{AB} = 0.877

Stiffness Factors K_{BA} = 0.721 I / L and K_{AB} = 0.877 I / L

Page A11.16 Moment Distribution Method – Fixed Beam with Variable Inertia

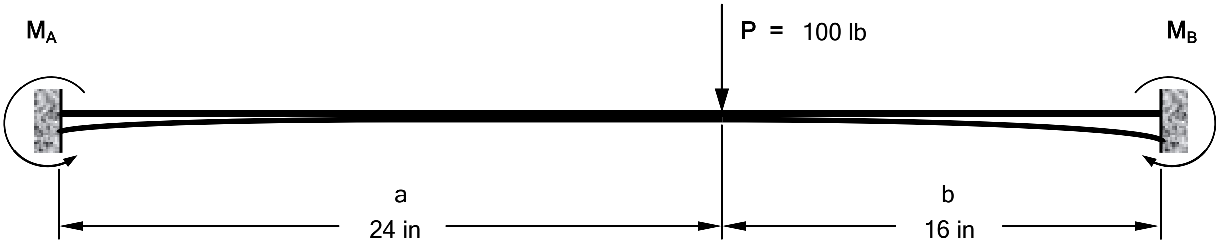


Figure A11.27

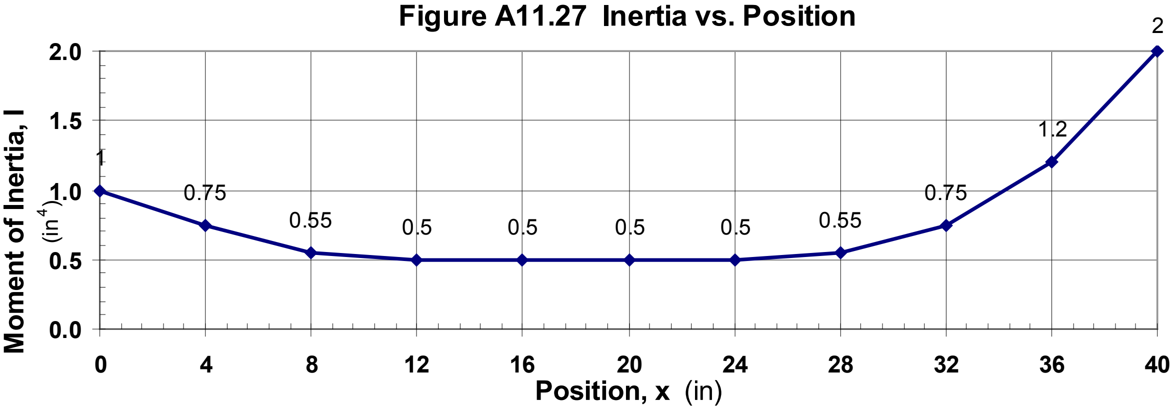


Figure A11.28

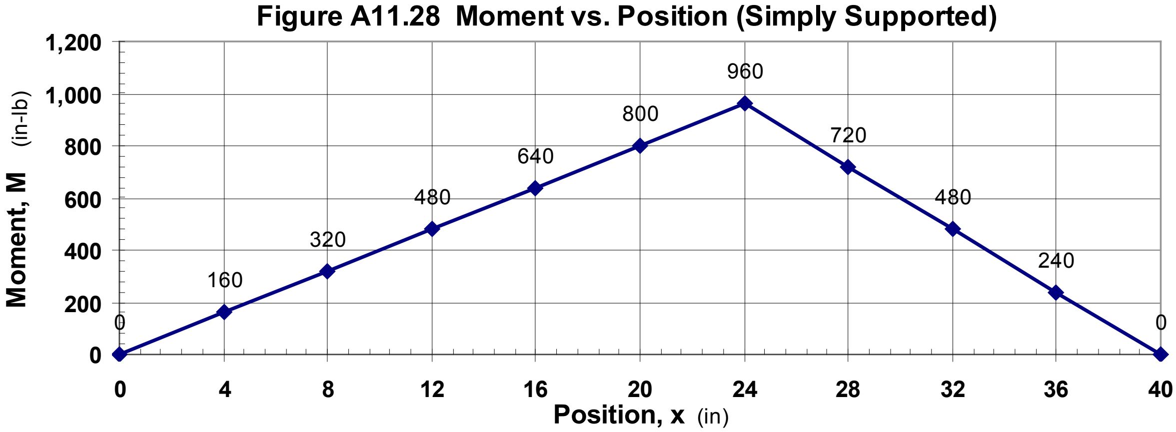
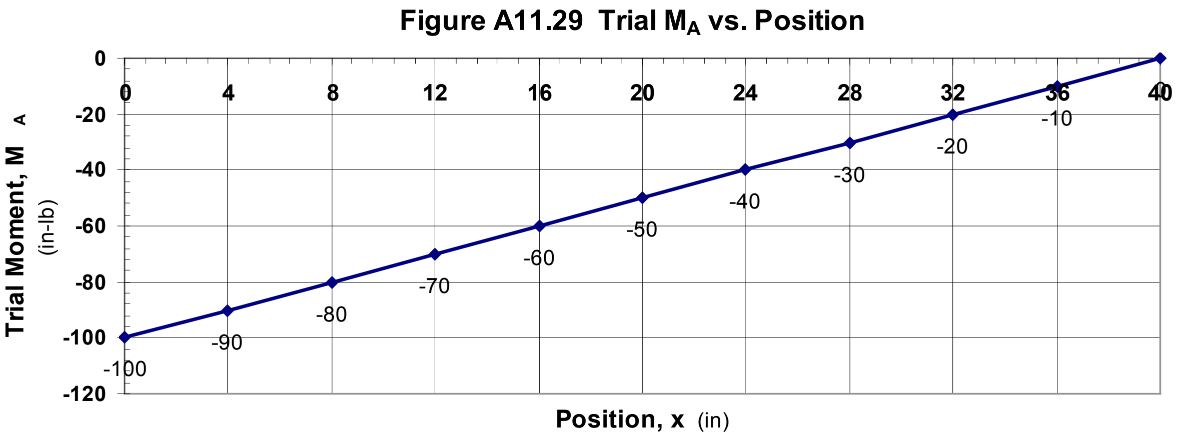


Figure A11.29



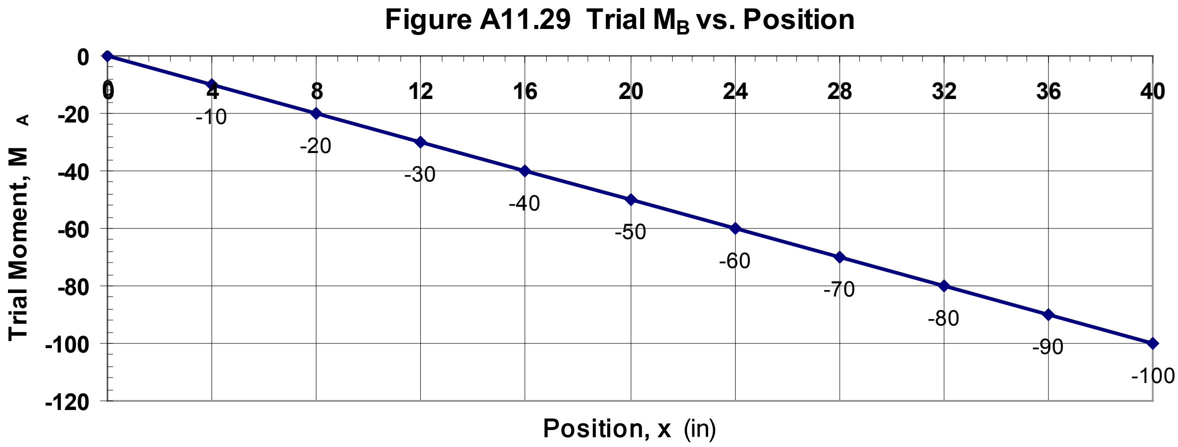


Figure A11.30a

Moment / Inertia at x = 24 inches should be 1,920 not 1,420

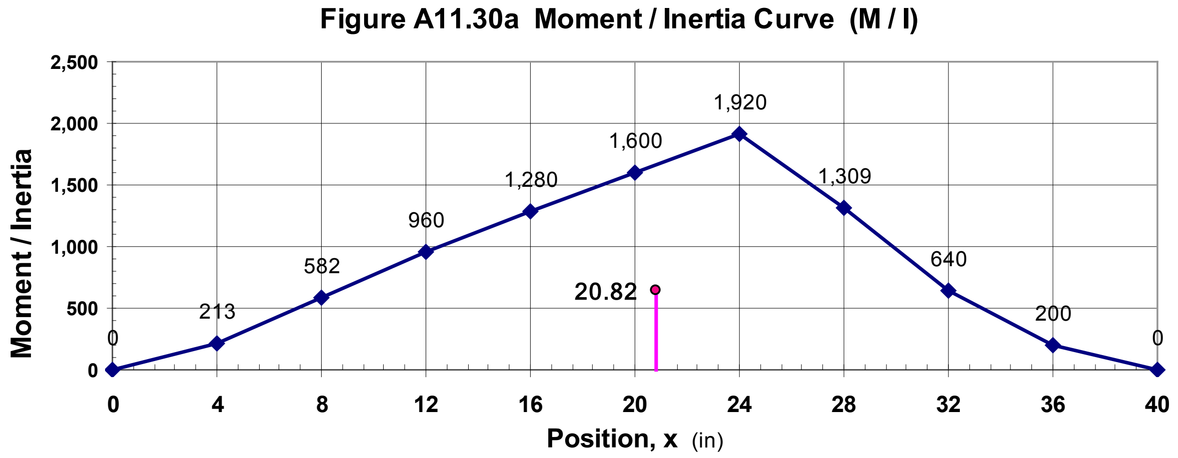


Figure A11.30b

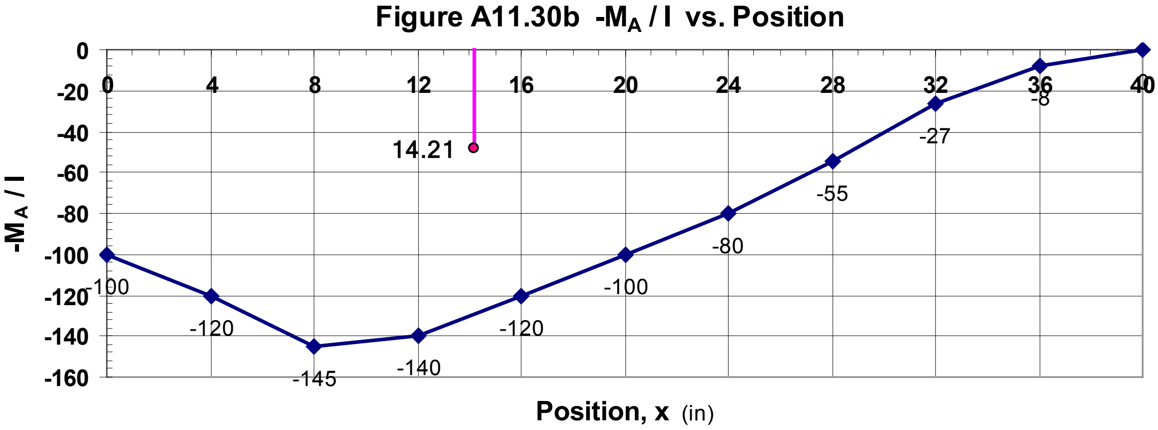
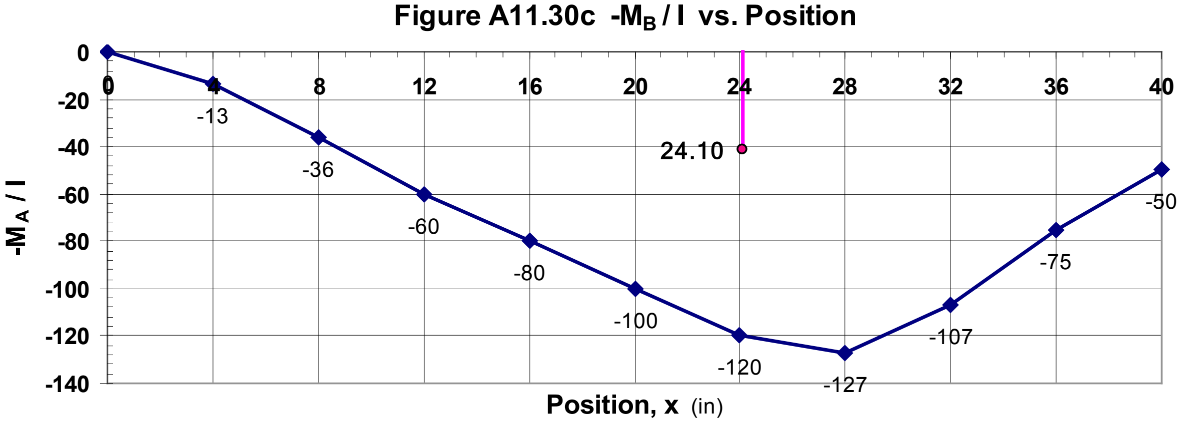
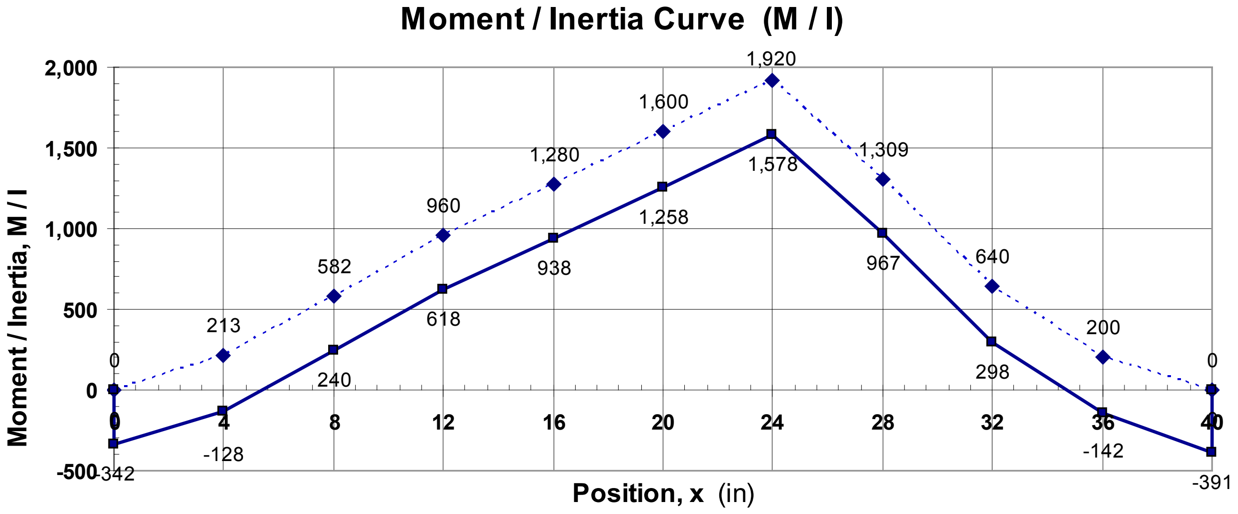


Figure A11.30c



Moment / Inertia Diagram



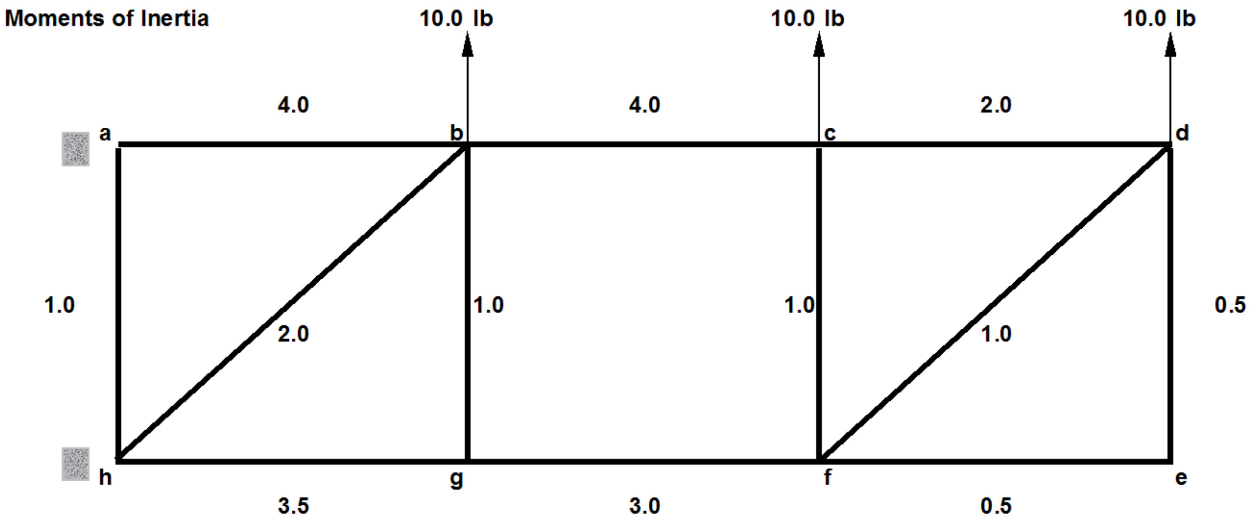
Page A11.21 Moment Distribution Method – Frame with Missing Diagonal Shear Member

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* page A11.21

Example Problem 3, Figure A11.42

Member bh is missing an inertia value. I use a value of “2” in my solution.

Moment Distribution (Hardy Cross) Method



	Joint a		Joint b		Joint c		Joint d		Joint e		Joint f		Joint g		Joint h	
Prop.	**ab**	**ba**	**bc**	**cb**	**cd**	**dc**	**de**	**ed**	**ef**	**fe**	**fg**	**gf**	**gh**	**hg**	**ah**	**ha**
I	4.0	4.0	4.0	4.0	2.0	2.0	0.5	0.5	0.5	0.5	3.0	3.0	3.5	3.5	1.0	1.0
L	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Σ K	0.083		0.174		0.117		0.053		0.017		0.087		0.125		0.099	
DF	ab ah	0.800 0.200	ba bc bg bh	0.3841 0.3841 0.0960 0.1358	cb cd cf	0.5714 0.2857 0.1429	dc de df	0.624 0.156 0.220	ed ef	0.500 0.500	fc fd fe fg	0.1920 0.1358 0.0960 0.5761	gb gf gh	0.1333 0.4000 0.4667	ha hb hg	0.1691 0.2391 0.5918
Σ DF		1		1		1		1		1		1		1		1

Member	**M. Iner. I (units)**	**Length L (inch)**
ab	**4.0**	**60**
bc	**4.0**	**60**
cd	**2.0**	**60**
de	**0.5**	**60**
ef	**0.5**	**60**
fg	**3.0**	**60**
gh	**3.5**	**60**
ah	**1.0**	**60**
bg	**1.0**	**60**
cf	**1.0**	**60**
bh	**2.0**	**84.85**
df	**1.0**	**84.85**

← Moment of inertia for member bh is missing. I use 2.0 units.

Prop.	**bg**	**gb**	**bh**	**hb**	**cf**	**fc**	**df**	**fd**
I	1.0	1.0	2.0	2.0	1.0	1.0	1.0	1.0
L	60	60	84.85	84.85	60	60	84.85	84.85

Continued

	ab	ba	bc	cb	cd	dc	de	ed	ef	fe	fg	gf	gh	hg	ah	ha
Order	8		5	**Start**	1		3		4		2		6		7	
Begin			**100**	**100**							***75***	***75***				
1			-28.571	-57.143	-28.571	-14.286										
2									-3.258	-6.516	-39.095	-19.547				
3					5.891	11.782	2.946	1.473								
4							0.446	0.893	0.893	0.446						
5	-13.718	-27.435	-27.435	-13.718												
6											-10.405	-20.809	-24.277	-12.139		
7													5.027	10.054	1.436	2.873
8	9.825	4.913													2.456	1.228
1			4.098	8.196	4.098	2.049										
2									0.329	0.658	3.947	1.974				
3					-0.923	-1.846	-0.462	-0.231								
4							-0.025	-0.049	-0.049	-0.025						
5	-1.454	-2.909	-2.909	-1.454												
6											-1.327	-2.655	-3.097	-1.549		
7													0.247	0.494	0.071	0.141
8	1.107	0.554													0.277	0.138
1			0.491	0.983	0.491	0.246										
2									0.075	0.149	0.896	0.448				
3					-0.102	-0.204	-0.051	-0.025								
4							-0.012	-0.025	-0.025	-0.012						
5	-0.135	-0.270	-0.270	-0.135												
6											-0.132	-0.265	-0.309	-0.154		
7													0.019	0.038	0.005	0.011
8	0.104	0.052													0.026	0.013
1			0.025	0.050	0.025	0.012										
2									0.008	0.017	0.100	0.050				
3					-0.004	-0.007	-0.002	-0.001								
4							-0.002	-0.004	-0.004	-0.002						
5	-0.008	-0.015	-0.015	-0.008												
6											-0.013	-0.027	-0.031	-0.016		
7													0.002	0.003	0.000	0.001
8	0.006	0.003													0.001	0.001
1			-0.0015	-0.0030	-0.0015	-0.0008										
2									0.0008	0.0016	0.0098	0.0049				
3					0.0005	0.0009	0.0002	0.0001								
4							-0.0002	-0.0005	-0.0005	-0.0002						
5	0.0005	0.0009	0.0009	0.0005												
6											-0.0013	-0.0026	-0.0031	-0.0015		
7													0.0002	0.0004	0.0001	0.0001
8	-0.0004	-0.0002													-0.0001	-0.0001
1			-0.001	-0.001	-0.001	0.000										
2									0.000	0.000	0.001	0.000				
3					0.000	0.000	0.000	0.000								
4							0.000	0.000	0.000	0.000						
5	0.000	0.000	0.000	0.000												
6											0.000	0.000	0.000	0.000		
7													0.000	0.000	0.000	0.000
8	0.000	0.000													0.000	0.000
1			-0.0002	-0.0003	-0.0002	-0.0001										
2									0.0000	0.0000	0.0001	0.0000				
3					0.0000	0.0001	0.0000	0.0000								
4							0.0000	0.0000	0.0000	0.0000						
5	0.0001	0.0001	0.0001	0.0001												
6											0.0000	0.0000	0.0000	0.0000		
7													0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000													0.0000	0.0000
7 Iter.	-4.27	-25.11	45.41	36.77	-19.10	-2.25	2.84	2.03	-2.03	-5.28	28.98	34.17	-22.42	-3.27	4.27	4.41

	ab	ba	bc	cb	cd	dc	de	ed	ef	fe	fg	gf	gh	hg	ah	ha
1st Iter.	-3.89	-22.52	43.99	29.14	-22.68	-2.50	3.39	2.37	-2.37	-6.07	25.50	34.64	-19.25	-2.08	3.89	4.10
2nd Iter.	-4.240	-24.878	45.183	35.881	-19.506	-2.301	2.906	2.085	-2.085	-5.436	28.120	33.962	-22.101	-3.140	4.240	4.380
3rd Iter.	-4.271	-25.096	45.404	36.729	-19.116	-2.259	2.842	2.035	-2.035	-5.299	28.884	34.146	-22.391	-3.256	4.271	4.404
4th Iter.	-4.273	-25.108	45.414	36.771	-19.095	-2.254	2.839	2.031	-2.031	-5.284	28.971	34.169	-22.420	-3.269	4.273	4.405
5th Iter.	-4.2730	-25.1076	45.4131	36.7683	-19.0961	-2.2539	2.8387	2.0304	-2.0304	-5.2829	28.9796	34.1713	-22.4232	-3.2699	4.2730	4.4056
6th Iter.	-4.2729	-25.1072	45.4128	36.7671	-19.0966	-2.2539	2.8388	2.0304	-2.0304	-5.2828	28.9804	34.1714	-22.4235	-3.2700	4.2729	4.4056
7th Iter.	-4.2729	-25.1071	45.4128	36.7668	-19.0968	-2.2540	2.8388	2.0304	-2.0304	-5.2827	28.9804	34.1714	-22.4235	-3.2700	4.2729	4.4056
FINAL MOMENT (in-lb)	**-4.273**	**-25.107**	**45.413**	**36.767**	**-19.097**	**-2.254**	**2.839**	**2.030**	**-2.030**	**-5.283**	**28.980**	**34.171**	**-22.424**	**-3.270**	**4.273**	**4.406**

V_c = 1.3697 lb **V_f = 1.0525 lb**

$V_c = (M_{cb} + M_{bc}) / 60$ in **$V_c + V_f =$ 2.42 lb** **$V_f = (M_{fg} + M_{gf}) / 60$ in**

External Shear = 20 lb **Multiply by 8.26 to develop 20 lb shear reaction**

Moment	**ab**	**ba**	**bc**	**cb**	**cd**	**dc**	**de**	**ed**	**ef**	**fe**	**fg**	**gf**	**gh**	**hg**	**ah**	**ha**
M (in-lb)	**-35.3**	**-207.3**	**375.0**	**303.6**	**-157.7**	**-18.6**	**23.4**	**16.8**	**-16.8**	**-43.6**	**239.3**	**282.2**	**-185.2**	**-27.0**	**35.3**	**36.4**

Continued

Order	bg	gb	bh	hb	cf	fc	df	fd
Begin								
1					-14.286	-7.143		
2					-6.516	-13.032	-4.607	-9.215
3							4.166	2.083
4								
5	-6.859	-3.429	-9.700	-4.850				
6	-3.468	-6.936						
7			2.031	4.062				
8								
1					2.049	1.024		
2					0.658	1.316	0.465	0.930
3							-0.653	-0.326
4								
5	-0.727	-0.364	-1.028	-0.514				
6	-0.442	-0.885						
7			0.100	0.200				
8								
1					0.246	0.123		
2					0.149	0.299	0.106	0.211
3							-0.072	-0.036
4								
5	-0.067	-0.034	-0.095	-0.048				
6	-0.044	-0.088						
7			0.008	0.015				
8								
1					0.012	0.006		
2					0.017	0.033	0.012	0.024
3							-0.003	-0.001
4								
5	-0.004	-0.002	-0.005	-0.003				
6	-0.004	-0.009						
7			0.001	0.001				
8								
1					-0.0008	-0.0004		
2					0.0016	0.0033	0.0012	0.0023
3							0.0003	0.0002
4								
5	0.0002	0.0001	0.0003	0.0002				
6	-0.0004	-0.0009						
7			0.0001	0.0002				
8								
1					-0.0004	-0.0002		
2					0.0002	0.0003	0.0001	0.0002
3							0.0001	0.0001
4								
5	0.0001	0.0001	0.0002	8.82E-05				
6	0.0000	-0.0001						
7			1.55E-05	3.09E-05				
8								
1					-7.91E-05	-3.95E-05		
2					1.5E-05	3E-05	1.06E-05	2.12E-05
3							2.14E-05	1.07E-05
4								
5	2.83E-05	1.41E-05	4E-05	2E-05				
6	-6.49E-06	-1.3E-05						
7			3.44E-06	6.88E-06				
8								
	-11.62	-11.75	-8.69	-1.14	-17.67	-17.37	-0.58	-6.33

	bg	gb	bh	hb	cf	fc	df	fd
1st Iter.	-10.33	-10.37	-7.67	-0.79	-20.80	-20.17	-0.44	-7.13
2nd Iter.	-11.497	-11.614	-8.597	-1.102	-18.095	-17.834	-0.629	-6.528
3rd Iter.	-11.608	-11.736	-8.685	-1.135	-17.700	-17.413	-0.596	-6.353
4th Iter.	-11.616	-11.747	-8.690	-1.136	-17.671	-17.373	-0.587	-6.330
5th Iter.	-11.6167	-11.7479	-8.6894	-1.1357	-17.6697	-17.3702	-0.5851	-6.3280
6th Iter.	-11.6166	-11.7479	-8.6892	-1.1356	-17.6699	-17.3700	-0.5849	-6.3277
7th Iter.	-11.6166	-11.7479	-8.6891	-1.1356	-17.6699	-17.3701	-0.5849	-6.3277
FINAL MOMENT (in-lb)	**-11.617**	**-11.748**	**-8.689**	**-1.136**	**-17.670**	**-17.370**	**-0.585**	**-6.328**

Moment	bg	gb	bh	hb	cf	fc	df	fd
M (in-lb)	**-95.9**	**-97.0**	**-71.7**	**-9.4**	**-145.9**	**-143.4**	**-4.8**	**-52.2**

Comparison of Moments from the Moment Distribution (Hardy Cross) Method

Five Iterations

Iteration	ab	ba	bc	cb	cd	dc	de	ed	ef	fe	fg	gf	gh	hg	ah	ha	bg
			100	**100**							***75***	***75***					
1	-13.718	-27.435	-28.571	-57.143	-28.571	-14.286	2.946	1.473	-3.258	-6.516	-39.095	-19.547	-24.277	-12.139	1.436	2.873	-6.859
2	9.825	4.913	-27.435	-13.718	5.891	11.782	0.446	0.893	0.893	0.446	-10.405	-20.809	5.027	10.054	2.456	1.228	-3.468
3	-1.454	-2.909	4.098	8.196	4.098	2.049	-0.462	-0.231	0.329	0.658	3.947	1.974	-3.097	-1.549	0.071	0.141	-0.727
4	1.107	0.554	-2.909	-1.454	-0.923	-1.846	-0.025	-0.049	-0.049	-0.025	-1.327	-2.655	0.247	0.494	0.277	0.138	-0.442
5	-0.135	-0.270	0.491	0.983	0.491	0.246	-0.051	-0.025	0.075	0.149	0.896	0.448	-0.309	-0.154	0.005	0.011	-0.067
Σ	**-4.37**	**-25.15**	**45.67**	**36.86**	**-19.01**	**-2.06**	**2.85**	**2.06**	**-2.01**	**-5.29**	**29.02**	**34.41**	**-22.41**	**-3.29**	**4.25**	**4.39**	**-11.56**

Eight Iterations

Iteration	ab	ba	bc	cb	cd	dc	de	ed	ef	fe	fg	gf	gh	hg	ah	ha	bg
			100	**100**							***75***	***75***					
1	-13.718	-27.435	-28.571	-57.143	-28.571	-14.286	2.946	1.473	-3.258	-6.516	-39.095	-19.547	-24.277	-12.139	1.436	2.873	-6.859
2	9.825	4.913	-27.435	-13.718	5.891	11.782	0.446	0.893	0.893	0.446	-10.405	-20.809	5.027	10.054	2.456	1.228	-3.468
3	-1.454	-2.909	4.098	8.196	4.098	2.049	-0.462	-0.231	0.329	0.658	3.947	1.974	-3.097	-1.549	0.071	0.141	-0.727
4	1.107	0.554	-2.909	-1.454	-0.923	-1.846	-0.025	-0.049	-0.049	-0.025	-1.327	-2.655	0.247	0.494	0.277	0.138	-0.442
5	-0.135	-0.270	0.491	0.983	0.491	0.246	-0.051	-0.025	0.075	0.149	0.896	0.448	-0.309	-0.154	0.005	0.011	-0.067
6	0.104	0.052	-0.270	-0.135	-0.102	-0.204	-0.012	-0.025	-0.025	-0.012	-0.132	-0.265	0.019	0.038	0.026	0.013	-0.044
7	-0.008	-0.015	0.025	0.050	0.025	0.012	-0.002	-0.001	0.008	0.017	0.100	0.050	-0.031	-0.016	0.000	0.001	-0.004
8	0.006	0.003	-0.015	-0.008	-0.004	-0.007	-0.002	-0.004	-0.004	-0.002	-0.013	-0.027	0.002	0.003	0.001	0.001	-0.004
Σ	**-4.27**	**-25.11**	**45.41**	**36.77**	**-19.10**	**-2.25**	**2.84**	**2.03**	**-2.03**	**-5.28**	**28.97**	**34.17**	**-22.42**	**-3.27**	**4.27**	**4.41**	**-11.62**

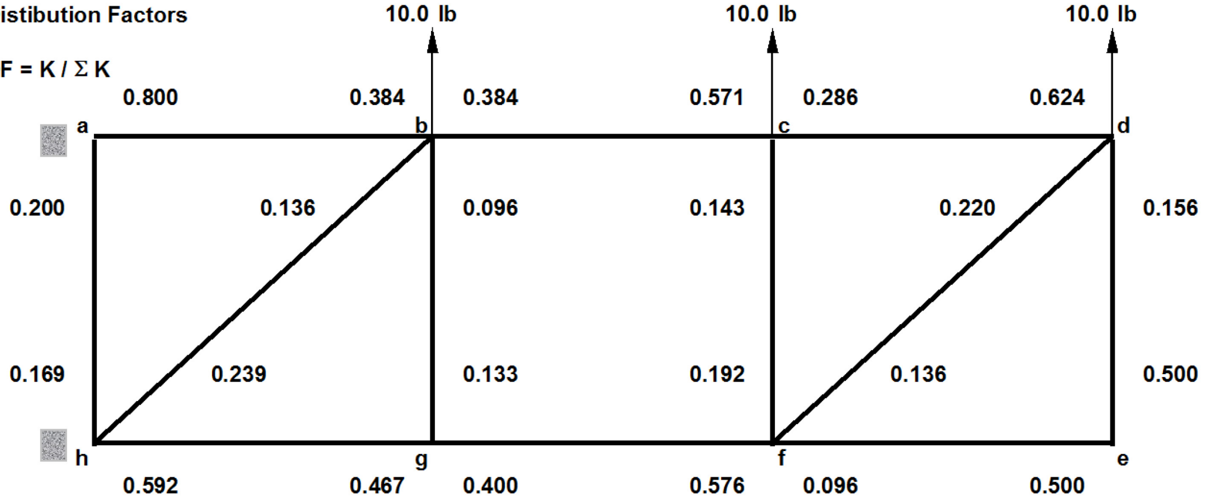
Fourteen Iterations

Iteration	ab	ba	bc	cb	cd	dc	de	ed	ef	fe	fg	gf	gh	hg	ah	ha	bg
			100	**100**							***75***	***75***					
1	-13.7175	-27.4350	-28.5714	-57.1429	-28.5714	-14.2857	2.9455	1.4728	-3.2579	-6.5158	-39.0949	-19.5475	-24.2775	-12.1387	1.4363	2.8725	-6.8588
2	9.8250	4.9125	-27.4350	-13.7175	5.8910	11.7820	0.4463	0.8926	0.8926	0.4463	-10.4046	-20.8093	5.0269	10.0538	2.4563	1.2281	-3.4682
3	-1.4544	-2.9088	4.0978	8.1956	4.0978	2.0489	-0.4615	-0.2308	0.3289	0.6579	3.9472	1.9736	-3.0972	-1.5486	0.0706	0.1411	-0.7272
4	1.1071	0.5535	-2.9088	-1.4544	-0.9231	-1.8461	-0.0245	-0.0491	-0.0491	-0.0245	-1.3274	-2.6547	0.2470	0.4940	0.2768	0.1384	-0.4425
5	-0.1349	-0.2697	0.4913	0.9826	0.4913	0.2457	-0.0509	-0.0255	0.0747	0.1494	0.8961	0.4481	-0.3086	-0.1543	0.0054	0.0108	-0.0674
6	0.1036	0.0518	-0.2697	-0.1349	-0.1019	-0.2038	-0.0123	-0.0246	-0.0246	-0.0123	-0.1323	-0.2645	0.0188	0.0376	0.0259	0.0129	-0.0441
7	-0.0077	-0.0155	0.0250	0.0499	0.0250	0.0125	-0.0019	-0.0009	0.0084	0.0167	0.1004	0.0502	-0.0313	-0.0157	0.0005	0.0009	-0.0039
8	0.0058	0.0029	-0.0155	-0.0077	-0.0037	-0.0075	-0.0019	-0.0037	-0.0037	-0.0019	-0.0134	-0.0268	0.0016	0.0032	0.0015	0.0007	-0.0045
9	0.0005	0.0009	-0.0015	-0.0030	-0.0015	-0.0008	0.0002	0.0001	0.0008	0.0016	0.0098	0.0049	-0.0031	-0.0015	0.0001	0.0001	0.0002
10	-0.0004	-0.0002	0.0009	0.0005	0.0005	0.0009	-0.0002	-0.0005	-0.0005	-0.0002	-0.0013	-0.0026	0.0002	0.0004	-0.0001	-0.0001	-0.0004
11	0.0002	0.0005	-0.0007	-0.0015	-0.0007	-0.0004	0.0001	0.0000	0.0001	0.0002	0.0009	0.0005	-0.0003	-0.0002	0.0000	0.0000	0.0001
12	-0.0002	-0.0001	0.0005	0.0002	0.0002	0.0003	0.0000	-0.0001	-0.0001	0.0000	-0.0001	-0.0003	0.0000	0.0001	-0.0001	0.0000	0.0000
13	0.0001	0.0001	-0.0002	-0.0003	-0.0002	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0001	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Σ	**-4.27**	**-25.11**	**45.41**	**36.77**	**-19.10**	**-2.25**	**2.84**	**2.03**	**-2.03**	**-5.28**	**28.98**	**34.17**	**-22.42**	**-3.27**	**4.27**	**4.41**	**-11.62**

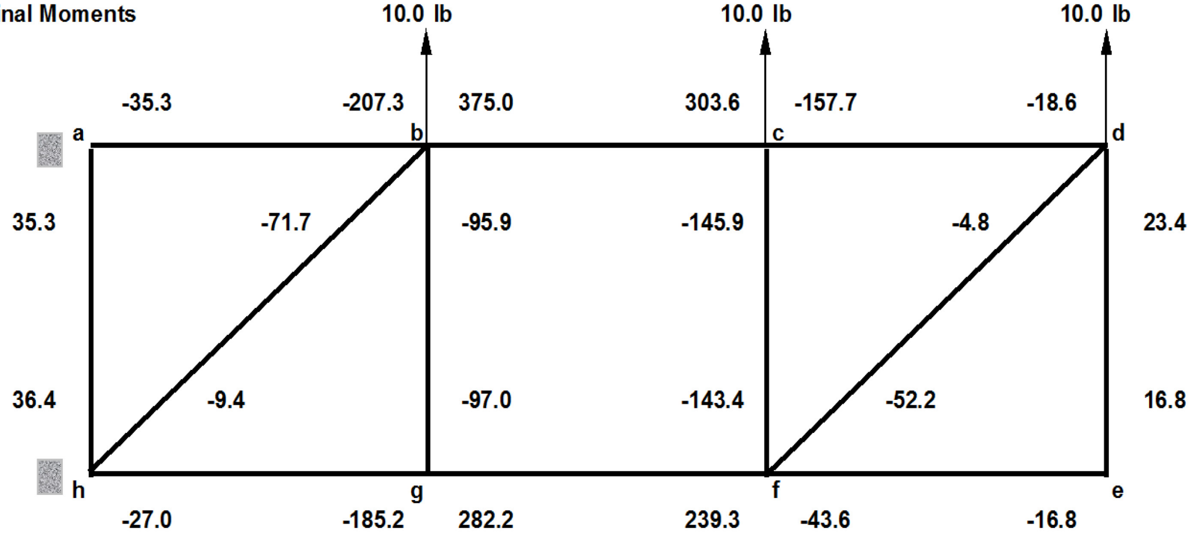
Summary

Distibution Factors

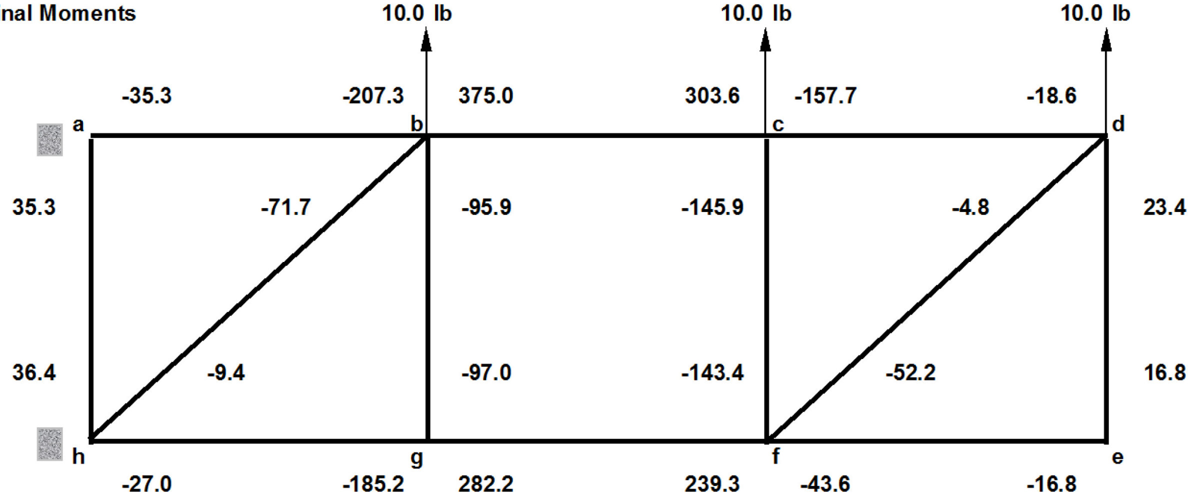
DF = K / Σ K



Final Moments



Final Moments



Page A11.23 Example Problem 12

Column 1

The distribution factor at joint C equals

$$\left(.00374 \,/\, .00374 \;+\; .01198 \right) \;=\; .238$$

should be

The distribution factor at joint C equals

$$0.00374 \,/ \left(0.00374 \;+\; 0.01198 \right) \;=\; 0.238$$

Thanks to Dr. Howard W. Smith.

Column 2 Add reference “NACA TN-534” to Figure A11.46 and Figure A11.47

Page A11.24 Fixed-End Moment Coefficient, Uniformly Varying Load – Figure A11.49

Figure A11.49 The curves are derived from NACA TN-534, Table C, page 44.

I have ended C_A at L / j = 4.1 (dashed curve to L / j = 4.5) and C_B at L / j = 4.6 per Graph IV on page 48 of NACA TN-534.

Page A11.25 Fixed-End Moment Coefficient, Concentrated Load – Figure A11.52

Figure A11.52 The curves are derived from NACA TN-534, Table D, page 45.

For L/j = 1.0 and a/L = 0.8 I am assuming that the value 1.2015 should be 1.0215.

For L/j = 1.0 and a/L = 0.9 I am assuming that the value .9654 should be .9854.

Page A11.25 Fixed-End Moment Coefficient for M_A, Concentrated Load – Figure A11.54

Figure A11.54 The curves are derived from NACA TN-534, Table E, page 46.

For L/j = 2.0 and a/L = 0.5 of M_A I am assuming that the value 1.0370 should be 1.0570.

Page A11.25 Fixed-End Moment Coefficient for M_B, Concentrated Load – Figure A11.55

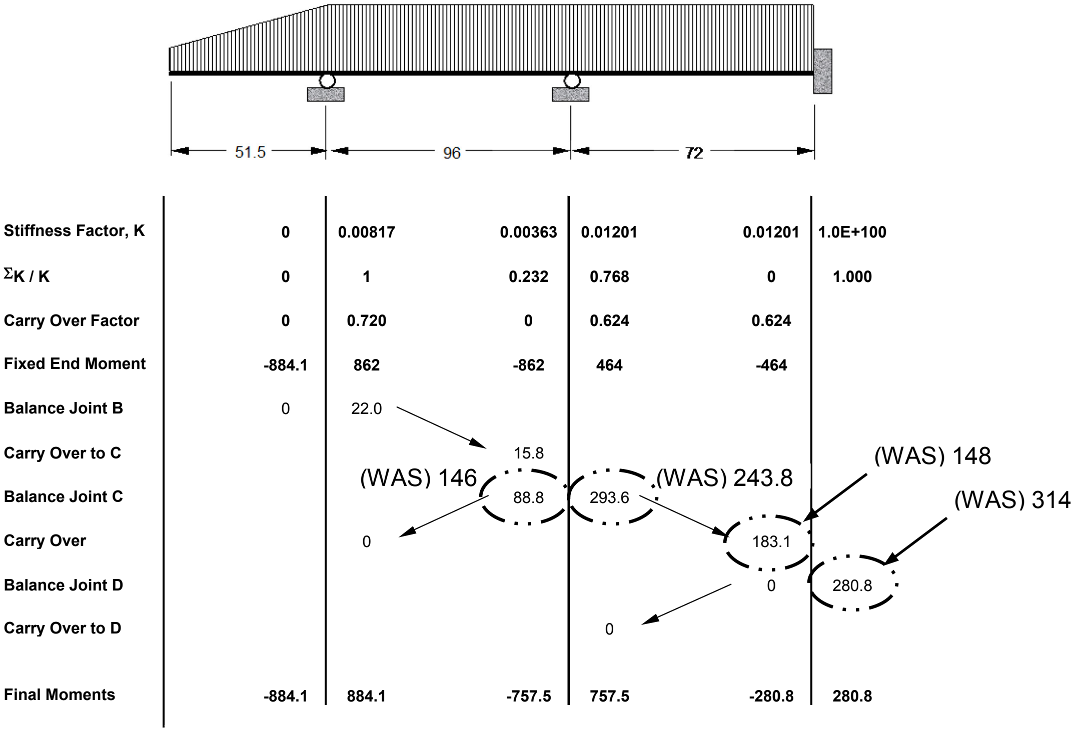
Figure A11.55 The curves are derived from NACA TN-534, Table E, page 46.

For L/j = 5.0 and a/L = 0.3 I am using the value 0.6846 instead of 0.6746.

Page A11.26 Moment Distribution Method – Continuous Beam

Figure A11.57 Effect of Axial Load on Moment Distribution

It looks like the Joint C balance is wrong and the errors follow through Joint D.



Stiffness Factor, K	**0**	**0.00817**	**0.00363**	**0.01201**	**0.01201**	**1.0E+100**
ΣK / K	**0**	**1**	**0.232**	**0.768**	**0**	**1.000**
Carry Over Factor	**0**	**0.720**	**0**	**0.624**	**0.624**	
Fixed End Moment	**-884.1**	**862**	**-862**	**464**	**-464**	
Balance Joint B	0	22.0				
Carry Over to C			15.8			
Balance Joint C			88.8	293.6		
Carry Over		0			183.1	
Balance Joint D					0	280.8
Carry Over to D				0		
Final Moments	**-884.1**	**884.1**	**-757.5**	**757.5**	**-280.8**	**280.8**

This yields values of 183.1 vs. 148, 280.8 vs. 314 and Final Moments of ±757.5 vs. ±705.8 and ±280.8 versus ± 314.

Page A11.26 Column Distribution Factor – Figure A11.56

Figure A11.56 The curve is derived from NACA TN-534, Table B, page 43.

I use the title for the abscissa in the NACA Technical Note of "Column Distribution Coefficient" instead of "Column Distribution Factor".

Page A11.27 Biplane Wing Example

Figure A11.58

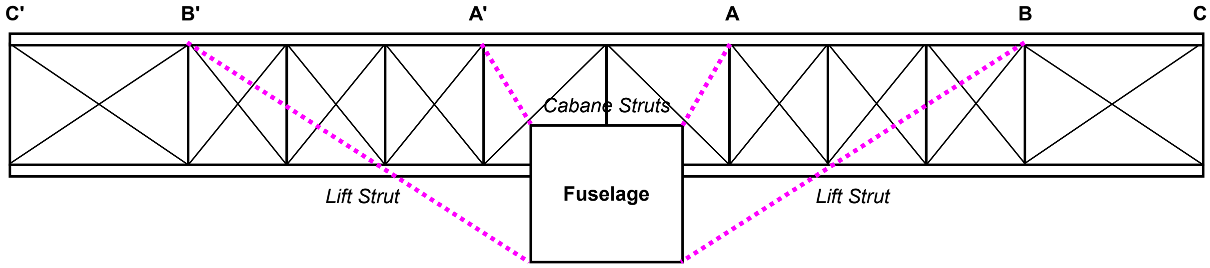
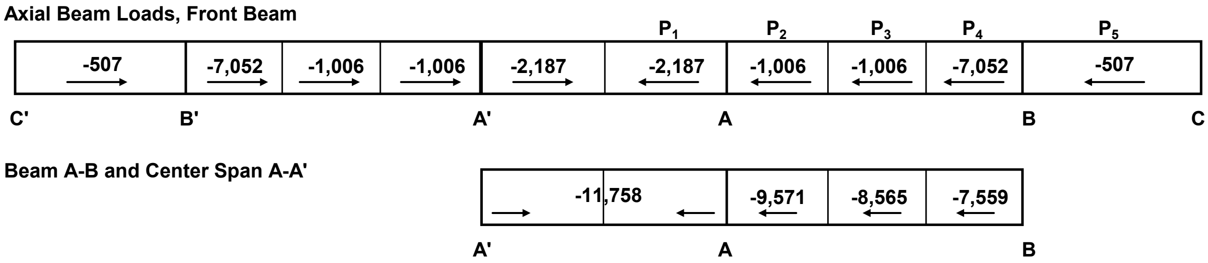
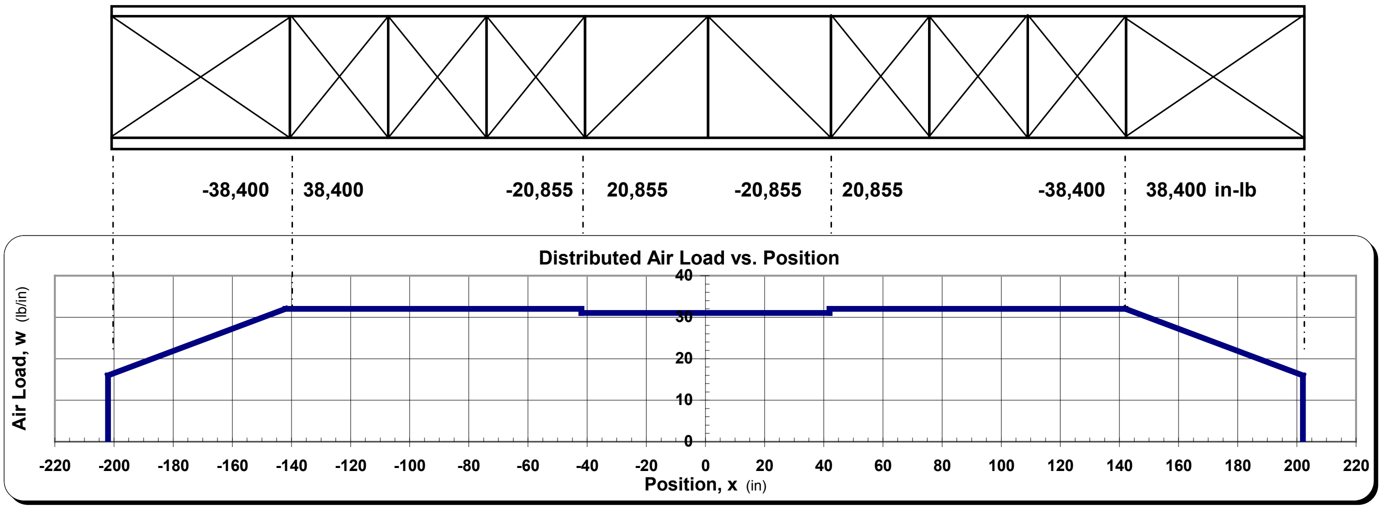


Figure A11.59





Bi-Plane Wing After ten iterations I get the following moments:

M = ±38,400 in-lb M = ±20,855 in-lb

Solution

K		**0**	**0.1498**		**0.0919**	**0.2355**		**0.2355**	**0.0919**		**0.1498**	**0**
ΣK / K		**0**	**1**		**0.281**	**0.719**		**0.719**	**0.281**		**1**	**0**
COF		**0**	**0.618**		**0**	**0.589**		**0.589**	**0**		**0.618**	**0**
Fixed End Moments		**-38,400**	**28,540**		**-28,540**	**19,175**		**-19,175**	**28,540**		**-28,540**	**38,400**
First Balancing		0	9,860		2,629	6,736		-6,736	-2,629		-9,860	0
Carry Over			0		6,091	-3,966		3,966	-6,091		0	
Second Balancing		0	0		-596	-1,528		1,528	596		0	0
Carry Over			0		0	900		-900	0		0	
Third Balancing		0	0		-253	-647		647	253		0	0
Carry Over			0		0	381		-381	0		0	
Fourth Balancing		0	0		-107	-274		274	107		0	0
Carry Over			0		0	161		-161	0		0	
Fifth Balancing		0	0		-45	-116		116	45		0	0
Carry Over			0		0	68		-68	0		0	
Sixth Balancing		0	0		-19	-49		49	19		0	0
Carry Over			0		0	29		-29	0		0	
Seventh Balancing		0	0		-8	-21		21	8		0	0
Carry Over			0		0	12		-12	0		0	
Eighth Balancing		0	0		-3	-9		9	3		0	0
Carry Over			0		0	5		-5	0		0	
Ninth Balancing		0	0		-1	-4		4	1		0	0
Carry Over			0		0	2		-2	0		0	
Tenth Balancing		0	0		-1	-2		2	1		0	0
Final Moments	Σ	**-38,400**	**38,400**	Σ	**-20,855**	**20,855**	Σ	**-20,855**	**20,855**	Σ	**-38,400**	**38,400**

Page A12.6 Slope Deflection Method – Elevator Beam Example

Column 2, $K = E\,I/L = \left(\,10{,}000{,}000\ x\ 0.03339\,\right)40 = 8347$

should be $K = E\ \ I\ /\ L = 10{,}000{,}000\left(\,0.03339\,\right)\ /\ 40 = 8{,}347$

Column 2, just above equation (4),

$$M_{4-3} = 2\,K\left(\,2\,\theta_4 - \theta_3 - 3\,\phi\,\right) + M_{F\,4-5}$$

should be $M_{4-3} = 2\,K\left(\,2\,\theta_4 + \theta_3 - 3\,\phi\,\right) + M_{F\,4-5}$

Page A13.3 Method 3 – Section Properties

Column 2, Equation 13

- (13) *should be* (13)

Thanks to Dr. Howard W. Smith.

Page A13.8 Beam Bending Stresses – Neutral Axis Method

Column 2, Stress on Stringer 1 There is a minus sign missing.

$$\sigma_{b1} = \left[\, 0.002355\, x \left(-38000\right) - \left(-.00046\, x\; 713000\right) \right] \ldots$$ *should be*

$$\sigma_{b1} = -\left[\, 0.002355 \left(-38{,}000\right) - \left(-0.00046\, x\; 713{,}000\right) \right] \ldots$$

Column 2, Stress on Stringer 9

$$\sigma_{b9} = -\left[\, 238.5 \,\right] 15.39 \;=\; \left[\, 3868 \,\right] 6.89 = -30320\ \# / in^2$$ *should be*

$$\sigma_{b9} = -\left[\, 238.5 \,\right] 15.39 \;-\; \left[\, 3{,}868 \,\right] 6.89 = -30{,}320\ lb / in^2$$

Page A13.12 Beam Bending Stresses

Column 1 …

Figure A13.17 Stainless Steel and Aluminum Alloy need to be reversed (flipped, exchanged) to match the analysis.

Note the difference between Fig.A13.17 and Fig.A13.18 when the stiffness ratios are calculated and substituted.

Thanks to Jeremy deNoyelles.

Column 2 …

In the equation for Iy, 0.165 should be 0.1615. I_x *= 28.27 should be 28.33*

See the slide rule disclaimer on p. A13.13, column 2.

Thanks to Chris Boshers.

Page A13.13 Beam Bending Stresses

-100 00 *should be* -10,000

-100,00 *should be* -10,000

$$\sigma_c = -\left[\,0.1797\;x\;5000 - (\,-0.0296\,)(\,-100\;00\,)\,\right]x - \left[\,(\,0.0403\,)(\,-100{,}00\,) - (\,-.0296\;x\;5000\,)\,\right]y$$

should be

$$\sigma_c = -\left[\,0.1797\;x\;5{,}000 - (\,-0.0296\,)(\,-10{,}000\,)\,\right]x - \left[\,(\,0.0403\,)(\,-10{,}000\,) - (\,-.0296\;x\;5{,}000\,)\,\right]y$$

Thanks to Chris Boshers.

Page A14.8 Shear Flow in a Zee Section

Column 2, Table A14.3

Portions 1 and 4: i_x = .000017 *should be* i_x = 0.000083

$\Sigma\, i_x$ = 0.0477 *should be* $\Sigma\, i_x$ = 0.0459 I_x = .6035 *should be* I_x = 0.6036

Page A14.9 Shear Flow in a Zee Section

Column 1 2 ϕ = 25° - 32.2' or ϕ = 12° - 46.1' *should be* 2 ϕ = 25° + 32.2' or ϕ = 12° + 46.1'

Column 2. Solution by Neutral Axis Method θ = 42° + 46' and in Figure A14.28 α = 87° + 17'

Page A14.10 Shear Flow in a Zee Section – Neutral Axis Method

V_n = 10000 x sin 45° - 29' = 7130 lb *should be* V_n = 10,000 sin (45° + 29') = 7,130 lb

Page A14.10 Shear Flow in a Zee Section – The "K" Method

k_1 = -7.406	I calculate	k_1 = -7.408
k_2 = 3.257	I calculate	k_2 = 3.258
k_3 = 34.25	I calculate	k_3 = 34.264

Page A15.10 Single Cell Wing Beam, Example 2

Bottom of column 2:

$$\sum M_o \;=\; 1000\;x\;2 + 400\;x\;3 + 17123 \;=\; 20323\;\; in.lb.$$ *should be*

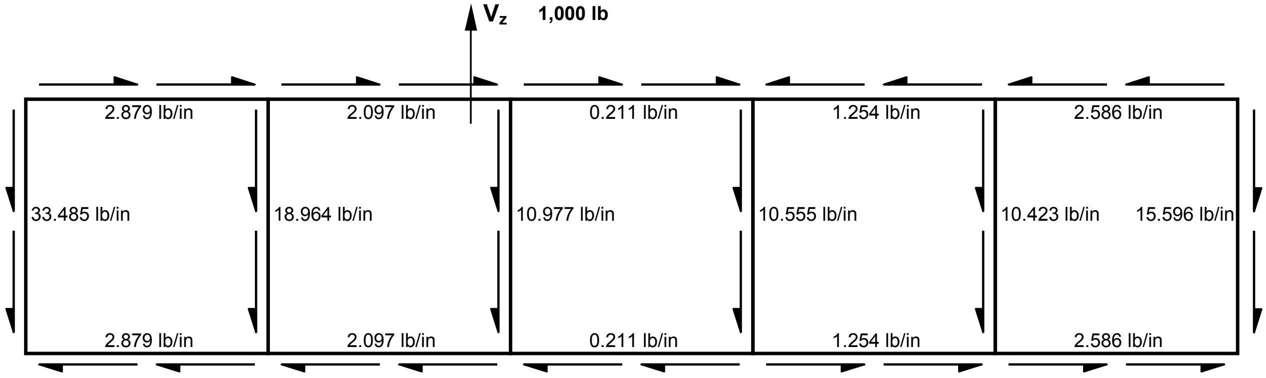
$$\sum M_o \;=\; 1000\,(\,2\,) \;+\; 400\,(\,3\,) \;+\; 32{,}257 \;=\; 35{,}457\;\; in - lb$$

Page A15.18 Shear Flow in a Symmetrical Five Cell Beam – Bending

Solution 1

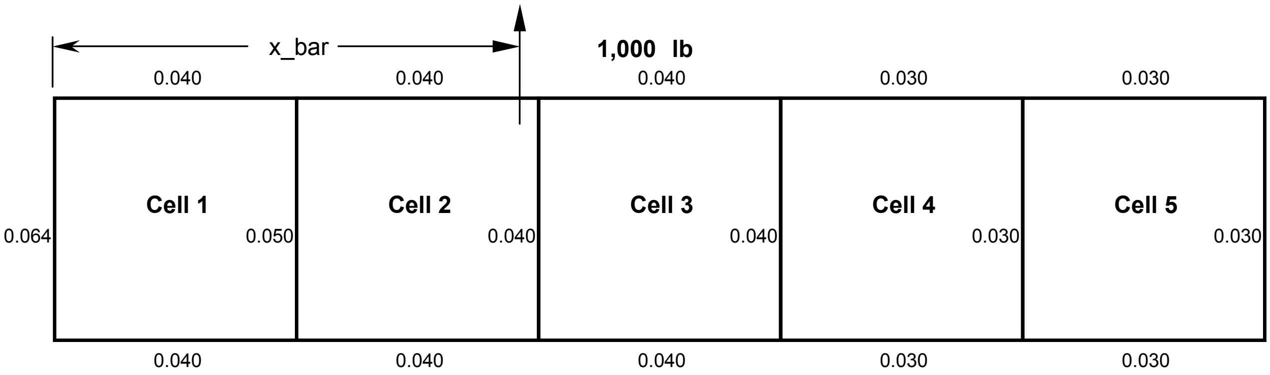
Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* page A15.18

Final Shear Flows



Flange and Web Data

Thickness, t (inches)



Shear Flow in a Symmetrical Five Cell Beam - Pure Bending

Assumed Static Condition for Shear Flow, q_s

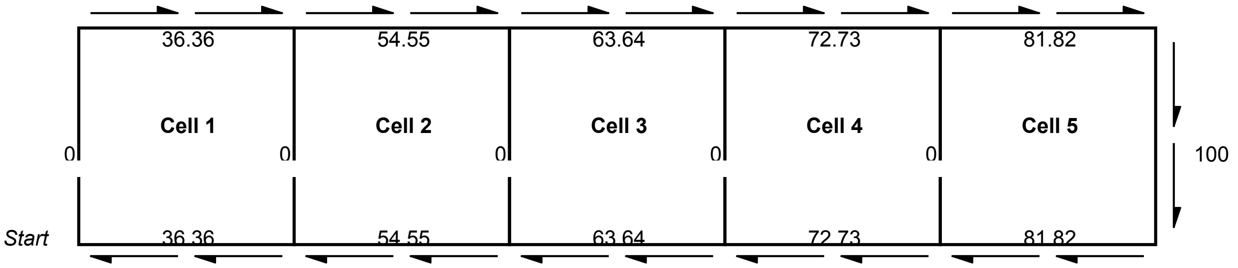


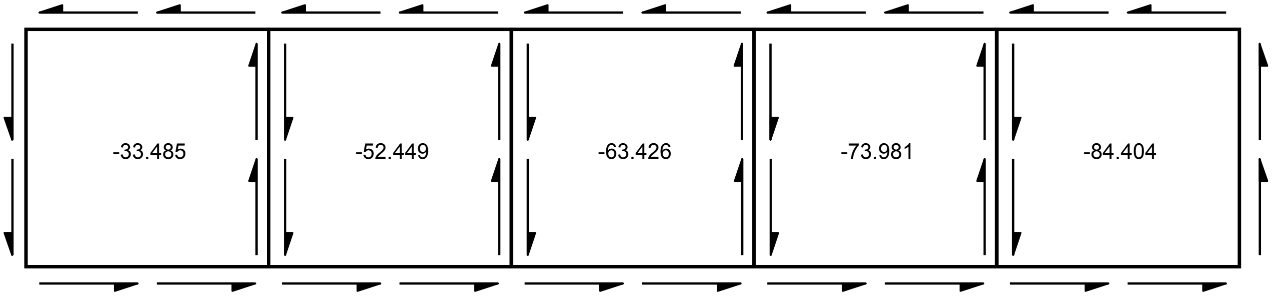
Table A15.2

1	**Σ q_s L / t for each Cell**	**18,182**	**27,273**		**31,818**		**48,485**		**87,879**
2	**Σ L / t for each Cell**	**856.25**	**950**		**1,000**		**1,250**		**1,333.33**
3	**L / t of Cell Web**		**200**		**250**		**250**		**333**
4	**Carry Over Factor, C**	0.2105	0.2336	0.2500	0.2632	0.2000	0.2500	0.2500	0.2667
5	**$q' = -\Sigma q_s\ L/t\ /\ \Sigma\ L/t$**	**-21.234**	**-28.708**		**-31.818**		**-38.788**		**-65.909**
6	**q" = Cq'**	-6.706	-4.470	-8.373	-7.177	-9.697	-6.364	-17.576	-9.697
7	**q"' = Cq"**	-3.000	-1.412	-4.441	-3.211	-5.985	-3.375	-2.586	-5.985
8	Carry Over	-1.367	-0.632	-2.420	-1.463	-1.490	-1.839	-1.596	-1.490
9	Carry Over	-0.713	-0.288	-0.777	-0.763	-0.859	-0.591	-0.397	-0.859
10	Carry Over	-0.249	-0.150	-0.427	-0.266	-0.247	-0.324	-0.229	-0.247
11	Carry Over	-0.135	-0.052	-0.135	-0.144	-0.138	-0.103	-0.066	-0.138
12	Carry Over	-0.044	-0.028	-0.074	-0.047	-0.042	-0.057	-0.037	-0.042
13	Carry Over	-0.024	-0.009	-0.023	-0.026	-0.023	-0.018	-0.011	-0.023
14	Carry Over	-0.008	-0.005	-0.013	-0.008	-0.007	-0.010	-0.006	-0.007
15	Carry Over	-0.004	-0.002	-0.004	-0.004	-0.004	-0.003	-0.002	-0.004
16	Carry Over	-0.0013	-0.0009	-0.0022	-0.0014	-0.0013	-0.0017	-0.0011	-0.0013
17	Carry Over	-0.0007	-0.0003	-0.0007	-0.0008	-0.0007	-0.0005	-0.0003	-0.0007
18	Carry Over	-0.0002	-0.0002	-0.0004	-0.0002	-0.0002	-0.0003	-0.0002	-0.0002
19	Carry Over	-0.0001	0.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
20	Carry Over	0.0000	0.0000	-0.0001	0.0000	0.0000	-0.0001	0.0000	0.0000
	Shear Flow q	**-33.485**	**-52.448**		**-63.425**		**-73.981**		**-84.404**

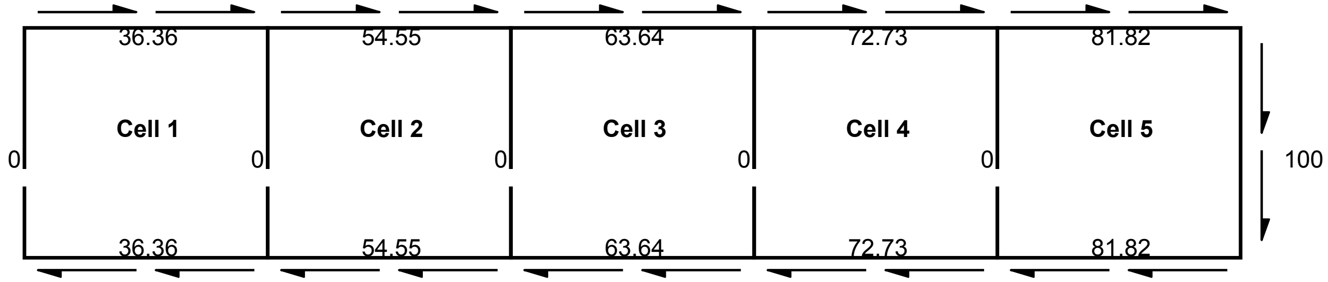
	Cell 1	**Cell 2**		**Cell 3**		**Cell 4**		**Cell 5**
Shear Flow q	**-33.485**	**-52.448**		**-63.425**		**-73.981**		**-84.404**
Carry Over Factor	0.2105	0.2336	0.2500	0.2632	0.2000	0.2500	0.2500	0.2667
Reiteration	-12.251	-7.049	-16.691	-13.112	-18.495	-12.685	-22.508	-18.495
$q' = -\Sigma q_s\ L/t\ /\ \Sigma\ L/t$	**-21.234**	**-28.708**		**-31.818**		**-38.788**		**-65.909**
Shear Flow q	**-33.485**	**-52.449**		**-63.425**		**-73.981**		**-84.404**
Carry Over Factor	0.2105	0.2336	0.2500	0.2632	0.2000	0.2500	0.2500	0.2667
Second Reiteration	-12.251	-7.049	-16.691	-13.112	-18.495	-12.685	-22.508	-18.495
$q' = -\Sigma q_s\ L/t\ /\ \Sigma\ L/t$	**-21.234**	**-28.708**		**-31.818**		**-38.788**		**-65.909**
Shear Flow q	**-33.485**	**-52.449**		**-63.426**		**-73.981**		**-84.404**

Shear Flow in a Symmetrical Five Cell Beam - Pure Bending

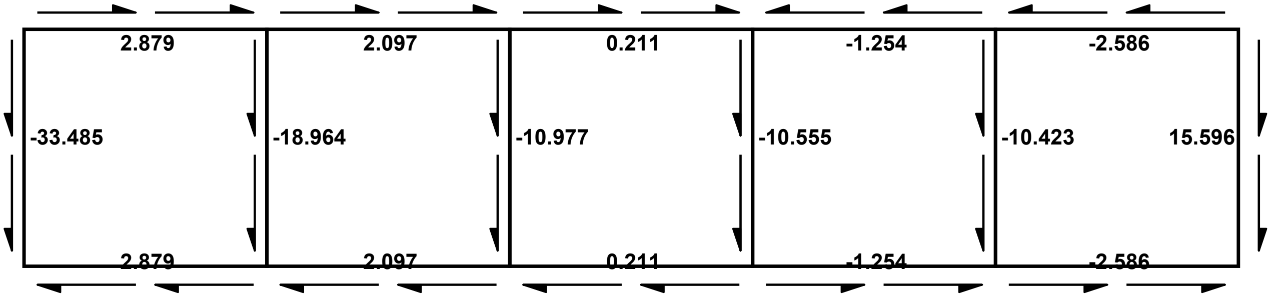
Closing Shear Flows



Assumed Static Condition



Final Shear Flows



Shear Flow in a Symmetrical Five Cell Beam - Pure Bending

Solution 2

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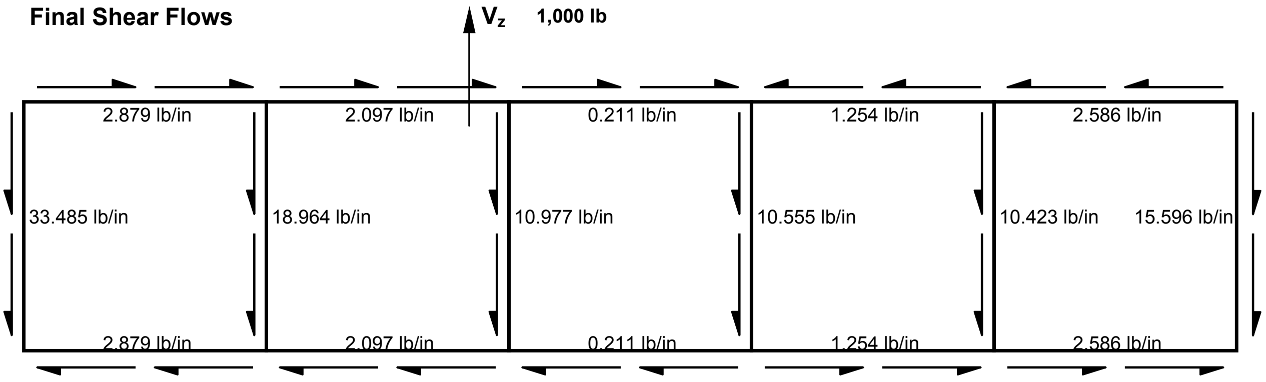
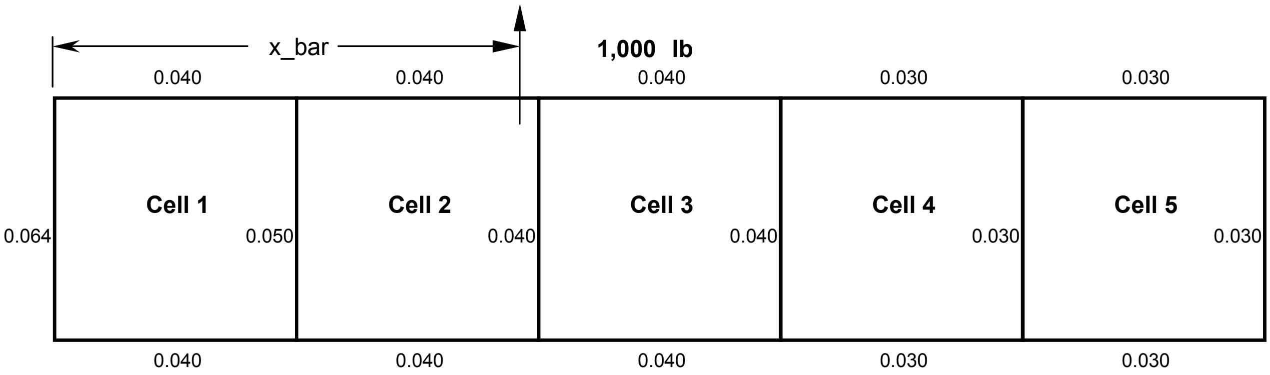
Final Shear Flows



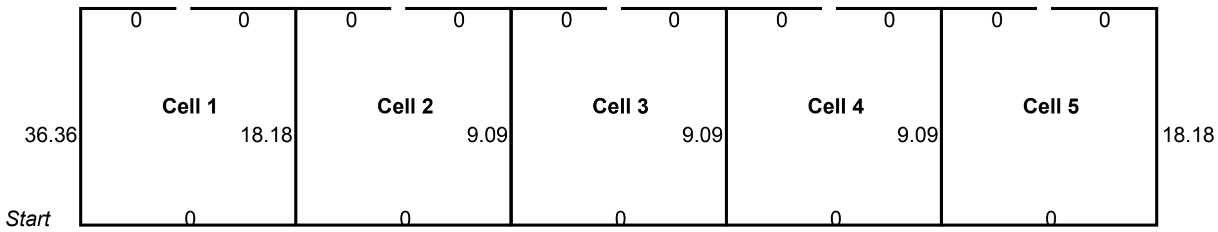
Flange and Web Data

Thickness, t (inches)



Shear Flow in a Symmetrical Five Cell Beam - Pure Bending

Assumed Static Condition for Shear Flow, q_s

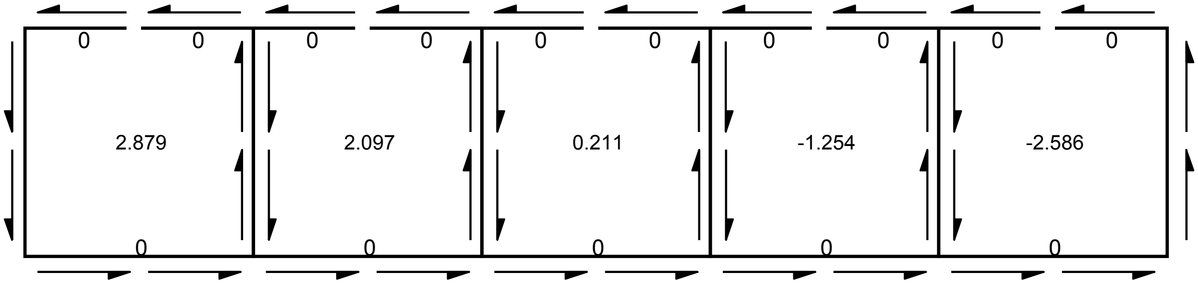


No.									
1	**Σ q_s L / t for each Cell**	**-2,045**	**-1,364**		**0**		**758**		**3,030**
2	**Σ L / t for each Cell**	**856.25**	**950**		**1,000**		**1,250**		**1,333.33**
3	**L / t of Cell Web**		**200**		**250**		**250**		**333**
4	**Carry Over Factor, C**	0.2105	0.2336	0.2500	0.2632	0.2000	0.2500	0.2500	0.2667
5	**q' = - Σq_s L/t / Σ L/t**	**2.389**	**1.435**		**0.000**		**-0.606**		**-2.273**
6	**q" = Cq'**	0.335	0.503	0.000	0.359	-0.152	0.000	-0.606	-0.152
7	**q'" = Cq"**	0.117	0.071	0.055	0.126	-0.152	0.041	-0.040	-0.152
8	Carry Over	0.029	0.025	-0.007	0.031	0.000	-0.005	-0.040	0.000
9	Carry Over	0.004	0.006	0.008	0.004	-0.011	0.006	0.000	-0.011
10	Carry Over	0.0034	0.0009	-0.0018	0.0036	0.0016	-0.0014	-0.0030	0.0016
11	Carry Over	-0.000218	0.000711	0.001371	-0.000234	-0.001105	0.001042	0.000425	-0.001105
12	Carry Over	0.000486	-0.000046	-0.000352	0.000520	0.000367	-0.000268	-0.000295	0.000367
13	Carry Over	-0.000093	0.000102	0.000234	-0.000100	-0.000141	0.000177	0.000098	-0.000141
14	Carry Over	0.000078	-0.000020	-0.000063	0.000084	0.000069	-0.000048	-0.000037	0.000069
15	Carry Over	-0.000019	0.000017	0.000040	-0.000021	-0.000021	0.000031	0.000018	-0.000021
16	Carry Over	0.000013	-0.000004	-0.000011	0.000014	0.000012	-0.000008	-0.000006	0.000012
17	Carry Over	-0.000004	0.000003	0.000007	-0.000004	-0.000004	0.000005	0.000003	-0.000004
18	Carry Over	0.000002	-0.000001	-0.000002	0.000002	0.000002	-0.000001	-0.000001	0.000002
19	Carry Over	-0.000001	0.000000	0.000001	-0.000001	-0.000001	0.000001	0.000001	-0.000001
20	Carry Over	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

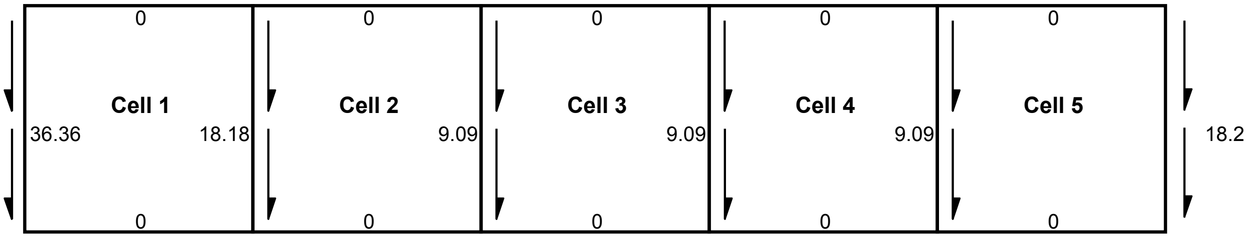
	Cell 1	**Cell 2**	**Cell 3**	**Cell 4**	**Cell 5**
Shear Flow q	**2.879**	**2.097**	**0.211**	**-1.254**	**-2.586**
Carry Over Factor	0.2105	0.2336 0.2500	0.2632 0.2000	0.2500 0.2500	0.2667
Reiteration	0.490	0.606 0.055	0.524 -0.313	0.042 -0.690	-0.313
q' = - Σq_s L/t / Σ L/t	**2.389**	**1.435**	**0.000**	**-0.606**	**-2.273**
Shear Flow q	**2.879**	**2.097**	**0.211**	**-1.254**	**-2.586**
Carry Over Factor	0.2105	0.2336 0.2500	0.2632 0.2000	0.2500 0.2500	0.2667
Second Reiteration	0.490	0.606 0.055	0.524 -0.313	0.042 -0.690	-0.313
q' = - Σq_s L/t / Σ L/t	**2.389**	**1.435**	**0.000**	**-0.606**	**-2.273**
Shear Flow q	**2.879**	**2.097**	**0.211**	**-1.254**	**-2.586**

Shear Flow in a Symmetrical Five Cell Beam - Pure Bending

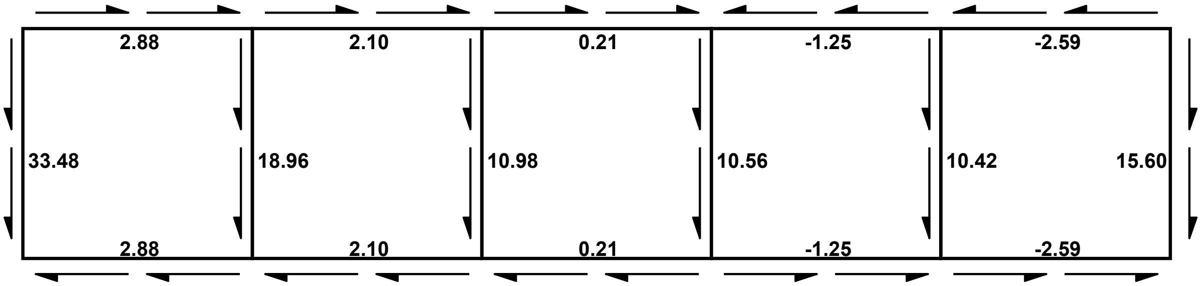
Closing Shear Flows



Assumed Static Condition



Final Shear Flows



Page A15.22 Shear Flow in a Symmetrical Ten-Cell Beam

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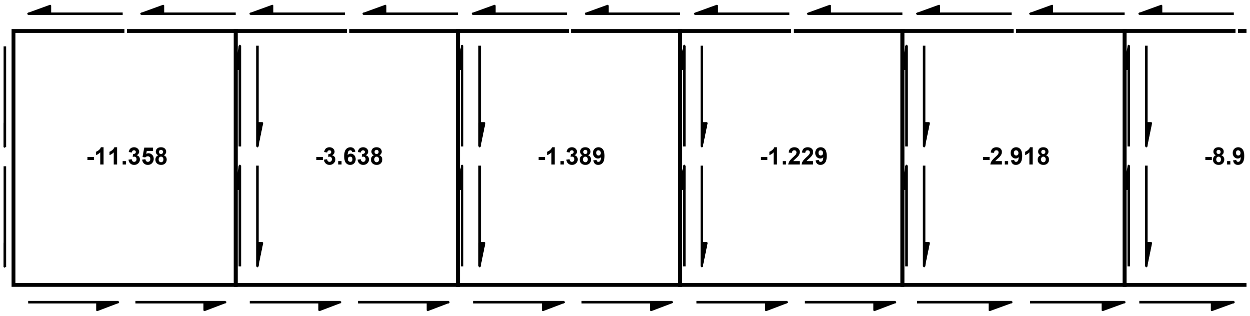
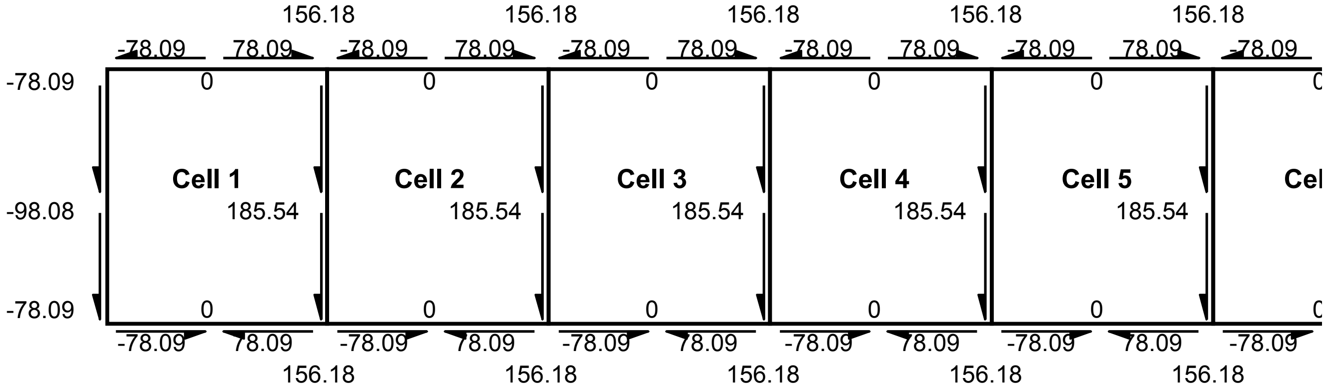
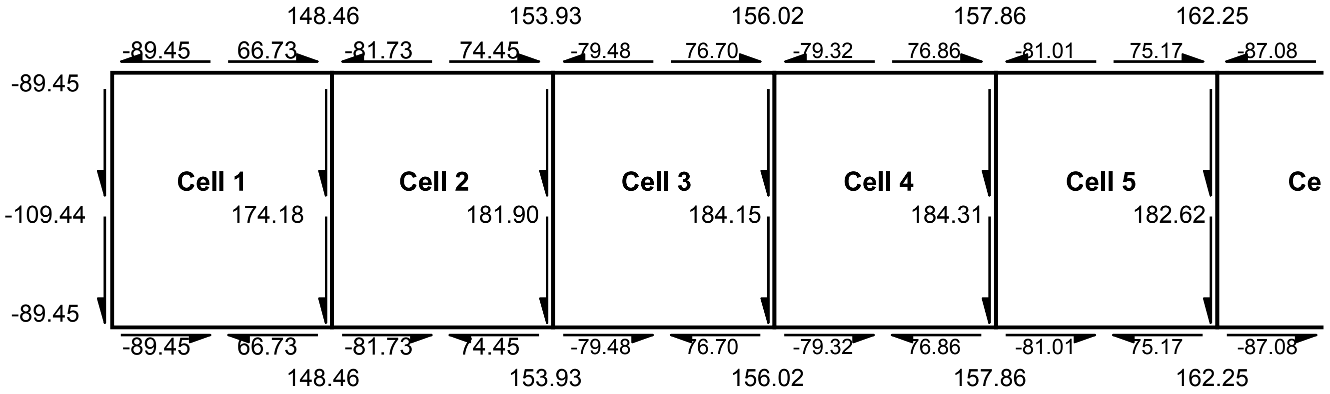
Closing Shear Flows – Left Half



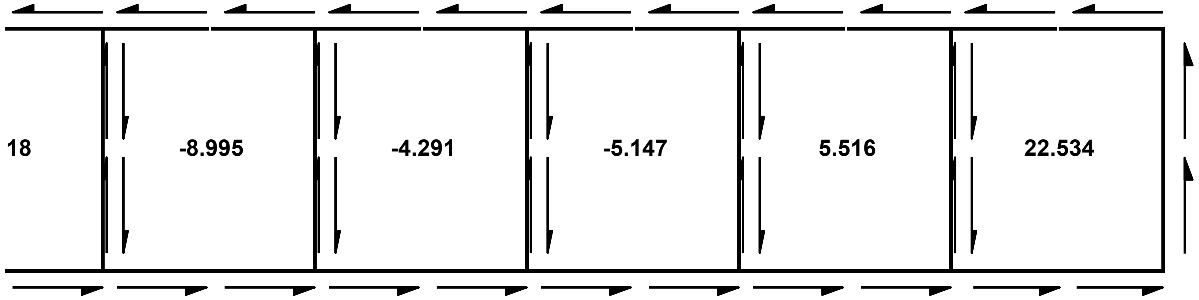
Assumed Static Condition – Left Half



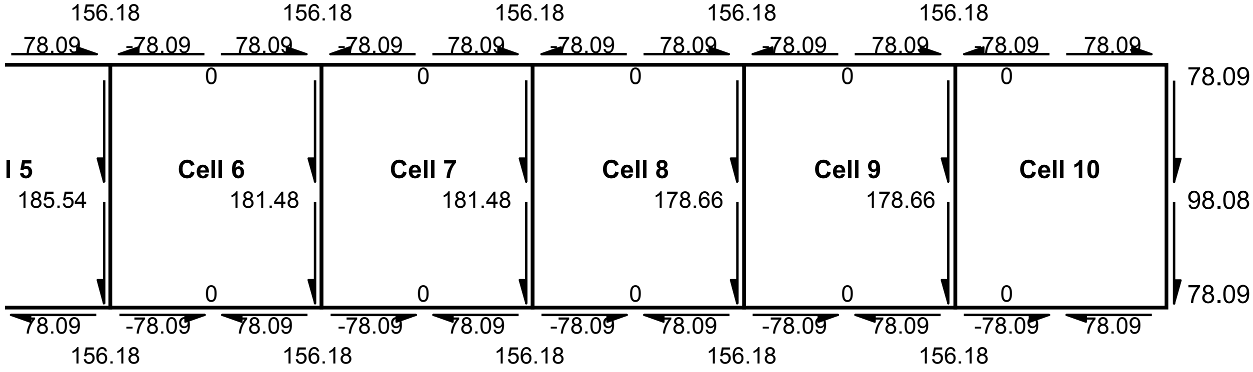
Final Shear Flow Values – Left Half



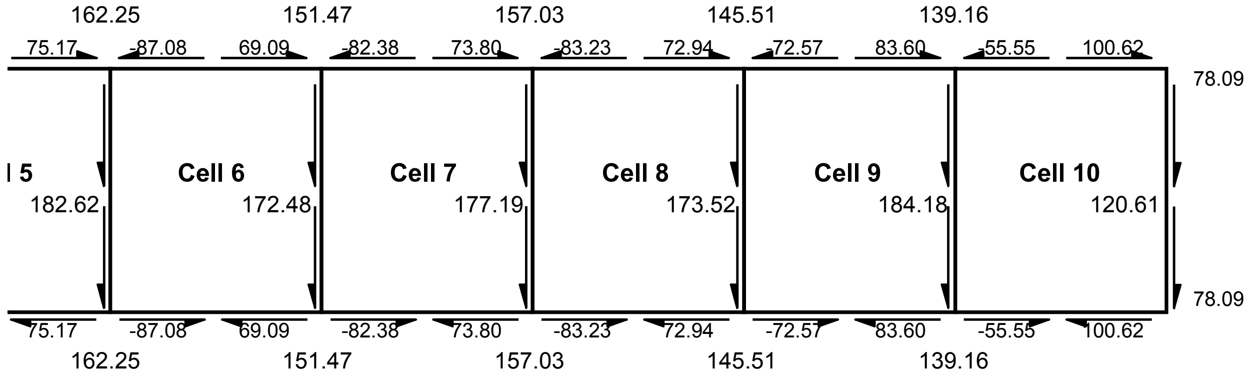
Closing Shear Flows – Right Half



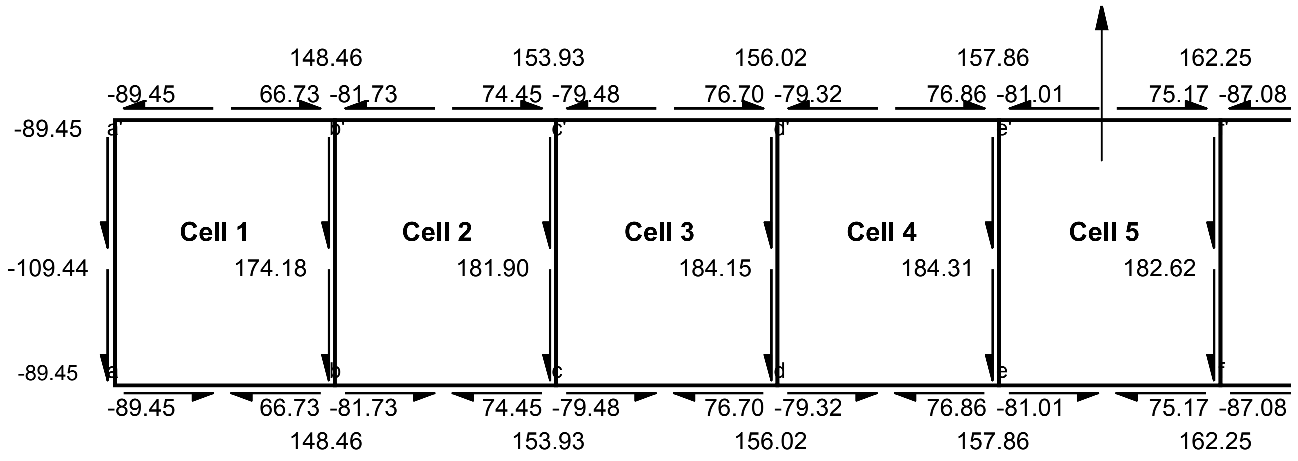
Assumed Static Condition – Right Half

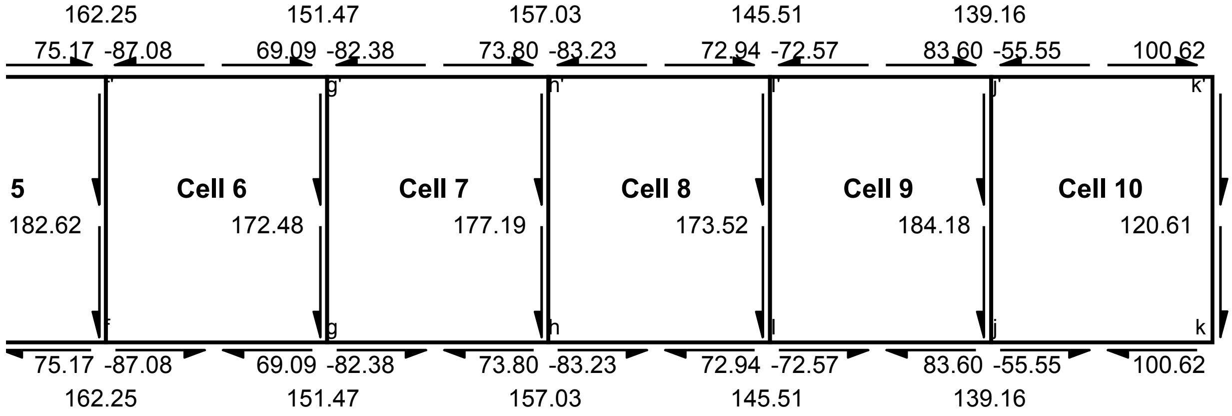


Final Shear Flow Values – Right Half

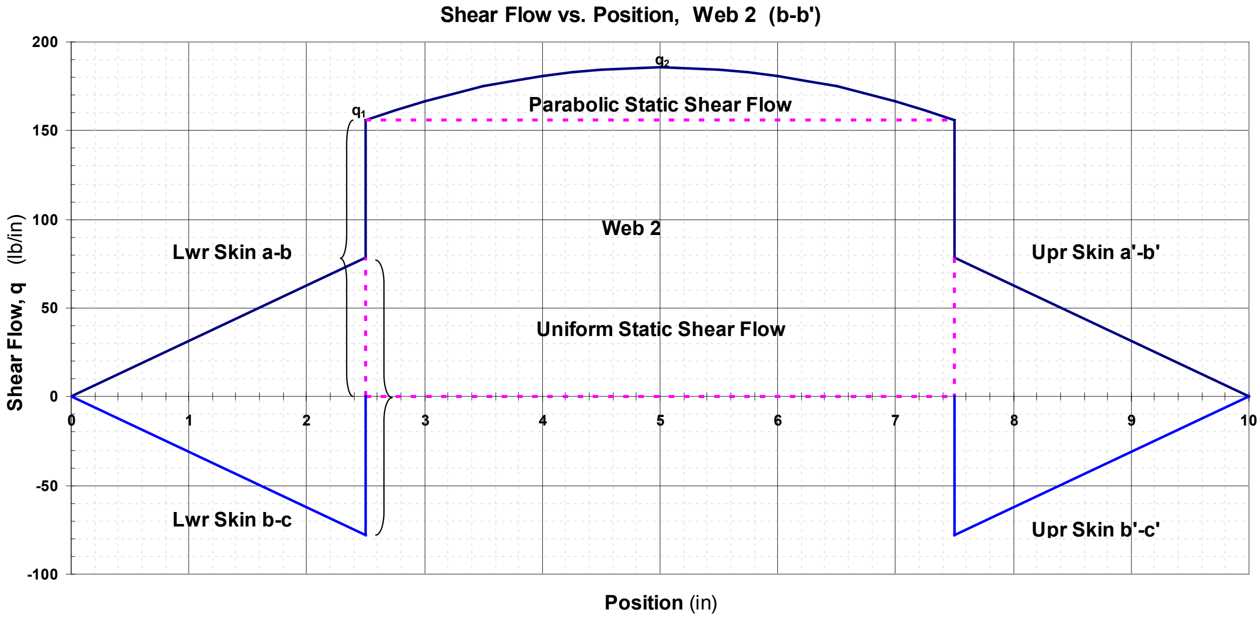


Final Shear Flow Values

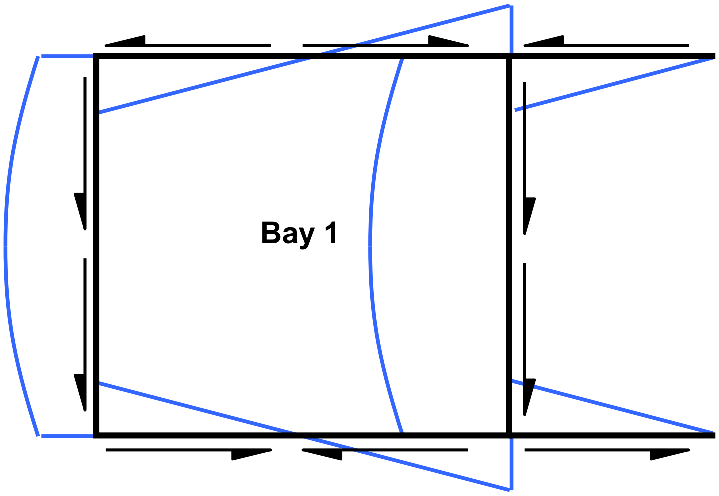




Shear Flow



Assumed Static Condition for Shear Flow q_s



Summary

Cell 1	Cell 2		Cell 3		Cell 4		Cell 5		Cell 6		Cell 7		Cell 8		Cell 9		Cell 10
2,207	**0**		**0**		**0**		**0**		**1,333**		**0**		**1,205**		**0**		**-4,745**
211.32	**186.38**		**186.38**		**186.38**		**186.38**		**194.92**		**203.46**		**211.17**		**218.89**		**227.57**
53.19		**53.19**		**53.19**		**53.19**		**53.19**		**61.73**		**61.73**		**69.44**		**69.44**	
0.2854	0.2517	0.2854	0.2854	0.2854	0.2854	0.2854	0.2854	0.2729	0.2854	0.3034	0.3167	0.2923	0.3034	0.3173	0.3289	0.3052	0.3173
-10.442	**0**		**0**		**0**		**0**		**-6.840**		**0**		**-5.707**		**0**		**20.850**
0	-2.980	0	0	0	0	0	0	-1.952	0	0	-2.075	-1.731	0	0	-1.810	6.615	0
-0.750	0	0	-0.850	0	0	-0.557	0	0	-0.533	-1.205	0	0	-1.113	1.580	0	0	1.466
0	-0.214	-0.243	0	-0.159	-0.243	0	-0.159	-0.496	0	0	-0.527	0.142	0	0	0.148	0.465	0
-0.115	0	-0.045	-0.130	-0.069	-0.045	-0.187	-0.069	0	-0.179	-0.122	0	0	-0.113	0.202	0	0	0.187
-0.011	-0.033	-0.057	-0.013	-0.066	-0.057	-0.020	-0.066	-0.086	-0.019	0	-0.091	0.027	0	0	0.028	0.059	0
-0.023	-0.0033	-0.023	-0.026	-0.022	-0.023	-0.043	-0.022	-0.005	-0.042	-0.020	-0.006	0.000	-0.019	0.029	0.000	0.000	0.027
-0.007	-0.006	-0.014	-0.007	-0.019	-0.014	-0.008	-0.019	-0.018	-0.007	-0.002	-0.019	0.003	-0.002	0.000	0.003	0.008	0.000
-0.005	-0.0019	-0.0075	-0.0057	-0.0061	-0.0075	-0.0104	-0.0061	-0.0026	-0.0100	-0.0050	-0.0028	-0.0005	-0.0046	0.0038	-0.0005	0.0000	0.0036
-0.002	-0.0014	-0.0034	-0.0027	-0.0051	-0.0034	-0.0025	-0.0051	-0.0043	-0.0024	-0.0011	-0.0045	-0.0002	-0.0010	-0.0002	-0.0002	0.0011	-0.0002
-0.0012	-0.00067	-0.0022	-0.0014	-0.0017	-0.0022	-0.0027	-0.0017	-0.0010	-0.0026	-0.0015	-0.0010	-0.0003	-0.0014	0.0003	-0.0004	-0.0001	0.00027
-0.0007	-0.00035	-0.00087	-0.00083	-0.00140	-0.00087	-0.00076	-0.00140	-0.00116	-0.00072	-0.00044	-0.00123	-0.00033	-0.00041	-0.00014	-0.00035	0.00009	-0.00013
-0.00031	-0.00021	-0.00063	-0.00035	-0.00046	-0.00063	-0.00073	-0.00046	-0.00033	-0.00070	-0.00050	-0.00035	-0.00016	-0.00046	-0.00009	-0.00017	-0.00004	-0.00008
-0.00021	-0.00009	-0.00023	-0.00024	-0.00039	-0.00023	-0.00023	-0.00039	-0.00034	-0.00022	-0.00016	-0.00036	-0.00017	-0.00015	-0.00007	-0.00017	-0.00003	-0.00006
-0.00008	-0.00006	-0.00018	-0.00009	-0.00013	-0.00018	-0.00021	-0.00013	-0.00011	-0.00020	-0.00017	-0.00012	-0.00007	-0.00015	-0.00007	-0.00007	-0.00002	-0.00006
-0.00006	-0.00002	-0.00006	-0.00007	-0.00011	-0.00006	-0.00007	-0.00011	-0.00010	-0.00007	-0.00006	-0.00011	-0.00007	-0.00005	-0.00003	-0.00007	-0.00002	-0.00003
Cell 1	**Cell 2**		**Cell 3**		**Cell 4**		**Cell 5**		**Cell 6**		**Cell 7**		**Cell 8**		**Cell 9**		**Cell 10**
-11.358	**-3.638**		**-1.389**		**-1.229**		**-2.918**		**-8.995**		**-4.290**		**-5.147**		**5.516**		**22.534**

Page A15.11 Single Cell Wing Beam – Multiple Stringers

Column 1

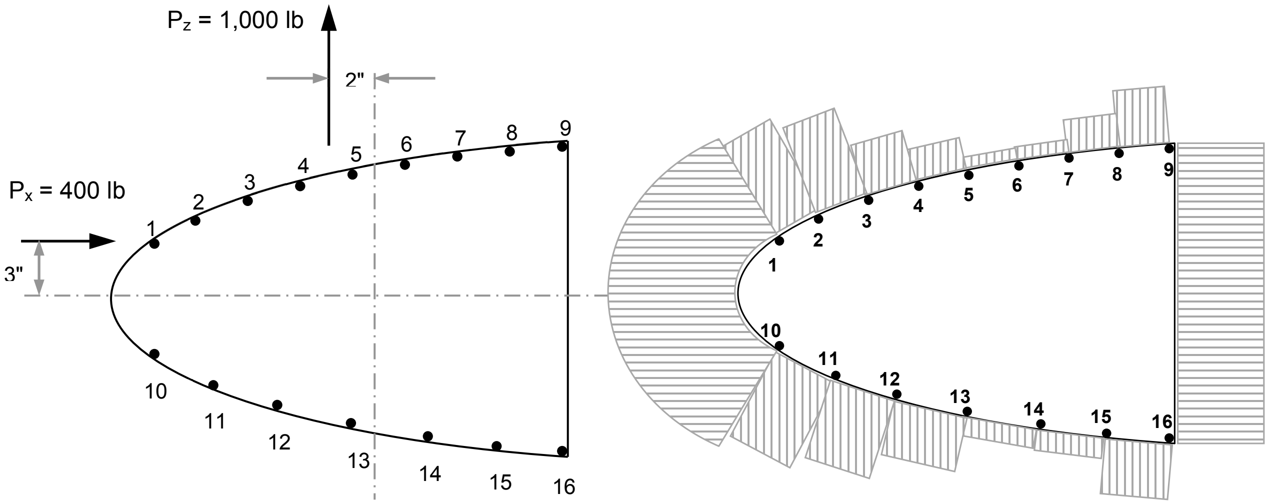
$$q \;=\; \frac{M}{2\;\;A} \;=\; \frac{20323}{2\;\;x\;\;493} \;=\; -20.6\;lb./in.$$ *should be*

$$q \;=\; -\frac{M}{2\;\;A} \;=\; -\frac{35{,}457}{2\;\left(\,493\,\right)} \;=\; -36\;lb/in$$

Figure A15-28

A = 125.9 ° “ *should be* A = 125.9 in^2

Figure A15-29



Final Shear Flow

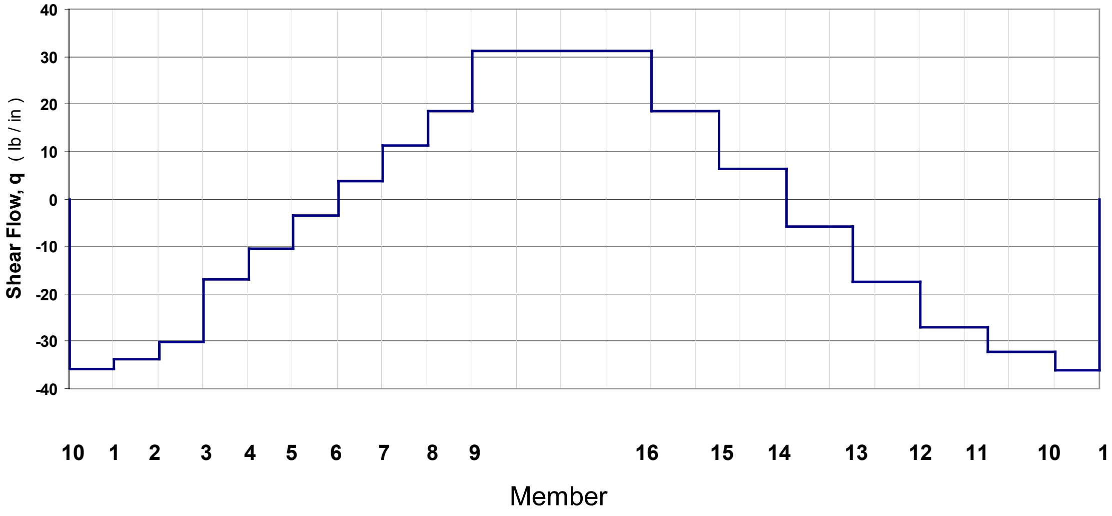


Table A15.1

Check Column 13 in Table A15.1.

q_y	q_z	A_{cell}	q
-0.485	-5.264	493.0	-35.96

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Member	**Total Area* A** (in^2)	**Arm Z** (in^2)	**ZA** (in^2)	**Arm X** (in)	**XA** (in^2)	**Σ ZA** (in^3)	**Σ XA** (in)	**q_y Σ xA** (in^2)	**q_z Σ zA** (in^3)	**q_{xz} Σ xA + Σ zA** (in^3)	**m** (in^2)	**q_{xz} m** (lb/in)	**Final q** (lb/in)
						0	0	0	0	0		0.0	-36
1	0.141	4.396	0.619	-17.41	-2.450								
						0.619	-2.450	1.189	-3.256	2.067	55.2	114.1	-33.9
2	0.141	6.446	0.907	-13.54	-1.905								
						1.526	-4.355	2.114	-8.031	5.917	44.2	261.5	-30.0
3	0.380	7.396	2.810	-9.11	-3.462								
						4.336	-7.817	3.795	-22.826	19.031	32.0	609.0	-16.9
4	0.168	7.766	1.308	-5.44	-0.916								
						5.644	-8.733	4.240	-29.710	25.471	38.2	973.0	-10.5
5	0.168	7.946	1.338	-0.86	-0.145								
						6.982	-8.878	4.310	-36.754	32.444	33.0	1,070.7	-3.5
6	0.168	7.896	1.330	3.14	0.529								
						8.312	-8.349	4.053	-43.754	39.700	33.2	1,318.1	3.7
7	0.168	7.696	1.296	7.14	1.202								
						9.608	-7.147	3.470	-50.576	47.106	40.2	1,893.7	11.1
8	0.168	7.296	1.229	11.74	1.977								
						10.836	-5.170	2.510	-57.044	54.534	32.6	1,777.8	18.6
9	0.290	6.896	2.000	15.39	4.463								
						12.836	-0.707	0.343	-67.571	67.228	251.8	**16,928.0**	31.3
16	0.350	-8.411	-2.944	15.39	5.387								
						9.892	4.680	-2.272	-52.074	54.346	46.0	2,499.9	18.4
15	0.310	-8.224	-2.549	9.64	2.988								
						7.343	7.668	-3.723	-38.654	42.376	48.2	2,042.5	6.4
14	0.310	-7.734	-2.398	3.32	1.029								
						4.945	8.697	-4.222	-26.033	30.255	46.4	1,403.8	-5.7
13	0.310	-7.004	-2.171	-2.96	-0.918								
						2.774	7.780	-3.777	-14.603	18.380	47.6	874.9	-17.6
12	0.280	-5.554	-1.555	-9.11	-2.551								
						1.219	5.229	-2.538	-6.417	8.956	36.6	327.8	-27.0
11	0.170	-4.504	-0.766	-13.54	-2.302								
						0.453	2.927	-1.421	-2.387	3.808	42.6	162.2	-32.2
10	0.170	-2.904	-0.494	-17.51	-2.977								
						-0.040	-0.050	0	0	0		0	-36
											Σ	**32,256.9**	

* From Table 9, page A3.11

Page A15.27 Shear Flow in Closed Thin-Wall Sections

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Problem 2, pages A15.27-A15.29

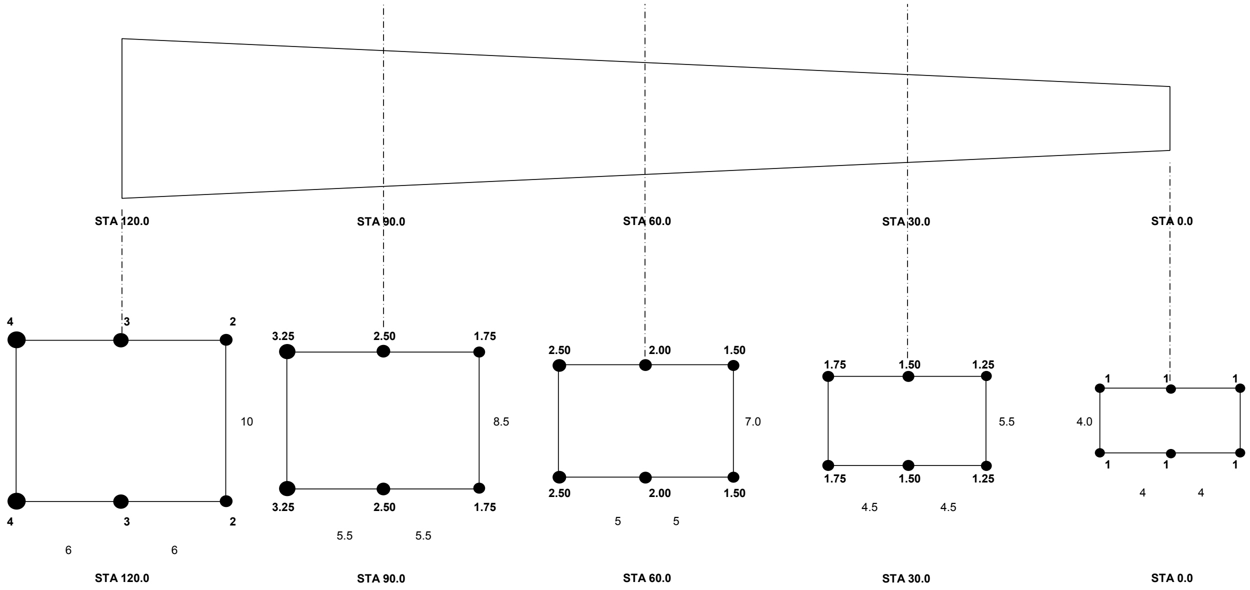


Figure A15-73

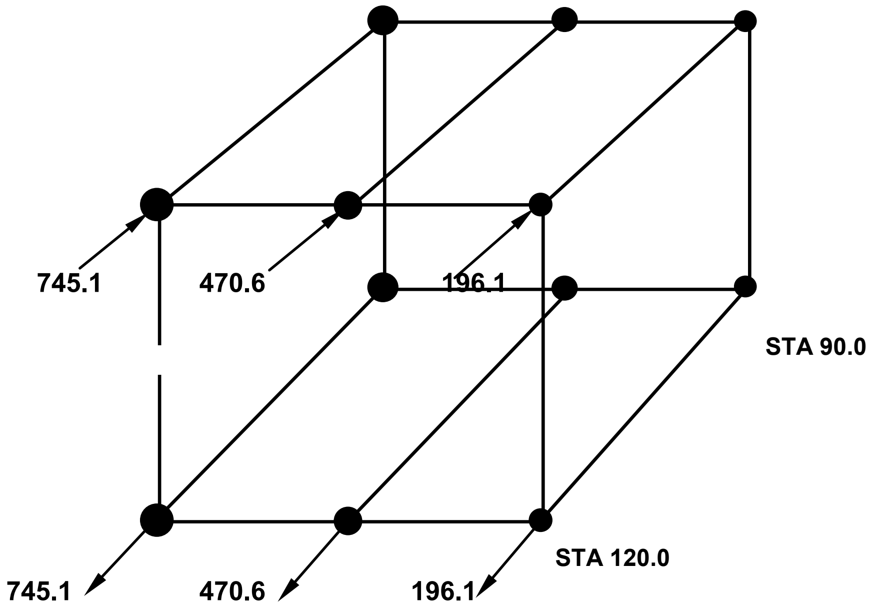
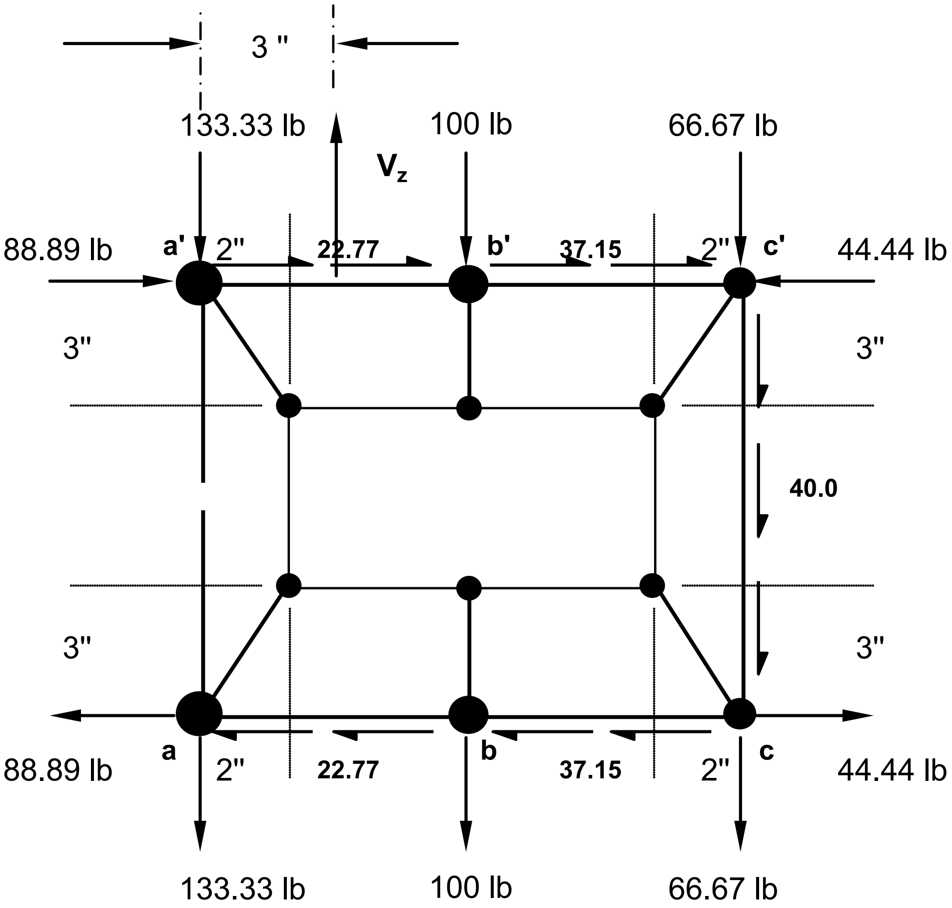


Figure A15-74



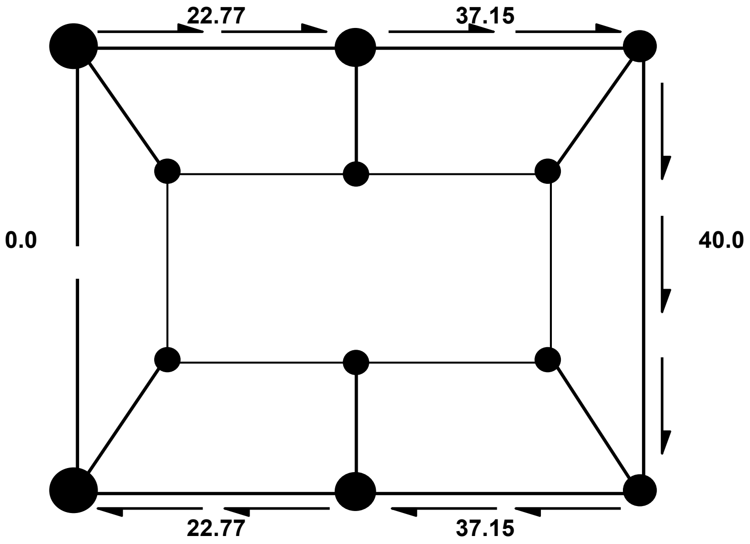


Figure A15-75

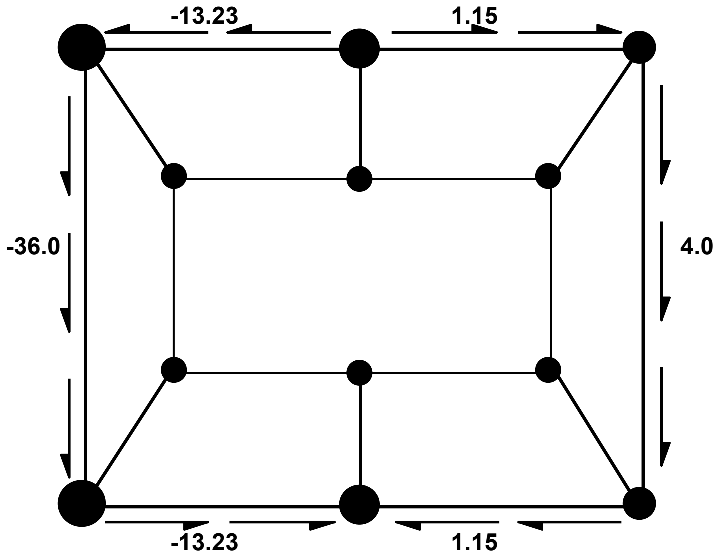
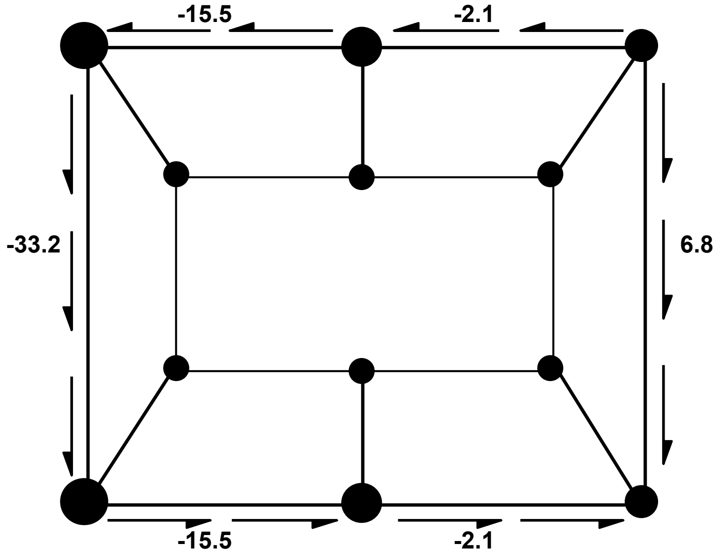


Figure A15-76



Page A16.3 Rectangular Plates Under Various Loadings

First Column

You may wish to add notes that stress = p R / 2 t is the Longitudinal Stress and p R / t is the Hoop Stress.

Thanks to Jim Baldwin.

Page A17.4 Rectangular Plates Under Various Loadings

Table A17.1

For the Roark reference, the pages are 458-476 in the 6th edition.

Thanks to Jim Baldwin.

Page A17.6 Membrane Stress and Deflection Coefficients

TABLE A17.2
Membrane Stress and Deflection Coefficients

a / b	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**4.0**	**5.0**
n_1	0.318	0.228	0.160	0.125	0.100	0.068	0.052
n_2	**0.411**	0.370	0.336	0.304	0.272	0.23	0.205

Coefficient n_2 for a/b = 1.0 should be 0.411 instead of 0.356. Reference NACA TM-965.

NACA TM-965 *Rectangular Shell Plating Under Uniformly Distributed Hydrostatic Pressure*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094451_1993094451.pdf

....M. Heubert and A. Sommer

Thanks to SparWeb on the www.eng-tips.com website.

Page A17.7 Large Deflection in Plates

Figure A17.3 on page A17.3 “a” is the long side

Figure A17.5 on page A17.5 “a” is the short side

Equation (20) is probably incorrect.

$$q = \frac{1}{\alpha} \frac{E \; t^{3}}{a^{4} \left(\frac{b}{a} \right)^{4}} w_{\max} + \frac{1}{n_{1}^{3}} \left(\frac{E \, t}{a^{4}} \right) w_{\max}{}^{3}$$

SparWeb suggests:

$$q = w_{\max} \left(\frac{E \; t^{3}}{a^{4}} \right) + w_{\max}{}^{3} \left(\frac{E \;\; t}{n_{1}^{3} \; b^{4}} \right)$$

Equation (21) is probably incorrect:

$$\frac{q \;\; a^{4}}{E \;\; t^{4}} = \frac{1}{\alpha} \left(\frac{w_{\max}}{t} \right)^{2} \left(\frac{a}{b} \right)^{4} + \frac{1}{n_{1}^{3}} \left(\frac{w_{\max}}{t} \right)^{3}$$

See *Theory of Plates and Shells* by Stephen Timoshenko.

Thanks to SparWeb on the www.eng-tips.com website.

Page A18.1 Combined Bending and Compression of Columns

Column 1, Figure A18.1

Add x ←

Column 2, Equation 4a

$$\left(0 \le z \; 1 - a \right)$$

should be

$$0 \le z \le \left(1 - a \right)$$

Thanks to Dr. Howard W. Smith.

Page A18.5 The Failure of Columns by Compression

Column 1, Just above Equation 10

“Fournier series” *should be* “Fourier series” Thanks to Dr. Howard W. Smith.

Page A18.10 Pure Bending of Thin Plates

Column 2, Figure 2b, Lower M_y

M_y *should be* M_x Thanks to Dr. Howard W. Smith.

Page A18.11 Pure Bending of Thin Plates

Column 2, Equation 4a

Ez *should be* E_z 2 places

Column 2, Equation 4b

σ_z *should be* σ_y

Ez *should be* E_y 2 places

Just above Equation 5a

M_x dy *should be* M_y dy

M_y dx *should be* M_x dx

Just below Equation 5b

$D = \frac{E\ h^2}{12\left(1 - \nu^2\right)}$ *should be* $D = \frac{E\ h^3}{12\left(1 - \nu^2\right)}$ Thanks to Dr. Howard W. Smith.

Page A19.6 Three Flange Single Cell Wing

Column 1

$V_x = 700$ lb *should be* $V_x = -700$ lb Thanks to Dr. Howard W. Smith.

Page A19.7 Three Flange Single Cell Wing

Column 1 Table

y x z *should be* Δy Δx Δz Thanks to Dr. Howard W. Smith.

Page A19.8 Analysis of Wing Structures

Cessna Aircraft Model 180 *should be* Cessna Aircraft Model 182 Thanks to Dr. Howard W. Smith.

Page A19.17 Bending and Shear Stress Analysis – Tapered Multiple Stringer Wing

Column 1, For Station 20 …

$$K_1 = -50 / \left(230.3\ x\ 10.30 - 50^2 \right) = -50 / 235500 = -.0002125$$

should be $K_1 = -50 / \left[230.3 \left(1{,}030 \right) \right] = -50 / 234{,}709 = -.0002130$

$K_2 = -1030 / 235500 = .004378$ *should be* $K_2 = 1{,}030 / 235{,}500 = -.004374$

Column 2,

$$\sigma_b = -\left[\ .00098\ x - 285000 - \left(-.0002125\ x\ 1300000 \right) \right]\ x - \left[\ .004378\ x\ 1{,}300{,}000 - \left(-.0002125\ x\ -285000 \right) \right]\ z$$

should be

$$\sigma_b = -\left[0.000978 \left(-285{,}000 \right) - \left(-.0002130 \right) 1{,}300{,}000 \right]\ x - \left[0.004374 \left(1{,}300{,}000 \right) - \left(-.0002130 \right) \left(-285{,}000 \right) \right]\ z$$

$\sigma_b = 3.3\ x - 5639\ z$ *should be* $\sigma_b = 1.83\ x - 5{,}625.5\ z$

<u>Stresses at Station 47.5</u>

$K_1 = 35.4 / \left(157.4\ x\ 700 - 35.4^2 \right) = -35.4 / 108950 = -.000324$ *should be*

$$K_1 = -35.4 / \left[157.4 \left(700 \right) - 35.4^2 \right] = -35.4 / 108{,}927 = -.000325$$

$K_2 = 700 / 108950 = .00643$ *should be* $K_2 = 700 / 108{,}927 = 0.006426$

$K_3 = 157.4 / 108750 = .001447$ *should be* $K_3 = 157.4 / 108{,}927 = 0.001445$

$$\sigma_b = -\left[\ .001447\ x - 215000 - \left(-.000324\ x\ 1{,}000{,}000 \right) \right]\ x - \left[\ .00643\ x\ 1{,}000{,}000 - \left(-.00324\ x\ -215000 \right) \right]\ z$$

should be

$$\sigma_b = -\left[0.001445 \left(-215{,}000 \right) - \left(-.000325 \right) 1{,}000{,}000 \right]\ x - \left[0.006426 \left(1{,}000{,}000 \right) - \left(-.000325 \right) \left(-215{,}000 \right) \right]\ z$$

$\sigma_b = -14.5\ x - 6360\ z$ *should be* $\sigma_b = -14.3\ x - 6{,}356\ z$

Page A19.19 Analysis for Shear Stresses in Webs and Skin

Column 2 Moment of External Loads about the Center of Gravity of Station 20

$$\sum M_{cg} = 12000\ x\ 33.3 - 2700\ x\ 11.8 - 390000 = 41800\ in.lb.$$

should be $\sum M_{cg} = 12{,}000\ x\ 33.3 - 2{,}700\ x\ 11.8 - 390{,}000 = -22{,}260\ in - lb$

Page A20.5 Fuselage Analysis – Effective Cross Section

Table A20.1

Table A20.1												
Trial No. 1 - Stringers						Trial No. 2 - Stringers						
1	2	2a	3	4	5	6	7	8	9	10	11	12
Stringer No.	Area A	Area A	Arm z'	$A z'$	$A z'^2$	z	σ_b	w	$w t$	A_{eff}	$A_{eff} z$	$A z^2$
2	**0.120**	***0.151***	**24.2**	3.65	88.27	27.55	-29,925	1.009	0.032	0.152	4.196	115.59
4	0.120	***0.151***	**22.0**	3.32	72.95	25.35	-27,535	1.052	0.034	0.154	3.896	98.75
6	0.120	***0.151***	**18.2**	2.74	49.92	21.55	-23,408	1.141	0.037	0.157	3.373	72.69
8	0.120	***0.151***	**13.3**	2.00	26.66	16.65	-18,085	1.298	0.042	0.162	2.690	44.78
10	0.120	***0.151***	**6.9**	1.04	7.18	10.25	-11,134	1.655	0.053	0.173	1.773	18.17
12	0.120	0.120	**0**	0	0	3.35	-3,639	2.894	0.093	0.213	0.712	2.39
13	**0.224**	0.224	**-3.2**	-0.72	2.29	0.15		Skin in Tension		0.224	0.034	0.01
14	0.120	0.120	**-6.9**	-0.83	5.71	-3.55				0.120	-0.426	1.51
15	**0.224**	0.224	**-10.1**	-2.26	22.85	-6.75				0.224	-1.512	10.21
16	0.120	0.120	**-13.3**	-1.60	21.23	-9.95				0.120	-1.194	11.88
17	**0.160**	0.160	**-15.8**	-2.53	39.94	-12.45				0.160	-1.992	24.80
18	0.120	0.120	**-18.2**	-2.18	39.75	-14.85				0.120	-1.782	26.46
19	**0.160**	0.160	**-20.3**	-3.25	65.93	-16.95				0.160	-2.712	45.97
20	0.120	0.120	**-22.0**	-2.64	58.08	-18.65				0.120	-2.238	41.74
21	**0.160**	0.160	**-23.7**	-3.79	89.87	-20.35				0.160	-3.256	66.26
22	0.120	0.120	**-24.2**	-2.90	70.28	-20.85				0.120	-2.502	52.17
23	**0.088**	0.088	**-24.9**	-2.19	54.56	-21.55				0.088	-1.896	40.87
	Σ	**2.49**	Σ	**-12.14**	**715.47**				Σ	**2.626**	**-2.838**	**674.2**

Table A20.2

Trial No. 1 - Buckled Sheets										
1	2	3	4	5	6	7	8	9	10	11
Buckled Sheet	b'	t	r	σ_{cr}	σ_b	K	A_{eff}	Arm z'	$A z'$	$A z'^2$
1	2.262	0.032	11	-8,989	-31,900	0.282	0.020	24.9	0.51	12.65
3	4.024	0.032	11	-8,989	-30,300	0.297	0.038	23.7	0.91	21.46
5	4.024	0.032	24	-4,120	-26,000	0.158	0.020	20.3	0.41	8.41
7	4.024	0.032	38	-2,602	-20,200	0.129	0.017	15.8	0.26	4.14
9	6.024	0.032	38	-2,602	-12,900	0.202	0.039	10.1	0.39	3.97
11	6.512	0.032	38	-2,602	-4,100	0.635	0.132	3.2	0.42	1.35
						Σ	**0.267**	Σ	**2.91**	**51.97**

Trial No. 2 - Buckled Sheets							
1	12	13	14	15	16	17	18
Buckled Sheet	z	σ_b	K	b'	$a = K t b'$	$a z$	$a Z^2$
1	28.25	-30,685	0.293	2.252	0.021	0.60	16.85
3	27.05	-29,382	0.306	3.981	0.039	1.05	28.52
5	23.65	-25,689	0.160	3.911	0.020	0.47	11.23
7	19.15	-20,801	0.125	3.789	0.015	0.29	5.56
9	13.45	-14,609	0.178	5.556	0.032	0.43	5.73
11	6.55	-7,115	0.366	4.930	0.058	0.38	2.48
				Σ	**0.185**	**3.22**	**70.36**

Page A20.6 Fuselage Analysis – Effective Cross Section

Table A20.3

Table A20.3			
1	2	3	4
Element No.	**a**	**z**	**a z**
1	0.021	28.250	0.604
2	0.153	27.367	4.178
3	0.039	27.050	1.068
4	0.154	25.167	3.877
5	0.020	23.650	0.481
6	0.157	21.367	3.354
7	0.015	19.150	0.294
8	0.162	16.467	2.669
9	0.032	13.450	0.431
10	0.174	10.067	1.750
11	0.058	6.550	0.382
12	0.216	3.167	0.684
		Σ	**19.77**

Stress in Stringers		
1	2	3
Stringer Ref.	**z**	**σ_b**
2	27.367	-29,285
4	25.167	-26,931
6	21.367	-22,865
8	16.467	-17,621
10	10.067	-10,773
12	3.167	-3,389
13	-0.033	35
14	-3.733	3,994
15	-6.933	7,419
16	-10.133	10,843
17	-12.633	13,518
18	-15.033	16,086
19	-17.133	18,333
20	-18.833	20,153
21	-20.533	21,972
22	-21.033	22,507
23	-21.733	23,256

From my calculations:

Trial No.	**Σ A**	**Σ A z'**	**z_{bar}**	**I_{NA}**
1	5.513	-18.468	-3.3501	1,473.03
2	5.612	-17.704	-3.2142	1,489.19
3	5.636	-17.517	-3.1811	1,492.59
4	5.636	-17.467	-3.1722	1,494.38
5	5.637	-17.448	-3.1688	1,494.95
6	5.637	-17.442	-3.1677	1,495.13
7	5.637	-17.440	-3.1673	1,495.19
8	5.637	-17.439	-3.1672	1,495.21
9	5.637	-17.439	-3.1672	1,495.22
10	5.637	-17.439	-3.1671	1,495.22

Page A20.8 Fuselage Section, Ultimate Bending Strength

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* pages A20.7-A20.8

Table A20.4, Column 7, Stringer No. 8

Arm Z^1 = 10.0 *should be* 0.0

Column 2, Calculation of Ultimate Resisting Moment

Moment $$M_x = \left(36500 \; x \; 3252 \right) / \; 35.7 + 0.7$$

should be $$M_x = \frac{36{,}500 \left(3{,}252 \right)}{\left(35.7 + 0.7 \right)}$$

Table A20.4

Table A20.4												
1	2	3	4	5	6	7	8	9	10	11	12	13
Stringer Number	Type	**Stringer Area** (in^2)	**Linear Stress** σ_b	**Eff. Skin Area**	**Total Area** (in^2)	**Arm Z^1** (in)	**Strain** ε (in/in)	**True Stress** (psi)	**K**	**K A**	**K A Z^1**	**K A Z^2**
1	S_1	0.135	-36,500	0.033	0.168	35.7	-0.006	-36,500	1.00	0.168	5.99	213.7
2	S_1	0.135	-34,700	0.034	0.169	33.8	-0.0057	-36,500	1.05	0.177	5.99	202.5
3	S_2	0.180	-31,000	0.035	0.215	30.3	-0.0051	-39,100	1.26	0.272	8.23	249.5
4	S_1	0.135	-26,600	0.038	0.173	26.0	-0.0048	-36,000	1.35	0.235	6.10	158.5
5	S_3	0.080	-20,500	0.044	0.124	20.1	-0.0034	-31,500	1.54	0.190	3.82	76.7
6	S_3	0.080	-13,400	0.054	0.134	13.7	-0.0023	-24,000	1.79	0.240	3.29	45.0
7	S_3	0.080	-7,150	0.074	0.154	7.0	-0.0012	-12,500	1.75	0.269	1.88	13.2
8	S_3	0.080	0	0	0.080	0	0	0	1.00	0.080	0	0
9	S_3	0.080	6,130	0.216	0.296	-6.0	0.0010	10,000	1.63	0.483	-2.90	17.4
10	S_3	0.080	12,280	0.216	0.296	-12.0	0.0200	20,500	1.67	0.494	-5.93	71.2
11	S_3	0.080	16,800	0.216	0.296	-16.5	0.0028	30,000	1.79	0.529	-8.72	143.9
12	S_3	0.080	20,400	0.216	0.296	-20.0	0.0034	35,000	1.72	0.508	-10.16	203.1
13	S_3	0.080	21,700	0.216	0.296	-21.2	0.0036	38,000	1.75	0.518	-10.99	233.0
				Σ	2.696			Fig A20.5	Σ	**4.162**	**-3.40**	**1,627.7**

See Figure C11.43 on page C11.38 for additional stringer information.

For stringers in tension 6.75 inches apart, the effective skin area, w t = 0.032 inch (6.75 inch) = 0.216 in^2

For example, the effective skin width for Stringer 1 in compression:

$$w \;=\; 1.9 \;\; t \sqrt{\frac{E}{\sigma_{ST}}} \;=\; 1.9 \;\left(0.032 \; inch \right) \sqrt{\frac{10{,}300{,}000 \; psi}{36{,}500 \; psi}} \;=\; 1.021 \;\; inch$$

Effective Skin Area A = 1.021 inch (0.032 inch) = 0.0327 in^2

Page A20.9 Shear Flow Analysis for Tapered Fuselage – Beam Properties at One Section

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* pages A20.9-A20.10

Table A20.5

Table A20.5

1	2	3	4	5	6	7	8	9
Stringer No.	**Arm, z** Sta 0	**Area A**	σ_b	P_x Sta 0	**dz / dx**	**dy / dx**	P_z	P_y
1	15.00	0.05	-25,000	-1,250	-0.0333	0.0000	-41.67	0.00
2	13.86	0.10	-23,097	-2,310	-0.0308	-0.0128	-71.13	-29.46
3	10.61	0.10	-17,678	-1,768	-0.0236	-0.0236	-41.67	-41.67
4	5.74	0.10	-9,567	-957	-0.0128	-0.0308	-12.20	-29.46
5	0.00	0.10	0	0	0	-0.0333	0	0
6	-5.74	0.10	9,567	957	0.0128	-0.0308	-12.20	29.46
7	-10.61	0.10	17,678	1,768	0.0236	-0.0236	-41.67	41.67
8	-13.86	0.10	23,097	2,310	0.0308	-0.0128	-71.13	29.46
9	-15.00	*0.05*	25,000	1,250	0.0333	0	-41.67	0.00
							-333.33	**0**

Properties

Properties

10	11	12	13	14	15
Arm, y Sta 0	**Arm, y** Sta 150	**Arm, z** Sta 0	**Arm, z** Sta 150	**dz / dx**	**dy / dx**
0.00	0.00	15.00	10.00	-0.0333	0.0000
5.74	3.83	13.86	9.24	-0.0308	-0.0128
10.61	7.07	10.61	7.07	-0.0236	-0.0236
13.86	9.24	5.74	3.83	-0.0128	-0.0308
15.00	10.00	0	0	0	-0.0333
-13.86	-9.24	-5.74	-3.83	0.0128	-0.0308
-10.61	-7.07	-10.61	-7.07	0.0236	-0.0236
-5.74	-3.83	-13.86	-9.24	0.0308	-0.0128
0	0	-15.00	-10.00	0.0333	0

Shear Flow

See Figure A20.7 page A20.10

Stringer No.	q_{flexural}	q_{internal}	q_{total}
1-2	-5.56	-7.07	**-1.52**
2-3	-15.82	-7.07	**8.75**
3-4	-23.68	-7.07	**16.60**
4-5	-27.93	-7.07	**20.86**
5-6	-27.93	-7.07	**20.86**
6-7	-23.68	-7.07	**16.60**
7-8	-15.82	-7.07	**8.75**
8-9	-5.56	-7.07	**-1.52**

Stringer No.	q_{flexural}	q_{internal}	q_{total}
9-10	5.56	-7.07	**-12.63**
10-11	15.82	-7.07	**-22.89**
11-12	23.68	-7.07	**-30.75**
12-13	27.93	-7.07	**-35.00**
13-14	27.93	-7.07	**-35.00**
14-15	23.68	-7.07	**-30.75**
15-16	15.82	-7.07	**-22.89**
16-1	5.56	-7.07	**-12.63**

Page A20.10 Tapered Circular Fuselage with Asymmetrical Stringer Areas – Delta P Method

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* pages A20.10-A20.15

Table A20.6

Table A20.6

1	2	3	4	5	6	7	8	9	10	11	12	13
Stringer No.	**Area** Sta 0	**Area** Sta 30	**Arm, z** Sta 0	**Arm, z** Sta 30	**σ_b** Sta 0	**σ_b** Sta 30	**P_x** Sta 0	**P_x** Sta 30	**ΔP 30**	**K**	**ΔP K 30**	**q**
1	*0.05*	*0.05*	15.00	14	-25,000	-21,429	-1,250	-1,071	5.95	0.935	5.57	5.57
2	0.10	0.10	13.86	12.93	-23,097	-19,797	-2,310	-1,980	11.00	0.935	10.29	
3	0.10	0.10	10.61	9.90	-17,678	-15,152	-1,768	-1,515	8.42	0.935	7.87	15.86
4	0.10	0.10	5.74	5.36	-9,567	-8,200	-957	-820	4.56	0.935	4.26	23.73
5	0.10	0.10	0.00	0.00	0	0	0	0	0	0	0	27.99
6	0.10	0.10	-5.74	-5.36	9,567	8,200	957	820	-4.56	0.935	-4.26	27.99
7	0.10	0.10	-10.61	-9.90	17,678	15,152	1,768	1,515	-8.42	0.935	-7.87	23.73
8	0.10	0.10	-13.86	-12.93	23,097	19,797	2,310	1,980	-11.00	0.935	-10.29	15.86
9	*0.05*	*0.05*	-15.00	-14	25,000	21,429	1,250	1,071	-5.95	0.935	-5.57	5.57

Table A20.7

Table A20.7

Section Properties at Station 0									**Total Stringer Loads at Station 0**				
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Stringer Ref.	**Area A**	**Arm z'**	**Arm y'**	**A z'**	**$A z'^2$**	**A y'**	**$A y'^2$**	**A z' y'**	**z**	**y**	**σ_b**	**σ_c**	**P_s**
a	**0.60**	**10.50**	**-12.00**	6.30	66.15	-7.20	86.40	-75.60	11.352	-14.471	-15,111	-441.2	-9,331.4
b	**0.10**	**18.98**	**-8.48**	1.90	36.02	-0.85	7.19	-16.10	19.832	-10.951	-21,996	-441.2	-2,243.7
c	**0.10**	**22.50**	**0.00**	2.25	50.63	0.00	0.00	0.00	23.352	-2.471	-22,697	-441.2	-2,313.8
d	**0.10**	18.98	8.48	1.90	36.02	0.85	7.19	16.10	19.832	6.009	-16,785	-441.2	-1,722.6
e	**0.80**	10.50	12.00	8.40	88.20	9.60	115.20	100.80	11.352	9.529	-7,736	-441.2	-6,542.0
f	**0.80**	-10.50	12.00	-8.40	88.20	9.60	115.20	-100.80	-9.648	9.529	11,992	-441.2	9,241.0
g	**0.20**	-18.98	8.48	-3.80	72.05	1.70	14.38	-32.19	-18.128	6.009	18,877	-441.2	3,687.2
h	**0.20**	-22.50	0.00	-4.50	101.25	0.00	0.00	0.00	-21.648	-2.471	19,578	-441.2	3,827.5
I	**0.20**	-18.98	-8.48	-3.80	72.05	-1.70	14.38	32.19	-18.128	-10.951	13,666	-441.2	2,644.9
j	**0.30**	-10.50	-12.00	-3.15	33.08	-3.60	43.20	37.80	-9.648	-14.471	4,618	-441.2	1,252.9
	3.40			**-2.90**	**643.64**	**8.40**	**403.15**	**-37.80**					**-1,500.0**

Table A20.8

Table A20.8

Section Properties at Station 30									**Total Stringer Loads at Station 30**				
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Stringer Ref.	**Area A**	**Arm z'**	**Arm y'**	**A z'**	**$A z'^2$**	**A y'**	**$A y'^2$**	**A z' y'**	**z**	**y**	**σ_b**	**σ_c**	**P_s**
a	**0.50**	**9.80**	**-11.20**	4.90	48.02	-5.60	62.72	-54.88	10.905	-13.305	-14,840	-503.4	-7,671.8
b	**0.10**	**17.72**	**-7.92**	1.77	31.40	-0.79	6.27	-14.03	18.825	-10.025	-21,146	-503.4	-2,164.9
c	**0.10**	**21.00**	**0.00**	2.10	44.10	0.00	0.00	0.00	22.105	-2.105	-21,490	-503.4	-2,199.3
d	**0.10**	17.72	7.92	1.77	31.40	0.79	6.27	14.03	18.825	5.815	-15,672	-503.4	-1,617.6
e	**0.66**	9.80	11.20	6.47	63.39	7.39	82.79	72.44	10.905	9.095	-7,100	-503.4	-5,018.0
f	**0.66**	-9.80	11.20	-6.47	63.39	7.39	82.79	-72.44	-8.695	9.095	11,310	-503.4	7,132.7
g	**0.20**	-17.72	7.92	-3.54	62.80	1.58	12.55	-28.07	-16.615	5.815	17,616	-503.4	3,422.6
h	**0.20**	-21.00	0.00	-4.20	88.20	0.00	0.00	0.00	-19.895	-2.105	17,960	-503.4	3,491.4
I	**0.20**	-17.72	-7.92	-3.54	62.80	-1.58	12.55	28.07	-16.615	-10.025	12,142	-503.4	2,327.8
j	**0.26**	-9.80	-11.20	-2.55	24.97	-2.91	32.61	28.54	-8.695	-13.305	3,570	-503.4	797.3
	2.98			**-3.29**	**520.46**	**6.27**	**298.55**	**-26.34**					**-1,500.0**

Table A20.9

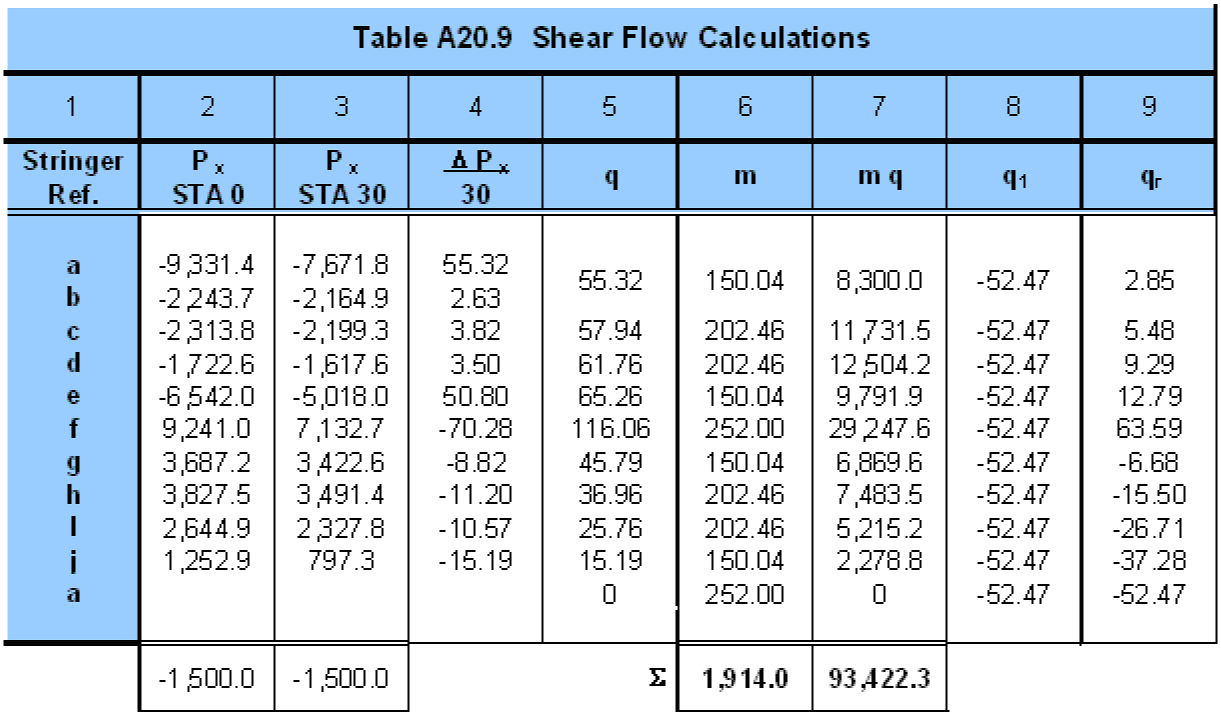


Table A20.9 Shear Flow Calculations

1	2	3	4	5	6	7	8	9
Stringer Ref.	**P_x STA 0**	**P_x STA 30**	**$\frac{\Delta P_x}{30}$**	**q**	**m**	**m q**	**q_1**	**q_r**
a	-9,331.4	-7,671.8	55.32					
b	-2,243.7	-2,164.9	2.63	55.32	150.04	8,300.0	-52.47	2.85
c	-2,313.8	-2,199.3	3.82	57.94	202.46	11,731.5	-52.47	5.48
d	-1,722.6	-1,617.6	3.50	61.76	202.46	12,504.2	-52.47	9.29
e	-6,542.0	-5,018.0	50.80	65.26	150.04	9,791.9	-52.47	12.79
f	9,241.0	7,132.7	-70.28	116.06	252.00	29,247.6	-52.47	63.59
g	3,687.2	3,422.6	-8.82	45.79	150.04	6,869.6	-52.47	-6.68
h	3,827.5	3,491.4	-11.20	36.96	202.46	7,483.5	-52.47	-15.50
I	2,644.9	2,327.8	-10.57	25.76	202.46	5,215.2	-52.47	-26.71
j	1,252.9	797.3	-15.19	15.19	150.04	2,278.8	-52.47	-37.28
a				0	252.00	0	-52.47	-52.47
	-1,500.0	-1,500.0		Σ	**1,914.0**	**93,422.3**		

Table A20.10

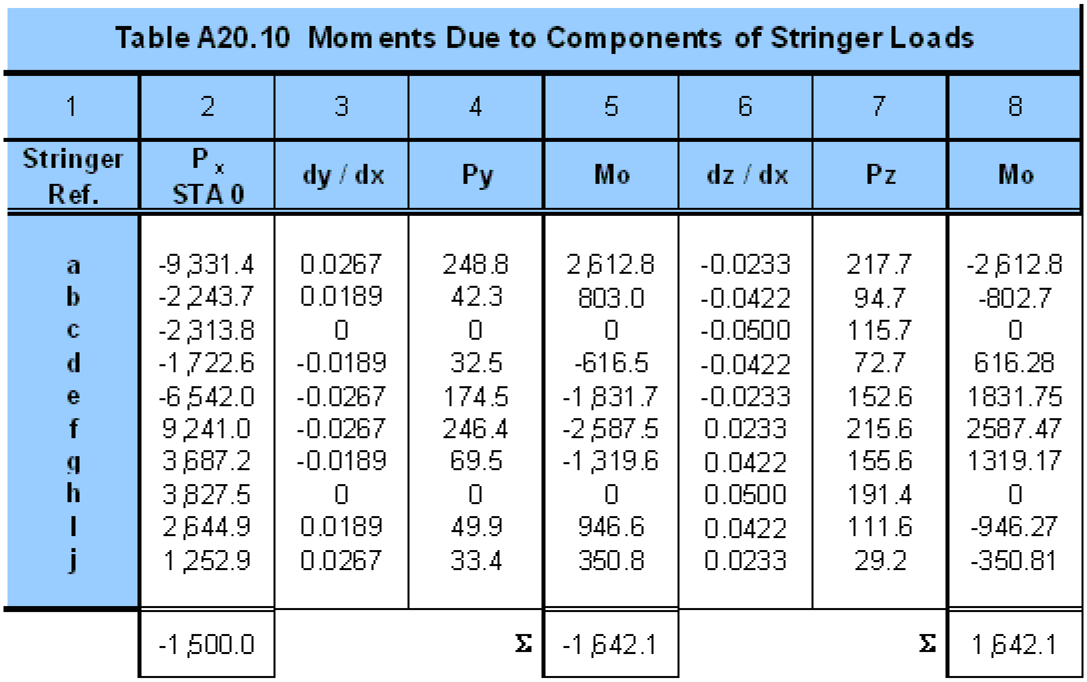


Table A20.10 Moments Due to Components of Stringer Loads

1	2	3	4	5	6	7	8
Stringer Ref.	**P_x STA 0**	**dy / dx**	**Py**	**Mo**	**dz / dx**	**Pz**	**Mo**
a	-9,331.4	0.0267	248.8	2,612.8	-0.0233	217.7	-2,612.8
b	-2,243.7	0.0189	42.3	803.0	-0.0422	94.7	-802.7
c	-2,313.8	0	0	0	-0.0500	115.7	0
d	-1,722.6	-0.0189	32.5	-616.5	-0.0422	72.7	616.28
e	-6,542.0	-0.0267	174.5	-1,831.7	-0.0233	152.6	1831.75
f	9,241.0	-0.0267	246.4	-2,587.5	0.0233	215.6	2587.47
g	3,687.2	-0.0189	69.5	-1,319.6	0.0422	155.6	1319.17
h	3,827.5	0	0	0	0.0500	191.4	0
I	2,644.9	0.0189	49.9	946.6	0.0422	111.6	-946.27
j	1,252.9	0.0267	33.4	350.8	0.0233	29.2	-350.81
	-1,500.0		Σ	-1,642.1		Σ	1,642.1

Comparison

Beam Properties at One Section

Stringer No.	$q_{flexural}$	$q_{internal}$	q_{total}
1-2	-5.56	-7.07	-1.52
2-3	-15.82	-7.07	8.75
3-4	-23.68	-7.07	16.60
4-5	-27.93	-7.07	20.86
5-6	-27.93	-7.07	20.86
6-7	-23.68	-7.07	16.60
7-8	-15.82	-7.07	8.75
8-9	-5.56	-7.07	-1.52
9-10	5.56	-7.07	-12.63
10-11	15.82	-7.07	-22.89
11-12	23.68	-7.07	-30.75
12-13	27.93	-7.07	-35.00
13-14	27.93	-7.07	-35.00
14-15	23.68	-7.07	-30.75
15-16	15.82	-7.07	-22.89
16-1	5.56	-7.07	-12.63

Delta P Method

Stringer No.	$q_{flexural}$	$q_{internal}$	q_{total}
1-2	5.57	-7.07	-1.51
2-3	15.86	-7.07	8.78
3-4	23.73	-7.07	16.66
4-5	27.99	-7.07	20.92
5-6	27.99	-7.07	20.92
6-7	23.73	-7.07	16.66
7-8	15.86	-7.07	8.78
8-9	5.57	-7.07	-1.51
9-10	5.57	-7.07	-12.64
10-11	15.86	-7.07	-22.93
11-12	23.73	-7.07	-30.80
12-13	27.99	-7.07	-35.06
13-14	27.99	-7.07	-35.06
14-15	23.73	-7.07	-30.80
15-16	15.86	-7.07	-22.93
16-1	5.57	-7.07	-12.64

Difference

Stringer No.	q_{total}
1-2	0.01
2-3	0.03
3-4	0.05
4-5	0.06
5-6	0.06
6-7	0.05
7-8	0.03
8-9	0.01
9-10	-0.01
10-11	-0.03
11-12	-0.05
12-13	-0.06
13-14	-0.06
14-15	-0.05
15-16	-0.03
16-1	-0.01

Page A22.7 Shear Lag Analysis of Box Beams

$$\left[\alpha_{rs}^{-1} \right] = 10^{-5} \begin{bmatrix} 3.285 & -1.505 & -0.656 & -0.183 \\ -1.505 & 4.219 & -1.000 & -0.279 \\ -0.656 & -1.000 & 4.687 & -0.495 \\ -0.183 & -0.279 & -0.495 & 5.519 \end{bmatrix}$$

should be

$$\left[\alpha_{rs}^{-1} \right] = 10^{-5} \begin{bmatrix} 3.286 & -1.506 & -0.656 & -0.183 \\ -1.506 & 4.219 & -1.000 & -0.279 \\ -0.656 & -1.000 & 4.686 & -0.494 \\ -0.183 & -0.279 & -0.494 & 5.518 \end{bmatrix}$$

$$[\,G_{im}\,] = [\,g_{im}\,] - \underbrace{[\,g_{ir}\,]\left[\alpha_{rs}^{-1}\right][\,\alpha_{rs}\,]}$$

$$[\,G_{im}\,] = \left\{ \begin{matrix} 0.07692 \\ 0 \\ -0.92300 \\ 0.92300 \\ 0 \\ 0.07692 \\ 0 \\ -1.84610 \\ 1.84610 \\ 0 \\ 0.07692 \\ 0 \\ -2.76900 \\ 2.76900 \\ 0 \\ 0.07692 \\ 0 \\ -3.69200 \\ 3.692 \\ 0 \end{matrix} \right\} - \left\{ \begin{matrix} 0 \\ -.03453 \\ 0 \\ 0.41439 \\ -0.41439 \\ 0 \\ -0.03134 \\ 0 \\ 0.79046 \\ -0.79046 \\ 0 \\ -0.02402 \\ 0 \\ 1.07874 \\ -1.07874 \\ 0 \\ -0.01004 \\ 0 \\ 1.19928 \\ -1.19928 \end{matrix} \right\} = \left\{ \begin{matrix} 0.07692 \\ 0.03453 \\ -0.92300 \\ 0.50861 \\ 0.41439 \\ 0.07692 \\ 0.03134 \\ -1.84610 \\ 1.05564 \\ 0.79046 \\ 0.07692 \\ 0.02402 \\ -2.76900 \\ 1.69026 \\ 1.07874 \\ 0.07692 \\ 0.01004 \\ -3.69200 \\ 2.49272 \\ 1.19928 \end{matrix} \right\}$$

Page A22.13 Stresses in Inner Bays

$$\begin{Bmatrix} q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{19} \\ q_{21} \\ q_{22} \end{Bmatrix} = \begin{bmatrix} .1006 & .0295 & .00676 & -.00676 \\ .0791 & -.0208 & -.01753 & -.0095 \\ .0013 & -.0411 & .00401 & .0230 \\ .0422 & .1135 & -.00675 & .00675 \\ .2409 & .2409 & .00676 & -.00676 \\ 3.087 & .964 & -.9355 & -.3444 \\ -2.379 & -1.473 & -.2284 & .3625 \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ q_{12}" \\ q_{14}" \end{Bmatrix}$$

should be

$$\begin{Bmatrix} q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{19} \\ q_{21} \\ q_{22} \end{Bmatrix} = \begin{bmatrix} 0.10060 & 0.02948 & 0.00676 & -0.00676 \\ 0.07911 & -0.02084 & -0.01753 & -0.00949 \\ 0.00126 & -0.04108 & 0.00402 & 0.02301 \\ 0.04220 & 0.11342 & -0.00676 & 0.00676 \\ 0.24090 & 0.24058 & 0.00676 & -0.00676 \\ 3.08824 & 0.96255 & -0.93541 & -0.34450 \\ -2.37776 & -1.47345 & -0.22841 & 0.36250 \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ q_{12}" \\ q_{14}" \end{Bmatrix}$$

Page A23.11 Single Bay Pinned Truss

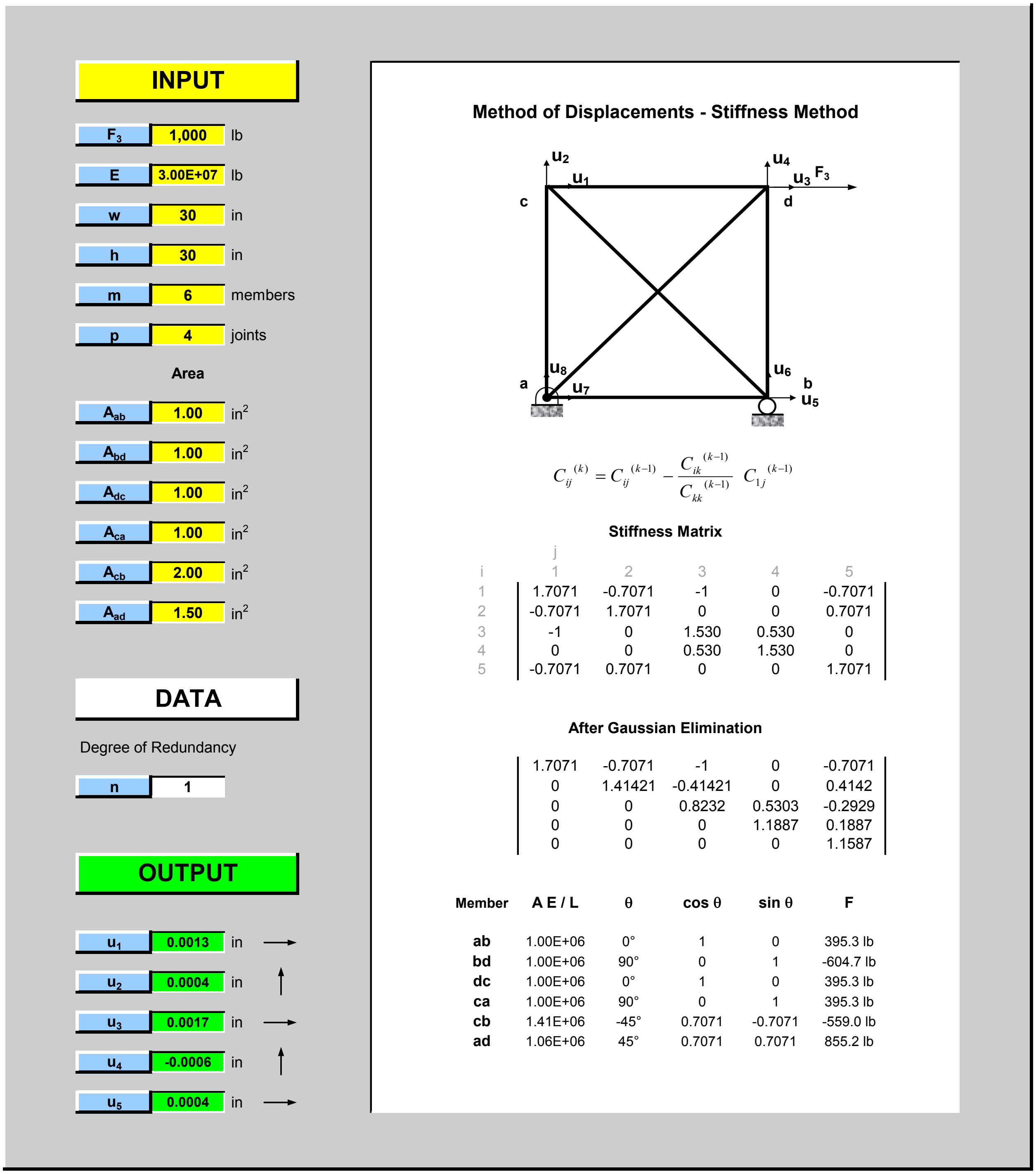
Table A23.3, AE/L = 1.607×10^6 should be 1.06066×10^6 if the areas are to be the same as Problem #1 on page A8.7 for E = 30E6 psi. I use cross-sectional areas, $A_{ab} = 1.0$ in^2 , $A_{bd} = 1.0$ in^2 , $A_{dc} = 1.0$ in^2 , $A_{ca} = 1.0$ in^2 , $A_{cb} = 2.0$ in^2 and $A_{ad} = 1.5$ in^2 .

For E = 10E6 psi. use cross-sectional areas, $A_{ab} = 3.0$ in^2 , $A_{bd} = 3.0$ in^2 , $A_{dc} = 3.0$ in^2 , $A_{ca} = 3.0$ in^2 , $A_{cb} = 6.0$ in^2 and $A_{ad} = 4.5$ in^2 . These will yield the same A E / L.

Method of Displacements – Stiffness Method

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures*, pages A23.11 - A23.12

Gaussian Elimination



INPUT

F_3	1,000	lb
E	3.00E+07	lb
w	30	in
h	30	in
m	6	members
p	4	joints

Area

A_{ab}	1.00	in^2
A_{bd}	1.00	in^2
A_{dc}	1.00	in^2
A_{ca}	1.00	in^2
A_{cb}	2.00	in^2
A_{ad}	1.50	in^2

DATA

Degree of Redundancy

n	1

OUTPUT

u_1	0.0013	in	→
u_2	0.0004	in	↑
u_3	0.0017	in	→
u_4	-0.0006	in	↑
u_5	0.0004	in	→

Method of Displacements - Stiffness Method

$$C_{ij}^{(k)} = C_{ij}^{(k-1)} - \frac{C_{ik}^{(k-1)}}{C_{kk}^{(k-1)}} C_{1j}^{(k-1)}$$

Stiffness Matrix

i \ j	1	2	3	4	5
1	1.7071	-0.7071	-1	0	-0.7071
2	-0.7071	1.7071	0	0	0.7071
3	-1	0	1.530	0.530	0
4	0	0	0.530	1.530	0
5	-0.7071	0.7071	0	0	1.7071

After Gaussian Elimination

1.7071	-0.7071	-1	0	-0.7071
0	1.41421	-0.41421	0	0.4142
0	0	0.8232	0.5303	-0.2929
0	0	0	1.1887	0.1887
0	0	0	0	1.1587

Member	A E / L	θ	cos θ	sin θ	F
ab	1.00E+06	0°	1	0	395.3 lb
bd	1.00E+06	90°	0	1	-604.7 lb
dc	1.00E+06	0°	1	0	395.3 lb
ca	1.00E+06	90°	0	1	395.3 lb
cb	1.41E+06	-45°	0.7071	-0.7071	-559.0 lb
ad	1.06E+06	45°	0.7071	0.7071	855.2 lb

Page 2

$$C_{ij}^{(k)} = C_{ij}^{(k-1)} - \frac{C_{ik}^{(k-1)}}{C_{kk}^{(k-1)}}\; C_{1j}^{(k-1)}$$

Stiffness Matrix

$$\begin{bmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ -0.70711 & 1.70711 & 0 & 0 & 0.70711 \\ -1 & 0 & 1.5303 & 0.5303 & 0 \\ 0 & 0 & 0.5303 & 1.5303 & 0 \\ -0.70711 & 0.70711 & 0 & 0 & 1.70711 \end{bmatrix}$$

k = 1

After Step 1

$$\begin{array}{c|ccccc} i & & & & & \\ 1 & C_{11}^{(1)} & C_{12}^{(1)} & C_{13}^{(1)} & C_{14}^{(1)} & C_{15}^{(1)} \\ \hline 2 & 0 & 1.414215 & -0.41421 & 0 & 0.414215 \\ 3 & 0 & -0.41421 & 0.944515 & 0.5303 & -0.41421 \\ 4 & 0 & 0 & 0.5303 & 1.5303 & 0 \\ 5 & 0 & 0.414215 & -0.41421 & 0 & 1.414215 \end{array} = \begin{bmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ 0 & 1.414215 & -0.41421 & 0 & 0.414215 \\ 0 & -0.41421 & 0.944515 & 0.5303 & -0.41421 \\ 0 & 0 & 0.5303 & 1.5303 & 0 \\ 0 & 0.414215 & -0.41421 & 0 & 1.414215 \end{bmatrix}$$

k = 2

After Step 2

$$\begin{array}{c|ccccc} i & & & & & \\ 1 & C_{11}^{(2)} & C_{12}^{(2)} & C_{13}^{(2)} & C_{14}^{(2)} & C_{15}^{(2)} \\ 2 & C_{21}^{(2)} & C_{22}^{(2)} & C_{23}^{(2)} & C_{24}^{(2)} & C_{25}^{(2)} \\ \hline 3 & 0 & 0 & 0.823194 & 0.5303 & -0.29289 \\ 4 & 0 & 0 & 0.5303 & 1.5303 & 0 \\ 5 & 0 & 0 & -0.29289 & 0 & 1.292894 \end{array} = \begin{bmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ 0 & 1.414215 & -0.41421 & 0 & 0.414215 \\ 0 & 0 & 0.82319 & 0.53030 & -0.29289 \\ 0 & 0 & 0.53030 & 1.53030 & 0 \\ 0 & 0 & -0.29289 & 0 & 1.292894 \end{bmatrix}$$

k = 3

After Step 3

$$\begin{array}{c|ccccc} i & & & & & \\ 1 & C_{11}^{(3)} & C_{12}^{(3)} & C_{13}^{(3)} & C_{14}^{(3)} & C_{15}^{(3)} \\ 2 & C_{21}^{(3)} & C_{22}^{(3)} & C_{23}^{(3)} & C_{24}^{(3)} & C_{25}^{(3)} \\ 3 & C_{31}^{(3)} & C_{32}^{(3)} & C_{33}^{(3)} & C_{34}^{(3)} & C_{35}^{(3)} \\ \hline 4 & 0 & 0 & 0 & 1.188682 & 0.188682 \\ 5 & 0 & 0 & 0 & 0.188682 & 1.188682 \end{array} = \begin{bmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ 0 & 1.41421 & -0.41421 & 0 & 0.41421 \\ 0 & 0 & 0.82319 & 0.53030 & -0.29289 \\ 0 & 0 & 0 & 1.18868 & 0.18868 \\ 0 & 0 & 0 & 0.18868 & 1.18868 \end{bmatrix}$$

k = 4

After Step 4

$$\begin{array}{c|ccccc} i & & & & & \\ 1 & C_{11}^{(4)} & C_{12}^{(4)} & C_{13}^{(4)} & C_{14}^{(4)} & C_{15}^{(4)} \\ 2 & C_{21}^{(4)} & C_{22}^{(4)} & C_{23}^{(4)} & C_{24}^{(4)} & C_{25}^{(4)} \\ 3 & C_{31}^{(4)} & C_{32}^{(4)} & C_{33}^{(4)} & C_{34}^{(4)} & C_{35}^{(4)} \\ 4 & C_{41}^{(4)} & C_{42}^{(4)} & C_{43}^{(4)} & C_{44}^{(4)} & C_{45}^{(4)} \\ \hline 5 & 0 & 0 & 0 & 0 & 1.158732 \end{array} = \begin{bmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ 0 & 1.41421 & -0.41421 & 0 & 0.41421 \\ 0 & 0 & 0.82319 & 0.53030 & -0.29289 \\ 0 & 0 & 0 & 1.18868 & 0.18868 \\ 0 & 0 & 0 & 0 & 1.15873 \end{bmatrix}$$

Stiffness Matrix — Displacements — Forces

$$\begin{bmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ 0 & 1.414215 & -0.41421 & 0 & 0.414215 \\ 0 & 0 & 0.823194 & 0.5303 & -0.29289 \\ 0 & 0 & 0 & 1.188682 & 0.188682 \\ 0 & 0 & 0 & 0 & 1.158732 \end{bmatrix} \quad \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{Bmatrix}$$

Page 3

$$
\begin{array}{ccccc}
C_{16}^{(1)} & C_{17}^{(1)} & C_{18}^{(1)} & C_{19}^{(1)} & C_{10}^{(1)} \\
\hline
0.41421 & 1 & 0 & 0 & 0 \\
0.58579 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0.414215 & 0 & 0 & 0 & 1
\end{array}
=
\begin{array}{ccccc}
F_1 & F_2 & F_3 & F_4 & F_5 \\
\hline
1 & 0 & 0 & 0 & 0 \\
0.41421 & 1 & 0 & 0 & 0 \\
0.58579 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0.41421 & 0 & 0 & 0 & 1
\end{array}
\qquad \text{5a}
$$

$$
\begin{array}{ccccc}
C_{16}^{(2)} & C_{17}^{(2)} & C_{18}^{(2)} & C_{19}^{(2)} & C_{10}^{(2)} \\
C_{26}^{(2)} & C_{27}^{(2)} & C_{28}^{(2)} & C_{29}^{(2)} & C_{20}^{(2)} \\
\hline
0.70711 & 0.29289 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0.29289 & -0.29289 & 0 & 0 & 1
\end{array}
=
\begin{array}{ccccc}
F_1 & F_2 & F_3 & F_4 & F_5 \\
\hline
1 & 0 & 0 & 0 & 0 \\
0.41421 & 1 & 0 & 0 & 0 \\
0.70711 & 0.29289 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0.29289 & -0.29289 & 0 & 0 & 1
\end{array}
\qquad \text{5b}
$$

$$
\begin{array}{ccccc}
C_{16}^{(3)} & C_{17}^{(3)} & C_{18}^{(3)} & C_{19}^{(3)} & C_{10}^{(3)} \\
C_{26}^{(3)} & C_{27}^{(3)} & C_{28}^{(3)} & C_{29}^{(3)} & C_{20}^{(3)} \\
C_{36}^{(3)} & C_{37}^{(3)} & C_{38}^{(3)} & C_{39}^{(3)} & C_{30}^{(3)} \\
\hline
-0.45552 & -0.18868 & -0.64420 & 1 & 0 \\
0.54448 & -0.18868 & 0.35580 & 0 & 1
\end{array}
=
\begin{array}{ccccc}
F_1 & F_2 & F_3 & F_4 & F_5 \\
\hline
1 & 0 & 0 & 0 & 0 \\
0.41421 & 1 & 0 & 0 & 0 \\
0.70711 & 0.29289 & 1 & 0 & 0 \\
-0.45552 & -0.18868 & -0.64420 & 1 & 0 \\
0.54448 & -0.18868 & 0.35580 & 0 & 1
\end{array}
\qquad \text{5c}
$$

$$
\begin{array}{ccccc}
C_{16}^{(4)} & C_{17}^{(4)} & C_{18}^{(4)} & C_{19}^{(4)} & C_{10}^{(4)} \\
C_{26}^{(4)} & C_{27}^{(4)} & C_{28}^{(4)} & C_{29}^{(4)} & C_{20}^{(4)} \\
C_{36}^{(4)} & C_{37}^{(4)} & C_{38}^{(4)} & C_{39}^{(4)} & C_{30}^{(4)} \\
C_{46}^{(4)} & C_{47}^{(4)} & C_{48}^{(4)} & C_{49}^{(4)} & C_{40}^{(4)} \\
\hline
0.61679 & -0.15873 & 0.45806 & -0.15873 & 1
\end{array}
=
\begin{array}{ccccc}
F_1 & F_2 & F_3 & F_4 & F_5 \\
\hline
1 & 0 & 0 & 0 & 0 \\
0.41421 & 1 & 0 & 0 & 0 \\
0.70711 & 0.29289 & 1 & 0 & 0 \\
-0.45552 & -0.18868 & -0.64420 & 1 & 0 \\
0.61679 & -0.15873 & 0.45806 & -0.15873 & 1
\end{array}
\qquad \text{5d}
$$

Stiffness Matrix

$$
\begin{array}{ccccc}
\hline
1.70711 & -0.70711 & -1 & 0 & -0.70711 \\
0 & 1.414215 & -0.41421 & 0 & 0.414215 \\
0 & 0 & 0.823194 & 0.5303 & -0.29289 \\
0 & 0 & 0 & 1.188682 & 0.188682 \\
0 & 0 & 0 & 0 & 1.158732
\end{array}
\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}
=
\begin{array}{ccccc}
F_1 & F_2 & \mathbf{F_3} & F_4 & F_5 \\
\hline
1 & 0 & 0 & 0 & 0 \\
0.41421 & 1 & 0 & 0 & 0 \\
0.70711 & 0.29289 & 1 & 0 & 0 \\
-0.45552 & -0.18868 & -0.64420 & 1 & 0 \\
0.61679 & -0.15873 & 0.45806 & -0.15873 & 1
\end{array}
$$

$$
\begin{array}{cccccccccccc}
 & & & & & & & & 1.158732 & \mathbf{u_5} & = & 0.45806 \quad F_3 \\
 & & & & & & 1.188682 & \mathbf{u_4} & 0.188682 & \mathbf{u_5} & = & -0.64420 \quad F_3 \\
 & & & & 0.823194 & \mathbf{u_3} & 0.5303 & \mathbf{u_4} & -0.29289 & \mathbf{u_5} & = & 1 \quad F_3 \\
 & & 1.414215 & \mathbf{u_2} & -0.41421 & \mathbf{u_3} & 0 & \mathbf{u_4} & 0.414215 & \mathbf{u_5} & = & 0 \quad F_3 \\
1.70711 & \mathbf{u_1} & -0.70711 & \mathbf{u_2} & -1 & \mathbf{u_3} & 0 & \mathbf{u_4} & -0.70711 & \mathbf{u_5} & = & 0 \quad F_3
\end{array}
$$

Inverse of the Stiffness Matrix

$$
\begin{vmatrix}
0.58579 & 0.29289 & 0.85898 & -0.38321 & 0.53230 \\
0 & 0.70711 & 0.35580 & -0.15873 & -0.13699 \\
0 & 0 & 1.21478 & -0.54194 & 0.39531 \\
0 & 0 & 0 & 0.84127 & -0.13699 \\
0 & 0 & 0 & 0 & 0.86301
\end{vmatrix}
$$

Displacements

$$
\begin{vmatrix}
1.8174 & 0.5323 & \mathbf{1.3497} & -0.4677 & 0.5323 \\
0.5323 & 0.8630 & \mathbf{0.3953} & -0.1370 & -0.1370 \\
1.3497 & 0.3953 & \mathbf{1.7450} & -0.6047 & 0.3953 \\
-0.4677 & -0.1370 & \mathbf{-0.6047} & 0.8630 & -0.1370 \\
0.5323 & -0.1370 & \mathbf{0.3953} & -0.1370 & 0.8630
\end{vmatrix}
$$

F_3 is the only applied load

Displacements

$\mathbf{u_5} = \boxed{\mathbf{0.3953}} \times 10^{-6} \ (F_3)$

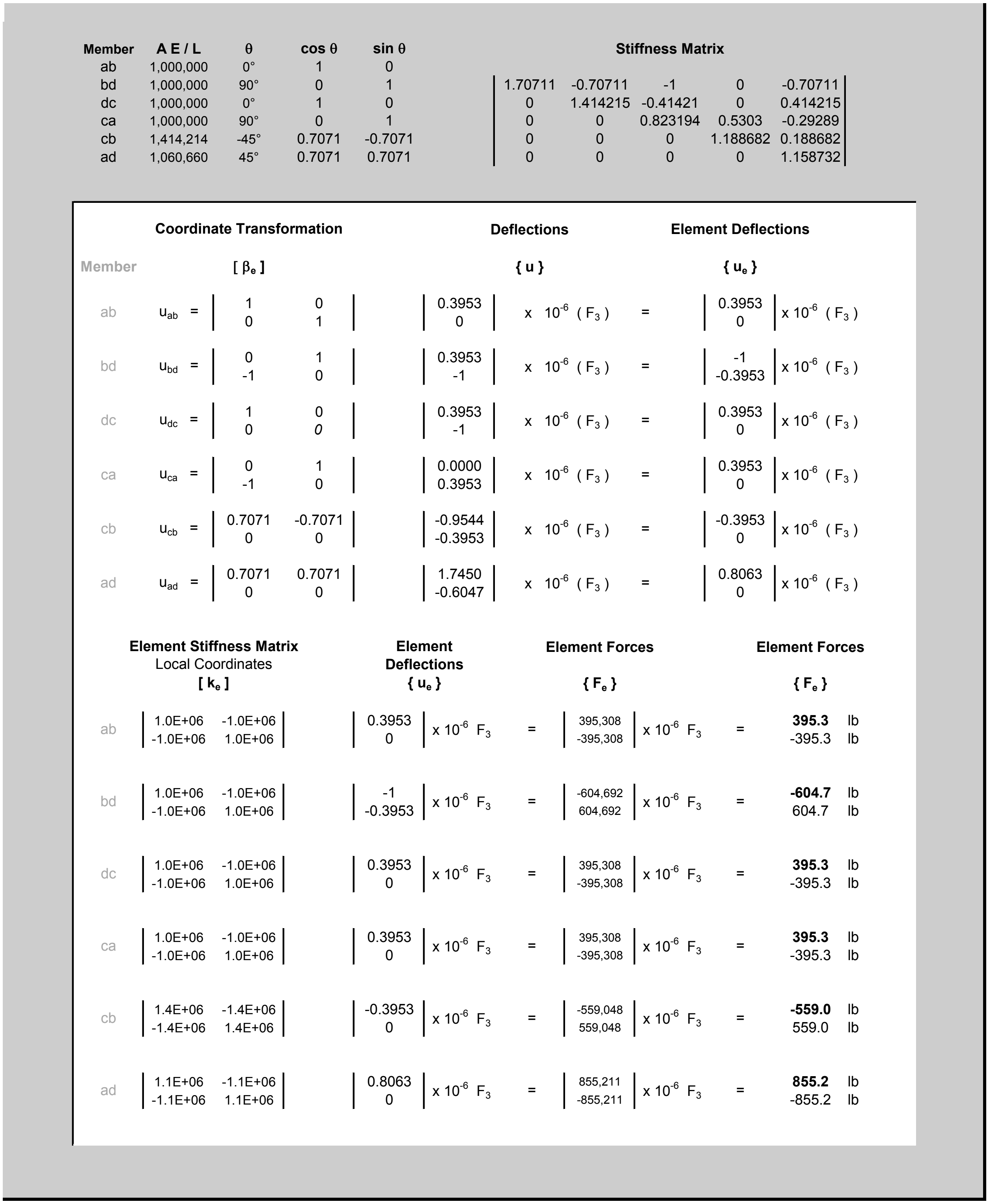
$\mathbf{u_4} = \boxed{\mathbf{-0.6047}} \times 10^{-6} \ (F_3)$

$\mathbf{u_3} = \boxed{\mathbf{1.7450}} \times 10^{-6} \ (F_3)$

$\mathbf{u_2} = \boxed{\mathbf{0.3953}} \times 10^{-6} \ (F_3)$

$\mathbf{u_1} = \boxed{\mathbf{1.3497}} \times 10^{-6} \ (F_3)$

Page 4



Member	A E / L	θ	cos θ	sin θ
ab	1,000,000	0°	1	0
bd	1,000,000	90°	0	1
dc	1,000,000	0°	1	0
ca	1,000,000	90°	0	1
cb	1,414,214	-45°	0.7071	-0.7071
ad	1,060,660	45°	0.7071	0.7071

Stiffness Matrix

$$\begin{vmatrix} 1.70711 & -0.70711 & -1 & 0 & -0.70711 \\ 0 & 1.414215 & -0.41421 & 0 & 0.414215 \\ 0 & 0 & 0.823194 & 0.5303 & -0.29289 \\ 0 & 0 & 0 & 1.188682 & 0.188682 \\ 0 & 0 & 0 & 0 & 1.158732 \end{vmatrix}$$

Coordinate Transformation — **Deflections** — **Element Deflections**

Member — $[\,\beta_e\,]$ — $\{\,u\,\}$ — $\{\,u_e\,\}$

ab
$$u_{ab} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ (F_3) = \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ (F_3)$$

bd
$$u_{bd} = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} 0.3953 \\ -1 \end{vmatrix} \times 10^{-6}\ (F_3) = \begin{vmatrix} -1 \\ -0.3953 \end{vmatrix} \times 10^{-6}\ (F_3)$$

dc
$$u_{dc} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} 0.3953 \\ -1 \end{vmatrix} \times 10^{-6}\ (F_3) = \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ (F_3)$$

ca
$$u_{ca} = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} 0.0000 \\ 0.3953 \end{vmatrix} \times 10^{-6}\ (F_3) = \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ (F_3)$$

cb
$$u_{cb} = \begin{vmatrix} 0.7071 & -0.7071 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} -0.9544 \\ -0.3953 \end{vmatrix} \times 10^{-6}\ (F_3) = \begin{vmatrix} -0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ (F_3)$$

ad
$$u_{ad} = \begin{vmatrix} 0.7071 & 0.7071 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} 1.7450 \\ -0.6047 \end{vmatrix} \times 10^{-6}\ (F_3) = \begin{vmatrix} 0.8063 \\ 0 \end{vmatrix} \times 10^{-6}\ (F_3)$$

Element Stiffness Matrix
Local Coordinates
$[\,k_e\,]$

Element Deflections
$\{\,u_e\,\}$

Element Forces
$\{\,F_e\,\}$

Element Forces
$\{\,F_e\,\}$

ab
$$\begin{vmatrix} 1.0E{+}06 & -1.0E{+}06 \\ -1.0E{+}06 & 1.0E{+}06 \end{vmatrix} \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{vmatrix} 395{,}308 \\ -395{,}308 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{matrix} \mathbf{395.3} & \text{lb} \\ -395.3 & \text{lb} \end{matrix}$$

bd
$$\begin{vmatrix} 1.0E{+}06 & -1.0E{+}06 \\ -1.0E{+}06 & 1.0E{+}06 \end{vmatrix} \begin{vmatrix} -1 \\ -0.3953 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{vmatrix} -604{,}692 \\ 604{,}692 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{matrix} \mathbf{-604.7} & \text{lb} \\ 604.7 & \text{lb} \end{matrix}$$

dc
$$\begin{vmatrix} 1.0E{+}06 & -1.0E{+}06 \\ -1.0E{+}06 & 1.0E{+}06 \end{vmatrix} \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{vmatrix} 395{,}308 \\ -395{,}308 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{matrix} \mathbf{395.3} & \text{lb} \\ -395.3 & \text{lb} \end{matrix}$$

ca
$$\begin{vmatrix} 1.0E{+}06 & -1.0E{+}06 \\ -1.0E{+}06 & 1.0E{+}06 \end{vmatrix} \begin{vmatrix} 0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{vmatrix} 395{,}308 \\ -395{,}308 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{matrix} \mathbf{395.3} & \text{lb} \\ -395.3 & \text{lb} \end{matrix}$$

cb
$$\begin{vmatrix} 1.4E{+}06 & -1.4E{+}06 \\ -1.4E{+}06 & 1.4E{+}06 \end{vmatrix} \begin{vmatrix} -0.3953 \\ 0 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{vmatrix} -559{,}048 \\ 559{,}048 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{matrix} \mathbf{-559.0} & \text{lb} \\ 559.0 & \text{lb} \end{matrix}$$

ad
$$\begin{vmatrix} 1.1E{+}06 & -1.1E{+}06 \\ -1.1E{+}06 & 1.1E{+}06 \end{vmatrix} \begin{vmatrix} 0.8063 \\ 0 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{vmatrix} 855{,}211 \\ -855{,}211 \end{vmatrix} \times 10^{-6}\ F_3 = \begin{matrix} \mathbf{855.2} & \text{lb} \\ -855.2 & \text{lb} \end{matrix}$$

Page A23.15 Pinned Truss

Line 2, Equation (75)

$A = 60\ t_2$ *should be* $A = 60\ t^2$

Page B1.8 Ramberg-Osgood Stress-Strain Curve

Column 1, Paragraph 1

"…when Ramsberg and Osgood …" *should be* "…when Ramberg and Osgood …"

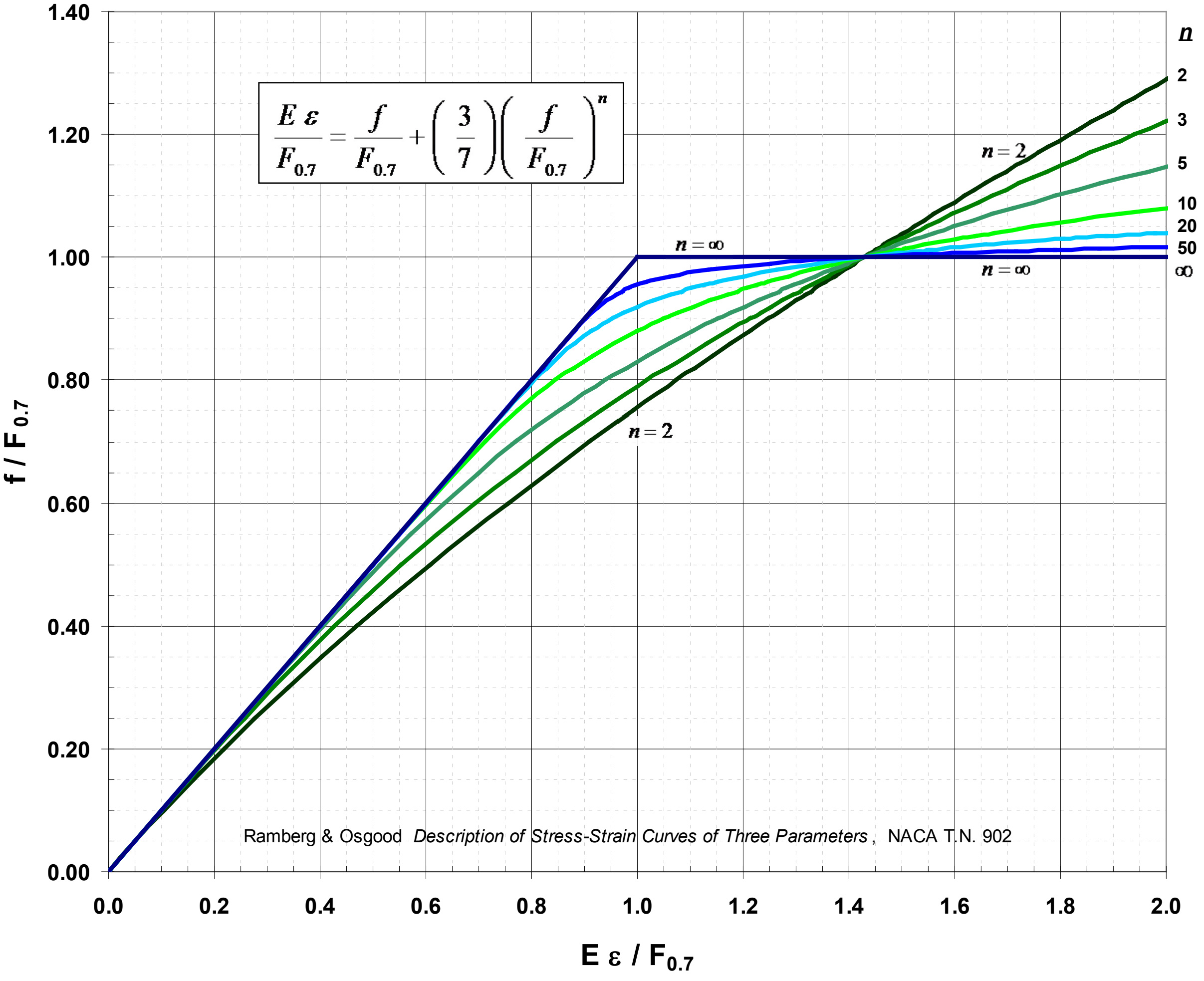
Column 1, Paragraph 2

"The Ramsberg and Osgood proposed …" *should be* "Ramberg and Osgood proposed a …"

Equation (3) $\dfrac{E\,\varepsilon}{F_{0.7}} = \dfrac{f}{F_{0.7}} + \left(\dfrac{f}{F_{0.7}}\right)^{n}$ *should be* $\dfrac{E\,\varepsilon}{F_{0.7}} = \dfrac{f}{F_{0.7}} + \dfrac{3}{7}\left(\dfrac{f}{F_{0.7}}\right)^{n}$

This is obvious when you plot the equation as Figure B1.14 on page B1.9.

Figure B1.14 ***Stress-Strain in the Inelastic Range***



Ramberg & Osgood *Description of Stress-Strain Curves of Three Parameters*, NACA T.N. 902

Page C1.5 Mohr's Circle for Principal Stresses

I believe that the angle theta should equal 19.33 degrees instead of 18 degrees 50 minutes.

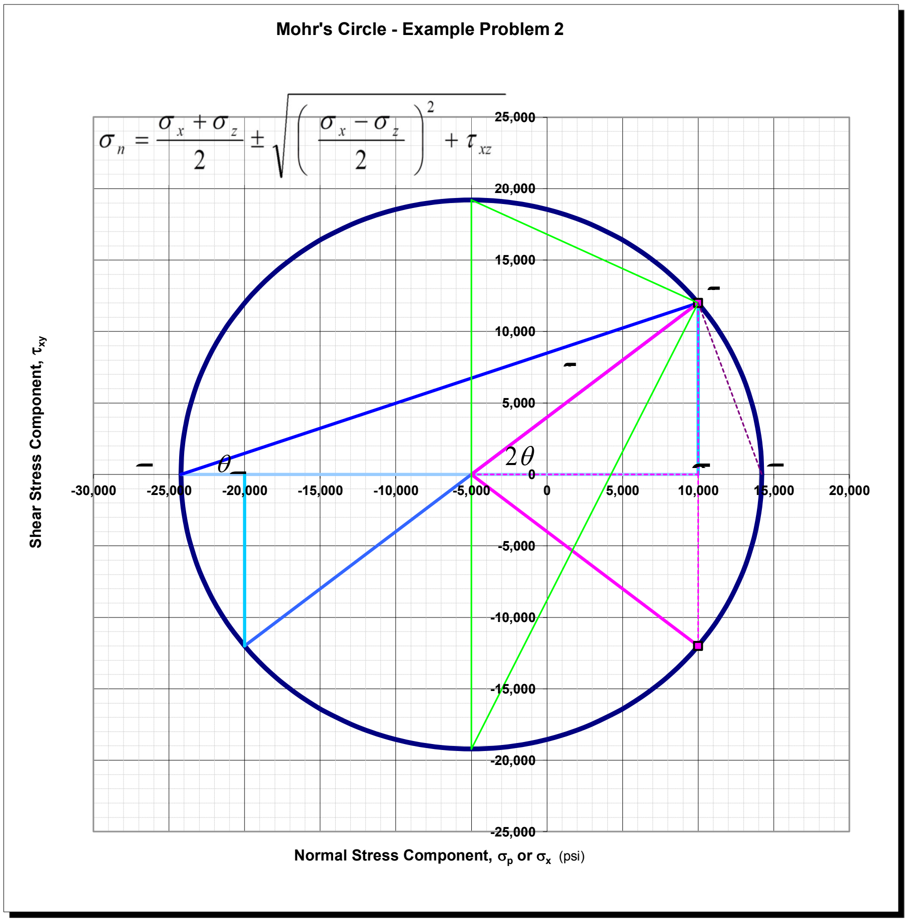
$$\tan 2\,\theta' \; \frac{2 \;\; x \;\; 12000}{1000 \; - \; \left(-20000 \right)} = .8$$

should be

$$\tan 2\,\theta' \; = \; \frac{2 \;\; \left(12{,}000 \right)}{10{,}000 \; - \; \left(-20{,}000 \right)} \; = \; 0.675 \;\; \mathit{radians}$$

θ = 19.33 degrees

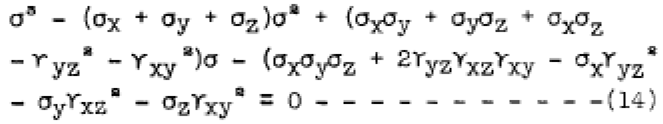
The figure for Mohr's circle on page C1.5 might look like:


Mohr's Circle - Example Problem 2

Page C1.5 Tri-Axial or Three Dimensional Stresses

Article C1.10

Missing τ_{xz}^2 term.



$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{yz}^2 - \tau_{xy}^2)\sigma - (\sigma_x\sigma_y\sigma_z + 2\tau_{yz}\tau_{xz}\tau_{xy} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2) = 0 \text{ - - - - - - - - - - - - (14)}$$

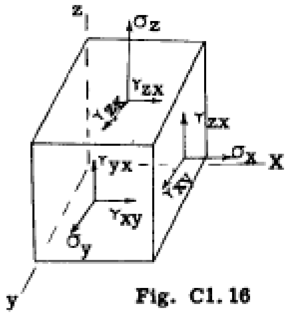


Fig. C1.16

Equation (14) is:

$$\sigma^3 - \left(\sigma_x + \sigma_y + \sigma_z\right)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2\right)\sigma$$
$$- \left(\sigma_x\sigma_y\sigma_z + 2\,\tau_{yz}\tau_{xz}\tau_{xy} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2\right) = 0$$

It should be:

$$\sigma^3 - \left(\sigma_x + \sigma_y + \sigma_z\right)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2\right)\sigma$$
$$- \left(\sigma_x\sigma_y\sigma_z + 2\,\tau_{yz}\tau_{xz}\tau_{xy} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2\right) = 0$$

Thanks to Chris Boshers.

Page C1.9 Octahedral Shear Stress Theory

Right Hand Column “For a triaxial stress system,”

$$\overline{f} = \frac{1}{\sqrt{2}}\sqrt{\left(f_x - f_z\right)^2 + \left(f_z - f_y\right)^2 + \left(f_y - f_x\right)^2 + 6\left(f_{s\,xz} + f_{s\,zy} + f_{s\,yx}\right)^2}$$

should be

$$\overline{f} = \frac{1}{\sqrt{2}}\sqrt{\left(f_x - f_z\right)^2 + \left(f_z - f_y\right)^2 + \left(f_y - f_x\right)^2 + 6\left(f_{s\,xz}{}^2 + f_{s\,zy}{}^2 + f_{s\,yx}{}^2\right)}$$

or another way to write the von Mises Effective Stress:

$$\overline{\sigma} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2\right)}$$

Page C2.2 Non-Dimensional Column Curves

Right Hand Column, Section C2.5

“… non-dimentional column curves have been derived by Cozzone and Melcon (See Ref. 3).”

should be

“… non-dimensional column curves have been derived by Cozzone and Melcon (See Ref. 2).”

Page C2.13 Strength of Columns using Column Curves

L’ = 24.9 *should be* 24.49 in

L’ / r = 41 *should be* 40.9 etc.

Page C2.13 Strength of Columns with Stable Cross Sections

Right Hand Column, Case 2

“subjected to a temperature of 600º F …” *should be* “subjected to a temperature of 300º F …”

Page C2.14 Strength of Columns with Stable Cross Sections

Column 1

Px = .618 *should be* $\rho_x = 0.618$ in

"L' = L, sin c = 1" *should be* "L' = L, since c = 1"

My guess is that Dr. Bruhn meant to use room temperature values (see page B1.11) for the first part of the example …

"From Table B1.1, n = 8.8, $E_c = 7{,}800{,}000$ and $F_{0.7} = 29{,}000$" *should be*

"From Table B1.1, n = 16.6, $E_c = 10{,}500{,}000$ and $F_{0.7} = 72{,}000$"

Room temperature values	B = 1.365	$F_c / F_{0.7} = 0.537$	$F_c = 38{,}635$ psi
Compare to 450° F values	B = 1.005	$F_c / F_{0.7} = 0.736$	$F_c = 21{,}344$ psi

Page C2.15 Strength of Stepped Column

Left Hand Column

"B becomes π^2 or 10 as shown …" *should be* "B becomes π^2 or 9.87 as shown …"

Right Hand Column

$L / \rho = 54.5$	54.9
P = 26,300	26,311
F2 = 59,500	59,556

For Portion 2

$f_1 / F_{0.7} = 0.826$	0.827	
$E_t / E = 0.675$	0.731	
$E_t = 0.675 \times 10{,}500{,}000 = 7{,}090{,}000$		$E_t = 0.731 \times 10{,}500{,}000 = 7{,}670{,}000$
$E I_1 / E I_2 = 4.7$	4.31	
B = 5.6	5.92	

Page C2.16 Column Strength With Known End Restraining Moment

Figure C2.34	Height of truss is missing. It should be 25 inches.
$\mu = 258{,}000$	(257,551)

$\mu L / E I = (258{,}000 \times 30) / 29{,}000{,}000 \times .0367 = 7.28$

should be $\mu L / E I = (257{,}551 \times 30) / 29{,}000{,}000 \times .03\mathbf{8}67 = 6.89$

$\mu L / E I = 2.58$ 2.56

Page C2.17 Column Strength With Known End Restraining Moment

Left Hand Column

$q = \frac{K^1\ L^3}{E\ I}$ *should be* $q = \frac{K'\ L^3}{E\ I}$

Page C2.17 Columns With Elastic Lateral Supports

Left Hand Column, Last Sentence

"… in the problem dealing with a *tapered* column." *should be* "… in the problem dealing with a *stepped* column."

Page C2.18 References

References

Cozzone Σ Melcon *should be* Cozzone & Melcon

Page C3.5 Bending Strength

Example Problem 2, column 1

F_b = 38000 + 23700 (1.17 - 1) = 40770 *should be*

F_b = 38,000 + 23,700 (1.17 - 1) = **42,029** (Actually K = 1.166 or 1.167)

M_{ult} = F_b (I / c) = 42,029 (0.358) = 15,046

Margin of Safety = M_{ult} / M - 1 = 15,046 / 14,000 - 1 = 0.07

Page C4.24 Ultimate Strength in Combined Compression, Bending and Torsion

Right Hand Column

"One" should be a "prime"

$R_{b^1}{}^2 + R_{st}{}^2 = \left(1 - R_c\right)^2$ *should be* $R_{b'}{}^2 + R_{st}{}^2 = \left(1 - R_c\right)^2$

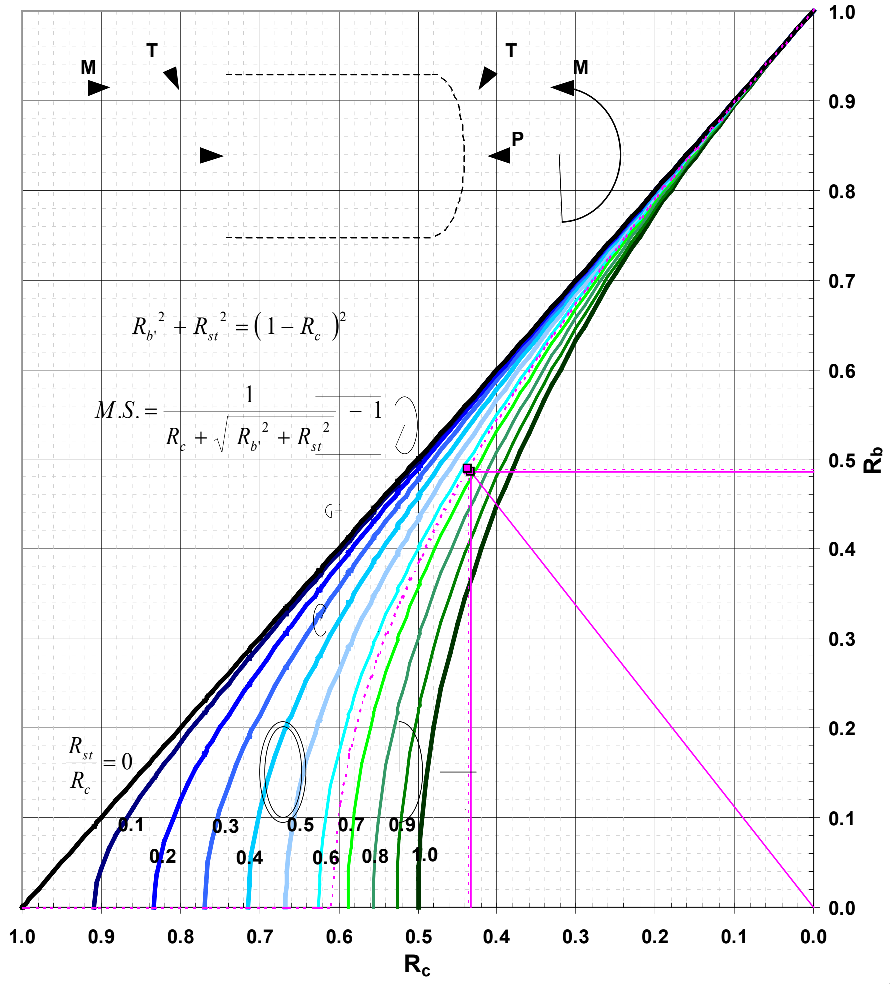
$R^{'}{}_{b} = .20$ *should be* $R_{b'} = 0.20$

Page C4.25 Ultimate Strength in Combined Compression, Bending and Torsion

Right hand column …

My solution gives M_{max} = 1,339.9 in-lb versus 1,347 in-lb which yields the following interaction curve:

Figure C4.36 ***Compression, Bending and Torsion***



Page C4.26 Combined Compression, Bending, Flexural Shear and Torsion

Figure C4.40

$R_c + R_{st} + \sqrt{R_b{}^2 + R_s{}^2 = 1.0}$ *should be* $R_c + R_{st} + \sqrt{R_b{}^2 + R_s{}^2} = 1$

Page C5.4 Chart of Non-Dimensional Compressive Buckling Stress

Figure C5.8 Chart of Non-Dimensional Compressive Buckling Stress – Clamped Edges

The equation listed under the figure is for a long plate with *simply supported* edges instead of clamped.

$$\eta = \left(\frac{E_s}{2\,E} \right) \left\{ 1 + 0.5 \left[1 + \left(\frac{3\,E_t}{E_s} \right) \right]^{1/2} \right\} \frac{\left(1 - \nu_e^{\,2} \right)}{\left(1 - \nu^2 \right)}$$

should be

$$\eta = \left(\frac{E_s}{E} \right) \left\{ 0.352 + 0.324 \left[1 + \left(\frac{3\,E_t}{E_s} \right) \right]^{1/2} \right\} \frac{\left(1 - \nu_e^{\,2} \right)}{\left(1 - \nu^2 \right)}$$

or if you wish

$$\eta = \left\{ 0.352 + 0.324 \left[1 + \left(\frac{3\,E_t}{E_s} \right) \right]^{1/2} \right\} \left(\frac{E_s}{E} \right) \frac{\left(1 - \nu_e^{\,2} \right)}{\left(1 - \nu^2 \right)}$$

or as it *should* be on page 49 of NACA TN 3781

“The plasticity reduction factor for a long plate with simply supported edges is

$$\eta = \left[\left(\frac{E_s}{E} \right) \frac{\left(1 - \nu_e^{\,2} \right)}{\left(1 - \nu^2 \right)} \right] \left\{ 0.500 + 0.250 \left[1 + \left(\frac{3\,E_t}{E_s} \right) \right]^{1/2} \right\}$$

while for a long clamped plate”

$$\eta = \left[\left(\frac{E_s}{E} \right) \frac{\left(1 - \nu_e^{\,2} \right)}{\left(1 - \nu^2 \right)} \right] \left\{ 0.352 + 0.324 \left[1 + \left(\frac{3\,E_t}{E_s} \right) \right]^{1/2} \right\}$$

Page C5.5 Simplified Cladding Reduction Factors

In Table C5.1, it is not obvious how f and β are defined.

They are defined as:

f = total clad thickness / (total core + clad thickness)

β = cladding proportional limit / nominal buckling stress

where the cladding proportional limit is 6 ksi for a 2000 series aluminum and 12 ksi for a 7000 series aluminum.

NACA TN-3781 *Handbook of Structural Stability Part I - Buckling of Flat Plates*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930084505_1993084505.pdf

.... George Gerard and Herbert Becker

Bruhn Figure C5.10 is Figure 12, page 78 in NACA TN-3781.

Bruhn Table C5.1 is Table 3, page 61 in NACA TN-3781.

Thanks to Jim Baldwin and Jeff Schmidt.

Page C5.7 Bending – Buckling Coefficient of Plates

Figure C5.15 Bending-Buckling Coefficient of Plates as a Function of a/b

Abscissa should range from 0.3 to 2.3 instead of 3 to 23.

Thanks to R.L. Hurwitz.

Page C5.8 Combined Bending and Shear

First column, Section C5.10

"R_s is the stress ratio due to torsional shear stress and R_{st} is the stress ratio for transverse or flexural shear stress."

should be …

"R_{st} is the stress ratio due to torsional shear stress and R_s is the stress ratio for transverse or flexural shear stress."

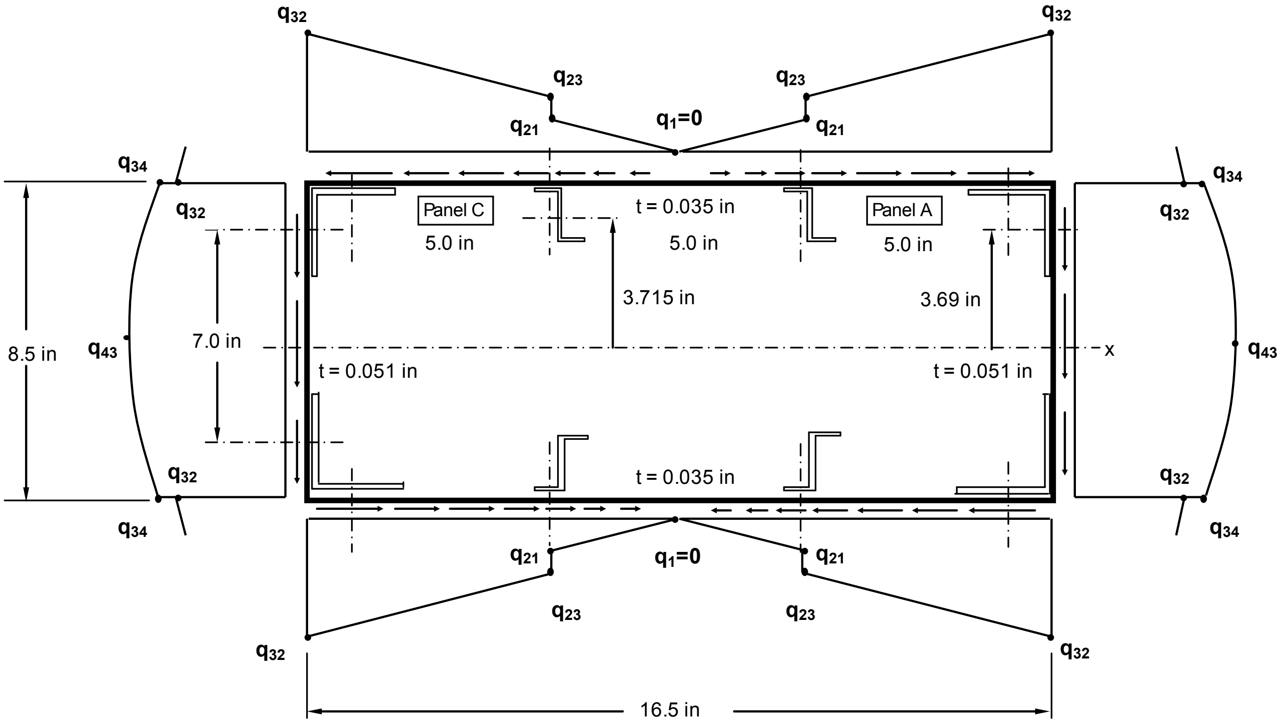
Page C5.11 Cantilever Wing – Skin, Ribs and Stiffeners

Bottom of Column 1 “From Art. C5.12 the interaction equation is $R_c + R_s^2 = 1$” … which is $R_L + R_s^2 = 1$

Bottom of Column 2

$$q = -\frac{200}{49.3} = 4.05 \sum Z\,A \qquad \textit{should be} \qquad q = -\frac{200}{49.3} \sum Z\,A = -4.05 \sum Z\,A$$

Bending, Cantilever Wing - Skin Buckling



Stringer Area, A = 0.18 in^2

Angle Area, A = 0.25 in^2

f_c = 2 P / A = 1,400 lb / 3.74 in^2 = 374 psi

f_c = M_x z / I_x = 5,170 in-lb 4.233 in / 49.30 in^4 = 444 psi

$q_1 = 0$ (Symmetry)

$q_{21} = (-4.0568)\,\Sigma\, Z\,A = -4.0568 (4.2325)\, 2.5 (0.035) = -1.50$ lb / in

$q_{23} = -1.50 - 4.0568\ \Sigma\, Z\,A = 1.50 - 4.0568 (3.715)\, 0.18 = -4.22$ lb / in

$q_{32} = -4.22 - 4.0568\ \Sigma\, Z\,A = -4.22 - 4.0568 (4.2325)\, 5.0 (0.035) = -7.22$ lb / in

$q_{34} = -7.22 - 4.0568\ \Sigma\, Z\,A = -7.22 - 4.0568 (3.69)\, 0.25 = -10.96$ lb / in

$q_{43} = -10.96 - 4.0568\ \Sigma\, Z\,A = -10.96 - 4.0568 (3.69 / 2)\, 0.051 (3.69) = -12.37$ lb / in

Page C5.12 Buckling Strength of Flat Sheet in Combined Stress Systems

Column 1, first equation

Total f_s = 163 + 854 = 917 psi *should be* Total f_s = 163 + 854 = 1,017 psi

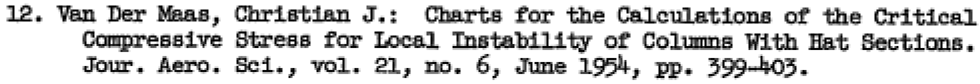
R_s = 0.368 $R_c + R_s^2$ = 1.042

$$M.S. = \frac{2}{0.906 + \sqrt{0.906^2 + 4\,(0.368)^2}} - 1 = -0.04$$

Page C6.3 Buckling Stress for Hat-Section Stiffeners

Figure C6.7 "Ref 12" *should be* "Ref 5"

It was Reference 12 in NACA TN-3782, page 17:



12. Van Der Maas, Christian J.: Charts for the Calculations of the Critical Compressive Stress for Local Instability of Columns With Hat Sections. Jour. Aero. Sci., vol. 21, no. 6, June 1954, pp. 399-403.

NACA TN-3782, Figure 6, page 31 Bruhn Figure C6.7, page C6.3

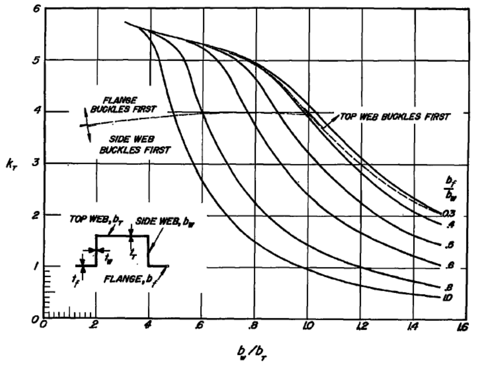


Figure 6.- Buckling stress for hat-section stiffeners. $t = t_f = t_w = t_T$; $\sigma_{cr} = \frac{k_T \pi^2 E}{12(1 - \nu_e^2)} \frac{t^2}{b_T^2}$. (Data of ref. 12.)

NACA TN-3782 *Handbook of Structural Stability Part II - Buckling of Composite Elements*

http://ntrs.nasa.gov/search.jsp?R=239713&id=4&qs=N%3D4294809390 .... Herbert Becker

Charts for the Calculation of the Critical Compressive Stress for Local Instability of Columns with Hat Sections

Journal of the Aeronautical Sciences, Volume 21, No. 6, June 1954 Van Der Maas, Christian J.

Page C6.4 Z-Section Stiffeners

Near the bottom of the right hand column

Fig. C *should be* Figures C6.9 and C6.10, page C6.5 Thanks to Jim Baldwin.

Page C7.6 Restraint Produced by Lips and Bulbs

Equation C7.8

$$2.73\ \frac{I_L}{b_f\ t^3} - \frac{A_L}{b_f\ t} \geq 5$$ *should be* $$2.73\ \frac{I_L}{b_f\ t_f^{\ 3}} - \frac{A_L}{b_f\ t_f} \geq 5$$

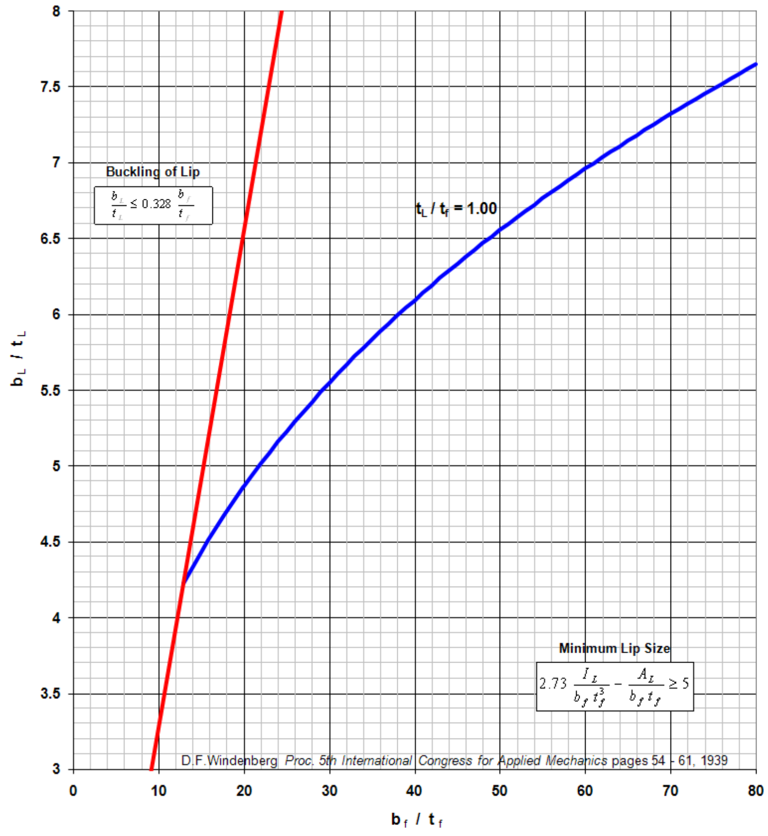
Equation C7.9

$$0.910\left(\frac{b_L}{t}\right)^3 - \frac{b_L}{t} = 5\ \frac{b_f}{t}$$ *should be* $$\boxed{0.910\left(\frac{b_L}{t_f}\right)^3 - \frac{b_L}{t_f} \geq 5\ \frac{b_f}{t_L}}$$

$$2.73\ \frac{I_L}{b_f\ t_f^{\ 3}} - \frac{A_L}{b_f\ t_f} \geq 5 \qquad 2.73\ \frac{t_L\ b_L^{\ 3}/3}{b_f\ t_f^{\ 3}} - \frac{b_L\ t_L}{b_f\ t_f} \geq 5$$

$$0.910\ \frac{b_L^{\ 3}}{t_f^{\ 3}} - \frac{b_L}{t_f} \geq 5\left(\frac{b_f}{t_L}\right) \qquad 0.910\left(\frac{b_L}{t_f}\right)^3 - \frac{b_L}{t_f} \geq 5\ \frac{b_f}{t_L}$$

Figure C7.11 Lip Criteria for Formed and Extruded Sections



D. F. Windenberg Proceedings. 5th International Congress for Applied Mechanics pages 54-61, 1939

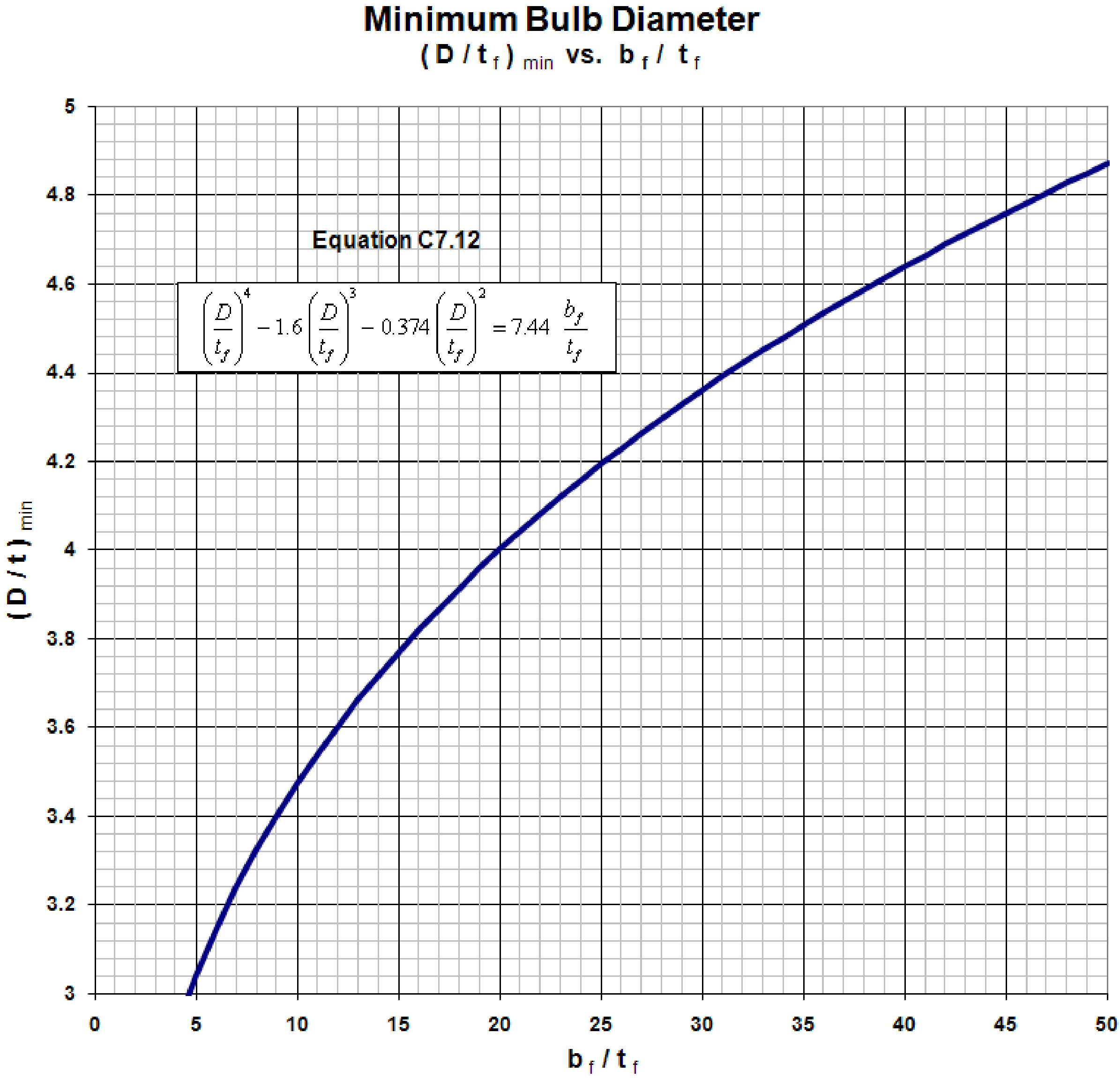
Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C7.11, page C7.6

Figure C7.12

Minimum bulb dimensions required for buckle as simply supported plate

should be

Minimum Bulb Dimensions Required for Flange to Buckle as a Simply Supported Plate



D. F. Windenberg Proceedings. 5th International Congress for Applied Mechanics pages 54-61, 1939

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C7.12, page C7.6

Page C7.6 Restraint Produced by Lips and Bulbs

Right Column, second and third line … instead of 0.388 and 3.617 …

Joe Zuklic calculates compressive buckling coefficients of 0.389 and 3.615.

For v = 0.3

Simply Supported Plate *K* = 4.0 From Figure C5.2C on page C5.2

$$f_{cr} = \frac{\pi^2 K\, E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 \qquad \frac{\pi^2\ K}{12\left(1-\upsilon^2\right)} = \frac{\pi^2\left(4.0\right)}{12\left(1-0.30^2\right)} = 3.615$$

Simple – Free Edges K = 0.43 From Figure C5.2E on page C5.2

(or 0.42 or 0.416 depending on the source)

$$f_{cr} = \frac{\pi^2 K\, E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 \qquad \frac{\pi^2\ K}{12\left(1-\upsilon^2\right)} = \frac{\pi^2\left(0.43\right)}{12\left(1-0.30^2\right)} = 0.3886$$

For v = 0.33

Simply Supported Plate *K* = 4.0 From Figure C5.2C on page C5.2

$$f_{cr} = \frac{\pi^2 K\, E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 \qquad \frac{\pi^2\ K}{12\left(1-\upsilon^2\right)} = \frac{\pi^2\left(4.0\right)}{12\left(1-0.33^2\right)} = 3.692$$

Simply Supported – Free K = 0.43 From Figure C5.2E on page C5.2

(or 0.42 or 0.416 depending on the source)

$$f_{cr} = \frac{\pi^2 K\, E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 \qquad \frac{\pi^2\ K}{12\left(1-\upsilon^2\right)} = \frac{\pi^2\left(0.43\right)}{12\left(1-0.33^2\right)} = 0.3969$$

Page C7.7 Crippling Stresses

Left column, Problem 1

$$\left(a + b\right) 2\ t = \frac{1.95}{0.10} = 19.5$$ *should be*

$$\frac{\left(a + b\right)}{2\ t} = \frac{1.95}{0.10} = 19.5$$ Thanks to Jim Baldwin.

Page C7.10 Sheet Effective Widths

Right Hand Side

Simply Supported (for $\upsilon = 0.3$)

K = 4.0 From Figure C5.2C on page C5.2

$$f_{cr} = \frac{\pi^2 K\, E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$

$$F_{cy} = \frac{\pi^2 \left(4.0\right) E}{12\left(1-\nu^2\right)}\left(\frac{t}{w_e}\right)^2$$

$$\left(\frac{t}{w_e}\right)^2 = \frac{12\left(1-\nu^2\right) F_{cy}}{4\,\pi^2\, E}$$

$$\left(\frac{t}{w_e}\right)^2 = 0.2766\;\frac{F_{cy}}{E}$$

$$\left(\frac{t}{w_e}\right) = 0.5259\sqrt{\frac{F_{cy}}{E}}$$

$$\left(\frac{w_e}{t}\right) = 1.90\sqrt{\frac{E}{F_{cy}}}$$

Effective Width

$$w_e = 1.90\;t\sqrt{\frac{E}{F_{cy}}}$$

From test data for light stringers

$$w_e = 1.70\;t\sqrt{\frac{E}{F_{cy}}}$$

Page C7.11 Sheet Effective Widths

Right Hand Side

Fixed – Free Edges (for $\upsilon = 0.3$)

K = 0.43 From Figure C5.2E on page C5.2

$$f_{cr} = \frac{\pi^2 K\, E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$

$$F_{cy} = \frac{\pi^2 \left(0.43\right) E}{12\left(1-\nu^2\right)}\left(\frac{t}{w_e}\right)^2$$

$$\left(\frac{t}{w_e}\right)^2 = \frac{12\left(1-\nu^2\right) F_{cy}}{0.43\,\pi^2\, E}$$

$$\left(\frac{t}{w_e}\right)^2 = 2.573\;\frac{F_{cy}}{E}$$

$$\left(\frac{t}{w_e}\right) = 1.604\sqrt{\frac{F_{cy}}{E}}$$

$$\left(\frac{w_e}{t}\right) = 0.623\sqrt{\frac{E}{F_{cy}}}$$

Effective Width

$$w_e = 0.623\;t\sqrt{\frac{E}{F_{cy}}}$$

Thanks to Joe Zuklic.

Page C7.15 Failure by Inter-Rivet Buckling

Right Column, Equation (C7.24)

$$F_{ir} = \frac{c\,\pi^2\,\eta\,\bar{\eta}}{12\left(1-\nu^2\right)}\left(\frac{t_s}{p}\right)^2 \qquad \textit{should be} \qquad F_{ir} = \frac{c\,\pi^2\,\eta\,\bar{\eta}\,E_t}{12\left(1-\nu^2\right)}\left(\frac{t_s}{p}\right)^2$$

Page C7.26 Column Strength of Stiffener With Effective Sheet

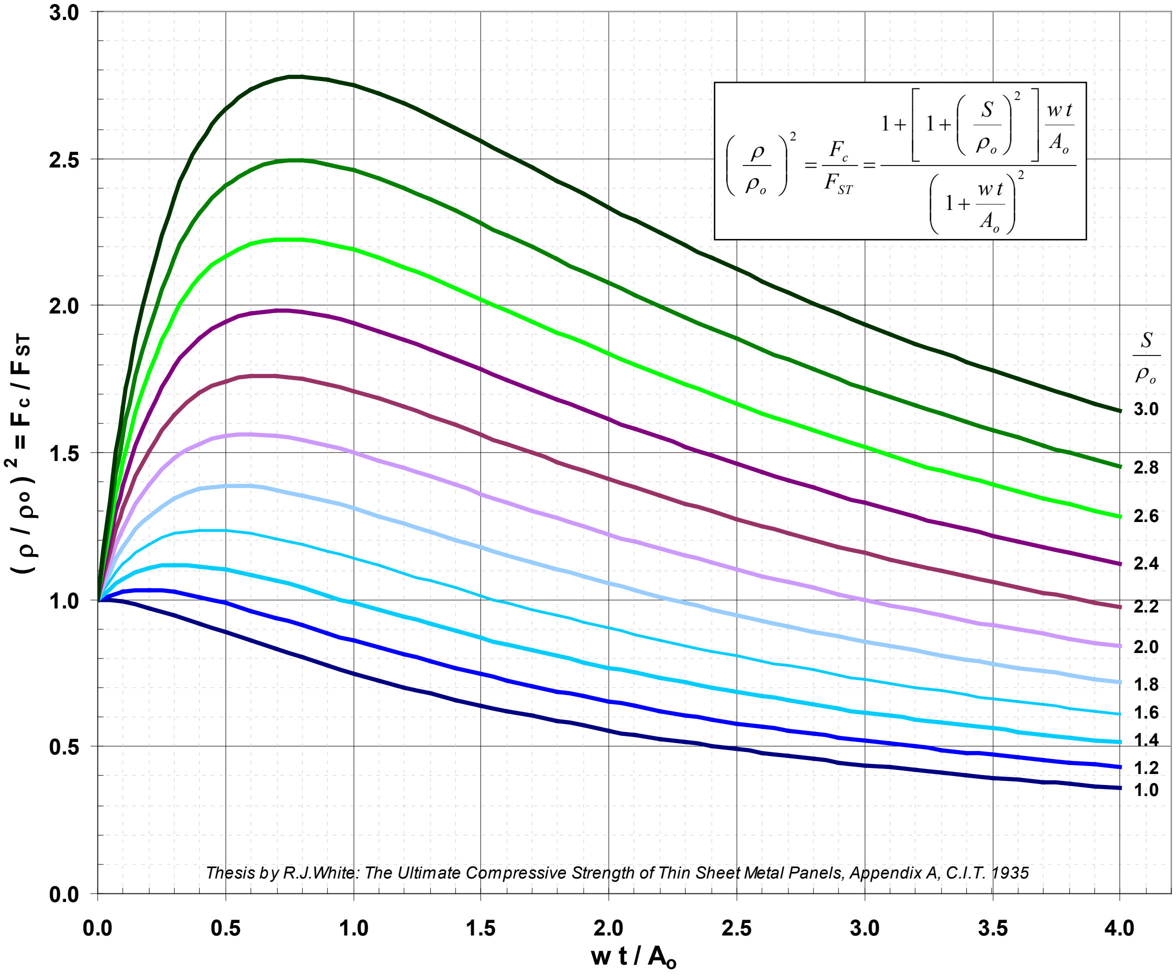
Left column w = .965 should be 0.960 in

Figure C7.36 The peak of the curve for S / ρ = 3.0 is too high.

The figure should look like …

Figure C7.36

Curves for the Determination of (ρ / ρ_o)^2 = F_c / F_{ST}



Thesis by R.J.White: The Ultimate Compressive Strength of Thin Sheet Metal Panels, Appendix A, C.I.T. 1935

Page C7.26 Column Strength of Stiffener With Effective Sheet

Right column

.2063	0.205
1.05	1.117
ρ = .548	0.565
L' / ρ = 44.8	43.4
Fc = 26,050	26,357 psi

Page C7.27 Column Strength of Stiffener With Effective Sheet

Left Column Revised effective width … w = .99 in *should be* w = 0.948 in

26,050 *should be* 26,357

Failing Load = 3675 *should be* 3,708

Page C8.24 Buckling Strength of Monocoque Cylinders

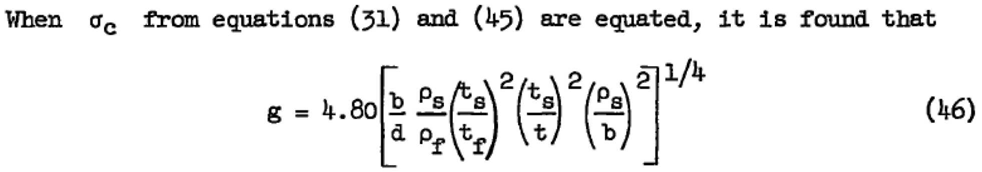
Figures C8.25 – C8.28, pages C8.24 and C8.25 are not applicable to general design. Thanks to SparWeb.

Page C9.12 Orthotropic Circular Cylinders

Table C9.1 Bending equation should be

$$g = 4.80 \left[\left(\frac{b}{d} \right) \left(\frac{\rho_s}{\rho_f} \right) \left(\frac{t_s}{t_f} \right)^2 \left(\frac{t_s}{t} \right)^2 \left(\frac{\rho_s}{b} \right)^2 \right]^{1/4}$$

NACA 3786, page 33, equation (46) …



When σ_c from equations (31) and (45) are equated, it is found that

$$g = 4.80\left[\frac{b}{d}\,\frac{\rho_s}{\rho_f}\left(\frac{t_s}{t_f}\right)^2\left(\frac{t_s}{t}\right)^2\left(\frac{\rho_s}{b}\right)^2\right]^{1/4} \qquad (46)$$

NACA 3786, Table 1, page 63 also lists the equation incorrectly. Thanks to SparWeb.

Page C9.14 References

Column 2, <u>References</u>

(2) Gerard & Becker *Handbook of Structural Stability Part III - Buckling of Curved Plates and Shells*

NACA Technical Note 3883 *should be* NACA Technical Note 3783

(3) NACA WRL-57 *should be* NACA WR-L-57

Page C10.7 Stiffener Size to Use with Non-Buckling Web

Equation C10.8, Section C10.10 is missing brackets. Thanks to Dave Kubala.

$I_v = \frac{2.29d}{t} \frac{V\ h_w}{33\ E}^{4/3}$ *should be* $I_v = \frac{2.29\ d}{t} \left(\frac{V\ h_w}{33\ E} \right)^{4/3}$

Page C10.8 Minimum Moment of Inertia Required in Stiffening Members

The equation in Figure C10.9 at the top of the page is difficult to read. It is:

$\frac{I_v}{d\ t^3} = \frac{0.0217}{\left(\frac{d}{h\ \sqrt{K_s}} \right)^{8/3}}$ Thanks to Jim Baldwin.

Page C10.14 Buckling Equation

Right Hand Column approximately halfway down.

$d\ /\ h\ \sqrt{k_s}$ *should be* $\frac{d}{h\ \sqrt{K_s}}$ see Figure C10.9, page C10.8

The required moment of inertia …where k_s as used in buckling equation must be multiplied by

$\pi^2\ /\ 1\ -\ .3^2\ =\ .905$ Thus,

$$d\ /\ h \sqrt{k_s} = 3.57\ /\ 7.125 \sqrt{6.4\ x\ .905} = 0.208$$

should be

The required moment of inertia …where K_s as used in the shear buckling equation must be multiplied by

$\frac{\pi^2}{12\ \left(1\ -\ \nu^2 \right)} = \frac{\pi^2}{12\ \left(1\ -\ 0.3^2 \right)} = 0.904$ Thus,

$$\frac{d}{h\ \sqrt{K_s}} = \frac{3.57}{7.125\ \sqrt{6.4\ \left(0.904 \right)}} = 0.208$$

Thanks to Jim Baldwin.

Page C11.11 Wagner Beam, Check of Web to Flange Rivet Attachment

$$P_r = \left[\left(\frac{V_{cr}}{h} \frac{I_F}{I} + \frac{V_{tu}}{h} \right)^2 + \left(\frac{V_{tu}}{h} \tan\alpha \right)^2 \right]^{1/2}$$

$$P_r = \left[\left(\frac{244}{28.56} \frac{210}{270.5} + \frac{12850}{28.56} \right)^2 + \frac{12850}{28.56} \; x \; .933^{\,3} \right]^{1/2}$$ *should be*

$$P_r = \left\{ \left[\frac{244}{28.56} \left(\frac{210}{270.5} \right) + \frac{12{,}850}{28.56} \right]^2 + \left[\left(\frac{12{,}850}{28.56} \right) 0.933 \right]^2 \right\}^{1/2}$$

Page C11.12 Lower Flange Bending Stresses

$$f_b = \frac{-12200 \;\; x \; - \; 17.42}{270.5} + \frac{-\;662800 \;\; x \;\; 19.06}{210}$$ *should be*

$$f_b = \frac{-12{,}200 \; \left(-17.42 \right)}{270.5} + \frac{-\,662{,}800 \; \left(-19.06 \right)}{210}$$

Page C11.12 Combined Flange Axial Stresses

$$f_c = -35167 \; -10770 \; = \; -45337 \; psi$$ *should be*

$$f_c = -35{,}167 \; -10{,}170 \; = \; -45{,}337 \; psi$$

Page C11.13 Crippling Stress for the Upper Flange

$$\frac{A}{g\ \ t^2}\left(\frac{F_{cy}}{E_c}\right)^{1/2} = \frac{0.675}{4\ \ x\ \ .1562^2}\left(\frac{70{,}000}{10{,}300{,}000}\right)^{1/2} = .572$$

From Fig. C7.7 we read Fcs / Fcy = .90, hence Fcs = 0.90 x 70000 = <u>63000</u> psi. compression.

For a “tee” section Figure C7.8 (page C7.5) is applicable and the cutoff is 0.80 F_{cy} . (Without bulbs, g = 3.)

From Figure C7.8 we read F_{cs} / F_{cy} = 0.839, hence F_{cs} = 0.839 (70,000 psi) = 58,730 psi compression.

Cutoff = 0.80 (70 ksi) = 56 ksi Therefore, F_{cs} = 56 ksi.

Page C11.14 NACA Symbols

Q	static moment about neutral axis of parts of cross-section as specified by subscripts. in.
	should be
Q	First Area Moment about Neutral Axis of Parts of Cross-Section as Specified by Subscripts (in^3)

Page C11.15 Thickness and Flange Flexibility Factor

t	thickness	Delete the extra parenthesis “)”
wd	flange flexibility factor	Delete the parenthesis “(“.
ωd	Flange Flexibility Factor defined by expression (19a) in NACA TN-2661	

Page C11.17 Average and Maximum Stress in Upright or Web Stiffener

Column 1, Section C11.20

$$f_u = \frac{k\ \ f_s\ \ \tan\ \alpha}{\dfrac{A_u}{dt} + .05\ (\ 1 - k\ )}$$

should be

$$f_u = \frac{k\ \ f_s\ \ \tan\ \alpha}{\dfrac{A_u}{dt} + 0.50\ (\ 1 - k\ )}$$

See NACA TN-2661, page 19, equation (30a). Thanks to Spero Papantos.

$$\sigma_U = -\frac{k\tau\ \tan\ \alpha}{\dfrac{A_{U_e}}{dt} + 0.5(1 - k)} \qquad (30a)$$

NACA TN-2661 *A Summary of Diagonal Tension Part I : Methods of Analysis*

http://naca.central.cranfield.ac.uk/report.php?NID=5043............... Paul Kuhn, James P. Peterson, L. Ross Levin

Page C11.18 Web Design

Section C11.23, column 2, eq 62. See NACA TN 2661, equation 33a, page 27. Thanks to Joe Zuklic.

$f_{s\ max} = f_s (1 + k C_1) (1 + k C_2)$ *should be* $f_{s\ max} = f_s (1 + k^2 C_1) (1 + k C_2)$

Page C11.19 Secondary Bending Moment in Flanges

Equation (69) $M = \frac{1}{12} k\ f_s\ t\ d^2\ C_3$ *should be* $M_{\max} = \frac{1}{12} k\ f_s\ t\ d^2\ C_3 \tan\alpha$

$$M_{max} = kC_3 \frac{S_W d^2 \tan \alpha}{12h}$$

See NACA TN 2661, page 50. Thanks to SuperStress on the www.eng-tips.com website.

Page C11.38 Allowable Stress in Fuselage Skin - Diagonal Tension

Example problem in Section C11.34. From page A20.8, the effective moment of inertia, I = 3,252 in^4 (3,257 in^4 by my calculations) instead of 2,382. This affects the stringer stresses ($f_{p\ max}$, $f_{p\ avg}$) and the shear flow in the panels (q_{2-3} and q_{3-4}).

The moment arms, z for the calculation of the first area moment, Q for stringers 2, 3 and 4 should be 34.56, 31.06 and 26.76 inches respectively instead of 38.3, 36.4 and 32.9 inches using $\bar{z} = 0.76\ in$ from page A20.8. (I calculate $\bar{z} = 0.81\ in$) This would impact the calculations for the shear flow in the panels.

Page C11.44 Longeron Type Fuselage Structure

Column 1, near the bottom …

"**Fig. C11.37 Example Problem**" *should be* "**C11.37 Example Problem**"

Page C11.49 References

Column 2, near the bottom … "Technical Note" should be "Technical Memorandum" … two places.

"**T.N. 838**" *should be* "**TM-838**"

"**T.N. 774**" *should be* "**TM-774**"

Thanks to Jim Baldwin.

Page C12.13 Hexagonal Cell Core Wrinkling - Biaxial and Shear Interaction

At the bottom of the right hand column it appears that a square root symbol is missing …

$$M.S. = \frac{2}{R_a + R_a^{\,2} + \left(4\ R_s^{\,2}\right)} - 1 = -0.18$$

should be

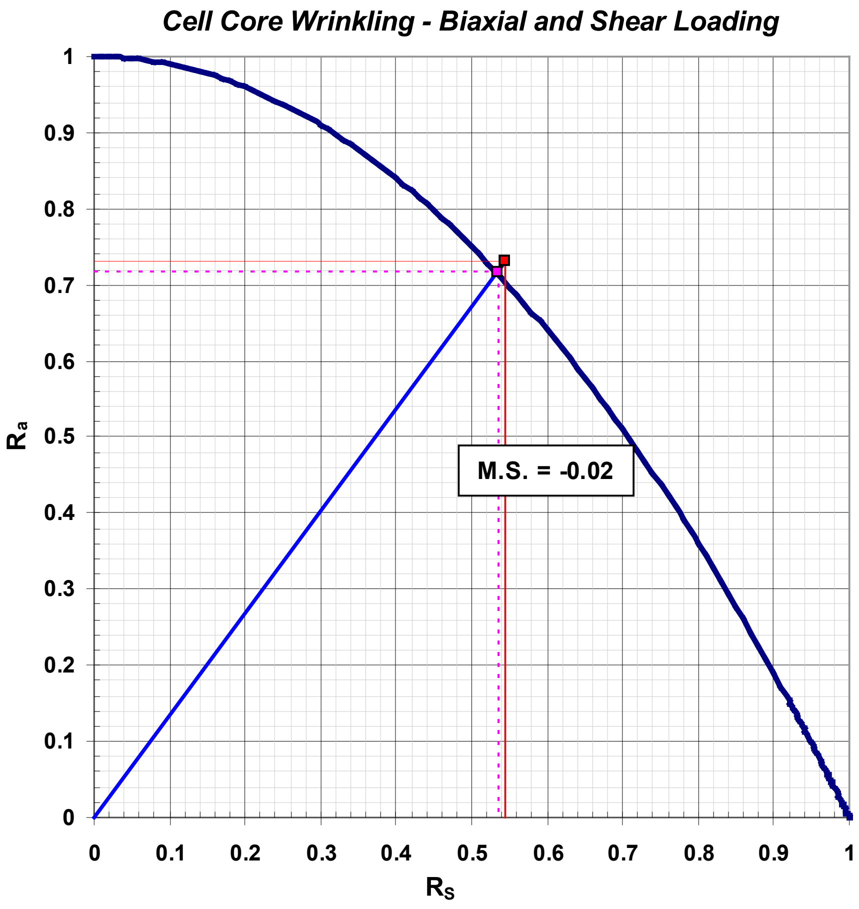
$$M.S. = \frac{2}{R_a + \sqrt{R_a^{\,2} + 4\ R_s^{\,2}}} - 1 = -0.02$$

$$M.S. = \frac{2}{0.762 + \sqrt{0.732^{\,2} + 4\ (0.543)^2}} - 1 = -0.02$$

Interaction Equations

$R_a + R_s^{\,2} = 1$ Cell Core Buckling Biaxial and Shear Interaction

$R_a + R_s^{\,2} = 1$ Cell Core Wrinkling Biaxial and Shear Interaction



Page C12.14 Shear Crimping

Column 1

“form Moore’s Circle” *should be* “from Mohr’s Circle” Thanks to Dr. Howard W. Smith.

Page D3.6 Splice with Filler

Column 1, Figure D3.18

“7075-T6 Slum.” *should be* “7075-T6 Alum.”

Bearing allowables “Bearing In .072 = 1630#” and “Bearing In .081 = 1840#”

are apparently based on F_{bru} = 145 ksi which seems high for 7075-T6 Sheet.

Case II

Using F_{bru} = 135 ksi I calculate:

“Bearing in 0.072 inch sheet = 1,519 lb” and “Bearing in 0.081 inch sheet = 1,709 lb”

The number of fasteners required does not change in this example if F_{bru} is greater than 132,128 psi.

Page D3.7 Framing Cutouts in Web

Column 1, last line, “as shown in Fig. C3.20” *should be* “as shown in Fig. D3.20”.

Thanks to Joe Zuklic.

Page D3.9 Framing Cutouts in Web

Don’t interpret the “dashes” as “minus” signs in any of the example cutouts.

Thanks to Chris Boshers.

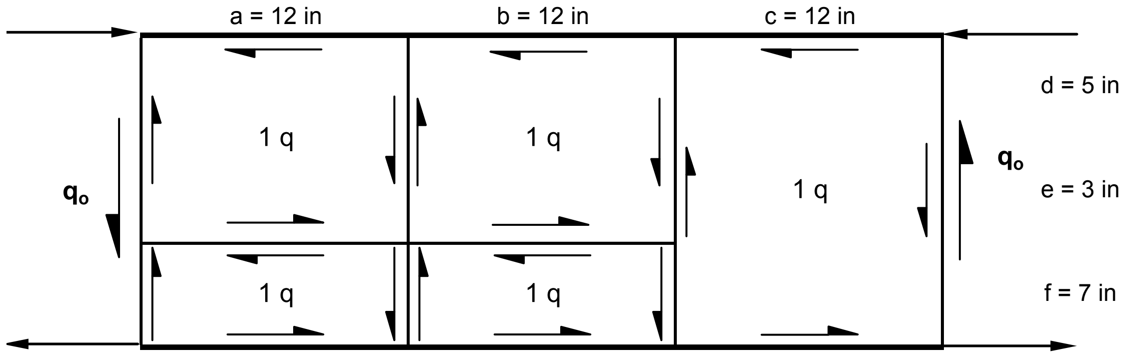
Figure D3.24

Figure D3.24 (b) Middle lower bay should have the shear flow, q_o reversed.

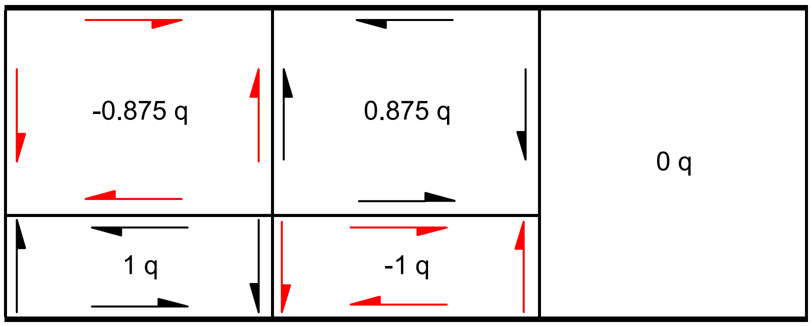
q_o *should be* $-q_o$

The upper and lower arrows in the bay should be reversed.

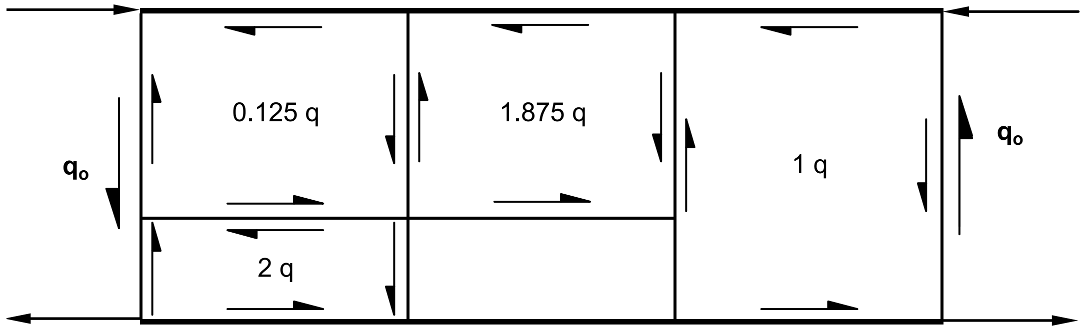
Before Cut-Out



Self-Balancing Internal Loads



Final Shear Flow Distribution



Page D3.10 Framing Cutouts With Doublers or Bents

First column, fourth paragraph from the bottom …

"Th se allowables apply…" *should be* "These allowables apply…"

2.0 Aircraft Structures

The first edition of *Aircraft Structures* by David J. Peery is another classic book on aircraft stress analysis. The following pages list topics from the book that are a good start to any stress analyst's education.

Errata – Analysis of Wing Ribs

Section 8.3, page 192

q_{ba} = 274 lb/in q_{ac} = 374 lb/in q_{bc} = 566 lb/in Incorrect

Three Equations, Three Unknowns

$$-222\,q_{ba} - 168\,q_{ac} = -90{,}000$$

$$-20\,q_{ba} + 20\,q_{ac} = 2{,}000$$

$$-4\,q_{ba} - 6\,q_{ac} - 10\,q_{bc} = -9{,}000$$

In Matrix Form

$$\begin{bmatrix} -222 & -168 & 0 \\ -20 & 20 & 0 \\ -4 & -6 & -10 \end{bmatrix} \begin{Bmatrix} q_{ba} \\ q_{ac} \\ q_{bc} \end{Bmatrix} = \begin{Bmatrix} -90{,}000 \\ 2{,}000 \\ -9{,}000 \end{Bmatrix}$$

Invert Matrix

$$\begin{bmatrix} -222 & -168 & 0 \\ -20 & 20 & 0 \\ -4 & -6 & -10 \end{bmatrix}^{-1} = \begin{bmatrix} -0.00256 & -0.02154 & 0 \\ -0.00256 & 0.02846 & 0 \\ 0.00256 & -0.00846 & -0.10 \end{bmatrix}$$

Solve

$$\begin{Bmatrix} q_{ba} \\ q_{ac} \\ q_{bc} \end{Bmatrix} = \begin{bmatrix} -0.00256 & -0.02154 & 0 \\ -0.00256 & 0.02846 & 0 \\ 0.00256 & -0.00846 & -0.10 \end{bmatrix} \begin{Bmatrix} -90{,}000 \\ 2{,}000 \\ -9{,}000 \end{Bmatrix} = \begin{Bmatrix} 187.7 \\ 287.7 \\ 652.3 \end{Bmatrix}$$

q_{ba} = 188 lb/in **q_{ac} = 288** lb/in **q_{bc} = 652** lb/in

Cutouts in Semi-Monocoque Structures

David J. Peery *Aircraft Structures* First Edition, Section 8.5, pages 202-210

Wing Loads

P_z	**30,000**	lb
P_x	**9,000**	lb

Wing Box Area

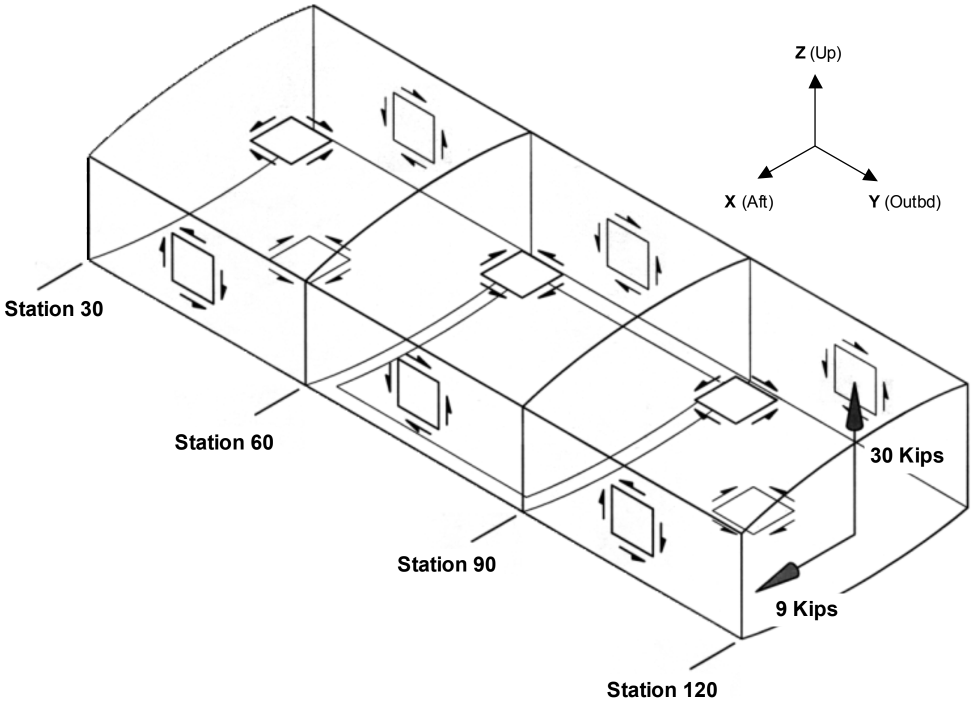
A_1	**75**	in^2
A_2	**200**	in^2
A_3	**90**	in^2
A_4	**90**	in^2
A_5	**150**	in^2
A_6	**150**	in^2

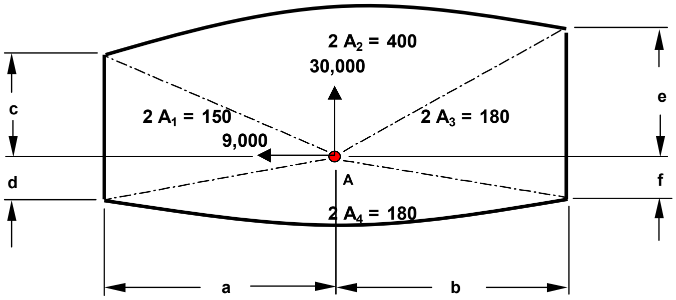
Geometry

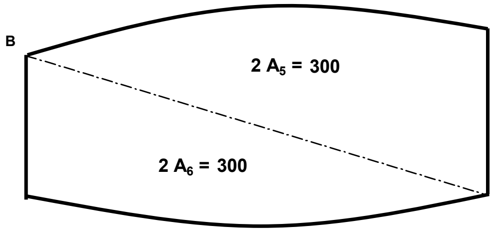
a	**15**	in
b	**15**	in
c	**7**	in
d	**3**	in
e	**9**	in
f	**3**	in

Wing Stations

WS	**120**	in
WS	**90**	in
WS	**60**	in
WS	**30**	in



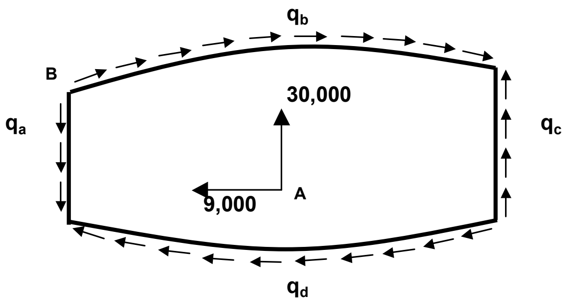




Cutouts in Semi-Monocoque Structures

Shear Flows for Wing Box *Without* Cutout

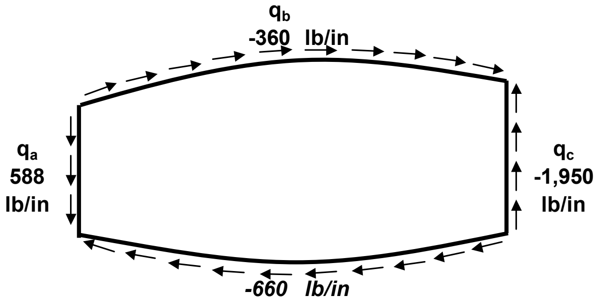
All Three Bays



← +	$\Sigma F_x = 0$	-30	q_b	+	30	q_d	+	9,000	=	0	
↑ +	$\Sigma F_z = 0$	$- q_a$	10	+	q_b	2	+	q_c	12	+	30,000
CCW +	$\Sigma M_A = 0$	q_a	150	-	q_b	400	+	q_c	180	- q_d	180
CCW +	$\Sigma M_B = 0$	q_c	300	-	q_d	300	+	30,000	15 -	9,000	7

$$\begin{vmatrix} 0 & -30 & 0 & 30 \\ -10 & 2 & 12 & 0 \\ 150 & -400 & 180 & -180 \\ 0 & 0 & 300 & -300 \end{vmatrix} \begin{vmatrix} q_a \\ q_b \\ q_c \\ q_d \end{vmatrix} = \begin{vmatrix} -9{,}000 \\ -30{,}000 \\ 0 \\ -387{,}000 \end{vmatrix}$$

$$\begin{vmatrix} 0.0842 & -0.2105 & -0.0074 & 0.0128 \\ 0.0316 & -0.0789 & -0.0053 & 0.0063 \\ 0.0649 & -0.0789 & -0.0053 & 0.0096 \\ 0.0649 & -0.0789 & -0.0053 & 0.0063 \end{vmatrix} \begin{vmatrix} -9{,}000 \\ -30{,}000 \\ 0 \\ -387{,}000 \end{vmatrix} = \begin{vmatrix} 588 \\ -360 \\ -1{,}950 \\ -660 \end{vmatrix}$$



$q_a = \boxed{588}$ lb/in

$q_b = \boxed{-360}$ lb/in

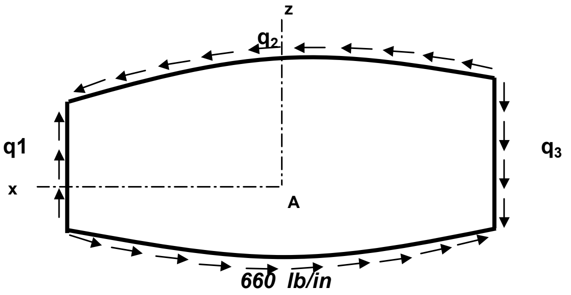
$q_c = \boxed{-1{,}950}$ lb/in

$q_d = \boxed{-660}$ *lb/in*

Cutouts in Semi-Monocoque Structures

Correcting Shear Flows

Wing Station, WS60 to WS90

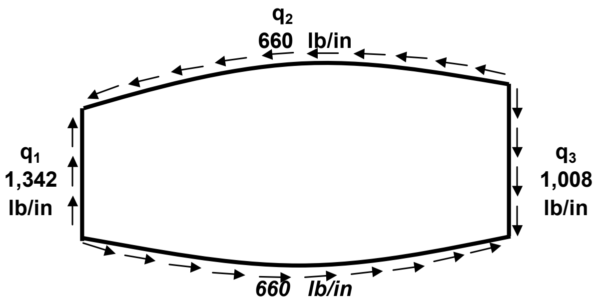


← +	$\Sigma F_x = 0$	30	q_2	+	-30	660	=	0				
↑ +	$\Sigma F_z = 0$	q_1	10	-	2	660	-	12	q_3	=	0	
CCW +	$\Sigma M_A = 0$	$-\ q_1$	150	+	400	q_2	-	180	q_3	+	180	660

$$\begin{vmatrix} 0 & 30 & 0 \\ 10 & 0 & -12 \\ -150 & 400 & -180 \end{vmatrix} \quad \begin{vmatrix} q_1 \\ q_2 \\ q_3 \end{vmatrix} = \begin{vmatrix} 19{,}800 \\ 1{,}320 \\ -118{,}800 \end{vmatrix}$$

$$\begin{vmatrix} 0.0444 & 0.0500 & -0.0033 \\ 0.0333 & 0 & 0 \\ 0.0370 & -0.0417 & -0.0028 \end{vmatrix} \quad \begin{vmatrix} 19{,}800 \\ 1{,}320 \\ -118{,}800 \end{vmatrix} = \begin{vmatrix} 1{,}342 \\ 660 \\ 1{,}008 \end{vmatrix}$$

Correcting Shear Flows **Wing Station, WS60 to WS90**



q_1 = $\boxed{1{,}342}$ lb/in

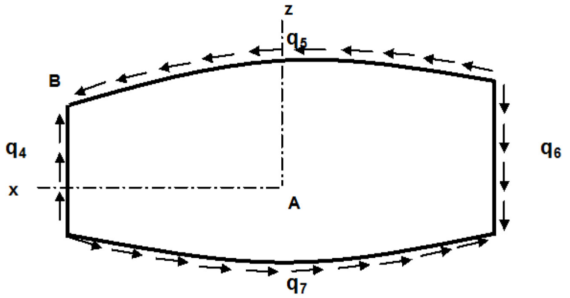
q_2 = $\boxed{660}$ lb/in

q_3 = $\boxed{1{,}008}$ lb/in

Cutouts in Semi-Monocoque Structures

Correcting Shear Flows

Wing Station, WS90 to WS120
Wing Station, WS30 to WS60



$$+\rightarrow \quad \Sigma F_x = 0 \quad -30\,q_5 + 30\,q_7 = 0$$

$$\uparrow + \quad \Sigma F_z = 0 \quad 10\,q_4 - 2\,q_5 - 12\,q_6 = 0$$

$$\text{CCW} + \quad \Sigma M_A = 0 \quad 150\,q_4 + 400\,q_5 + 180\,q_6 + 180\,q_7 = 0$$

$$\text{OUTBD} + \quad \Sigma F_y = 0 \quad -30\,q_4 + 30\,q_7 = 30{,}030$$

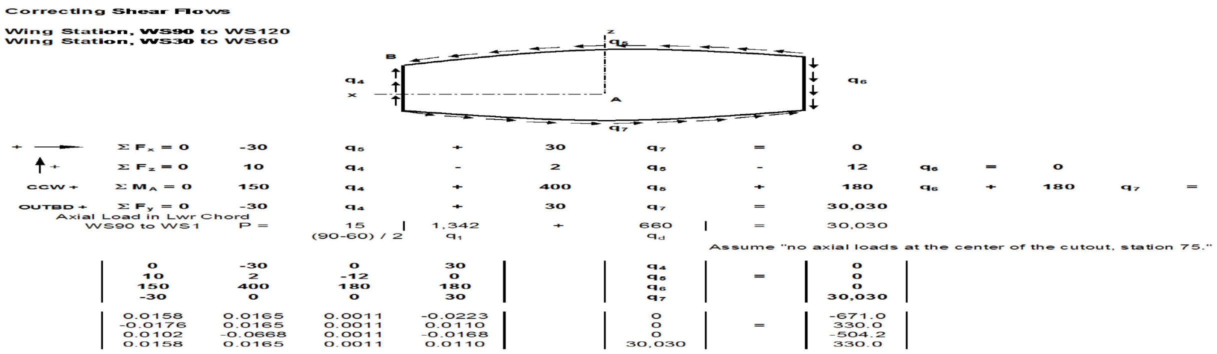
Axial Load in Lwr Chord

$$\text{WS90 to WS1} \quad P = \underset{(90-60)/2}{15}\left|\underset{q_1}{1{,}342} + \underset{q_d}{660}\right| = 30{,}030$$

Assume "no axial loads at the center of the cutout, station 75."

$$\begin{vmatrix} 0 & -30 & 0 & 30 \\ 10 & 2 & -12 & 0 \\ 150 & 400 & 180 & 180 \\ -30 & 0 & 0 & 30 \end{vmatrix} \quad \begin{vmatrix} q_4 \\ q_5 \\ q_6 \\ q_7 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 30{,}030 \end{vmatrix}$$

$$\begin{vmatrix} 0.0158 & 0.0165 & 0.0011 & -0.0223 \\ -0.0176 & 0.0165 & 0.0011 & 0.0110 \\ 0.0102 & -0.0668 & 0.0011 & -0.0168 \\ 0.0158 & 0.0165 & 0.0011 & 0.0110 \end{vmatrix} \quad \begin{vmatrix} 0 \\ 0 \\ 0 \\ 30{,}030 \end{vmatrix} = \begin{vmatrix} -671.0 \\ 330.0 \\ -504.2 \\ 330.0 \end{vmatrix}$$


Correcting Shear Flows

Wing Station, WS90 to WS120
Wing Station, WS30 to WS60

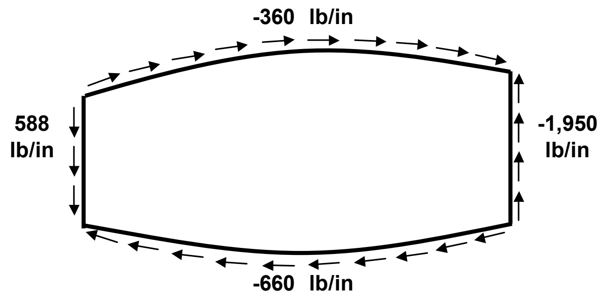
$\Sigma F_x = 0$ -30 q_5 + 30 q_7 = 0
$\Sigma F_z = 0$ 10 q_4 - 2 q_5 - 12 q_6 = 0
CCW + $\Sigma M_A = 0$ 150 q_4 + 400 q_5 + 180 q_6 + 180 q_7 =
OUTBD + $\Sigma F_y = 0$ -30 q_4 + 30 q_7 = 30,030
Axial Load in Lwr Chord
WS90 to WS1 P = 15 | 1,342 + 660 | = 30,030
(90-60) / 2 q_1 q_d
Assume "no axial loads at the center of the cutout, station 75."

0 -30 0 30 | q_4 | 0
10 2 -12 0 | q_5 = | 0
150 400 180 180 | q_6 | 0
-30 0 0 30 | q_7 | 30,030

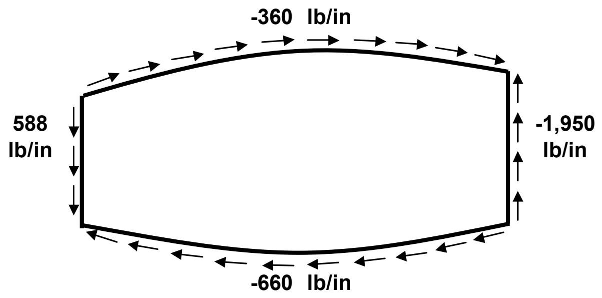
0.0158 0.0165 0.0011 -0.0223 | 0 | -671.0
-0.0176 0.0165 0.0011 0.0110 | 0 = | 330.0
0.0102 -0.0668 0.0011 -0.0168 | 0 | -504.2
0.0158 0.0165 0.0011 0.0110 | 30,030 | 330.0

Cutouts in Semi-Monocoque Structures

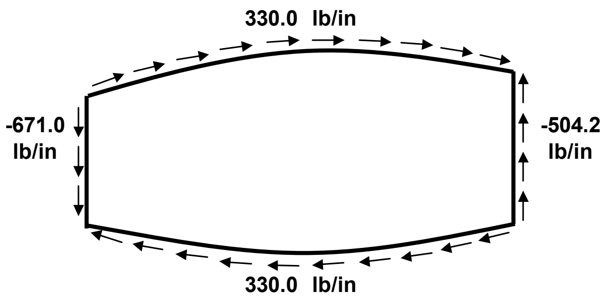
Shear Flows Without Cutout, WS30 to WS60



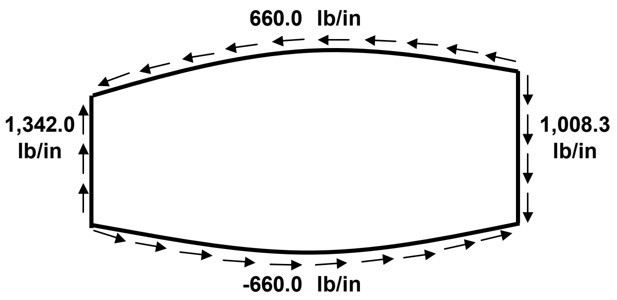
Shear Flows Without Cutout, WS60 to WS90



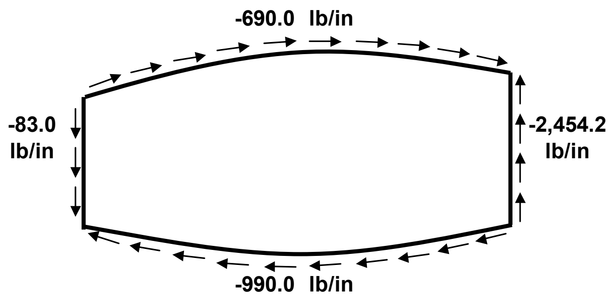
Correcting Shear Flows, WS30 to WS60



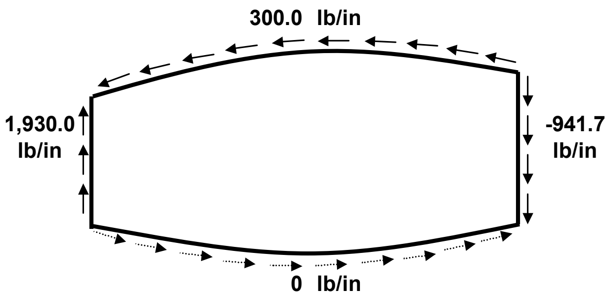
Correcting Shear Flows, WS60 to WS90



Final Corrected Shear Flows, WS30 to WS60

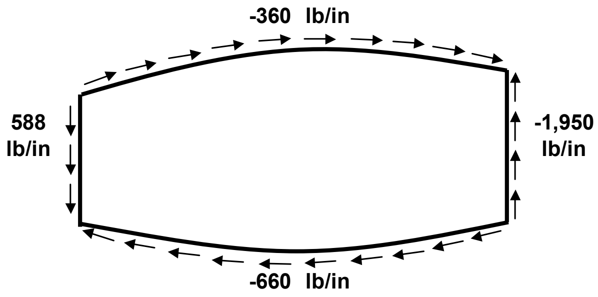


Final Corrected Shear Flows, WS60 to WS90

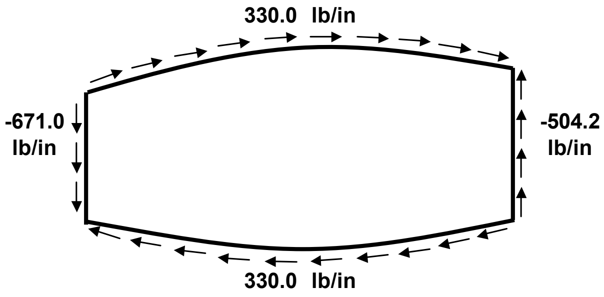


Cutouts in Semi-Monocoque Structures

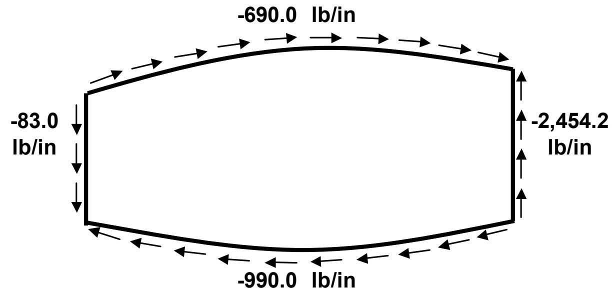
Shear Flows Without Cutout, WS90 to WS120



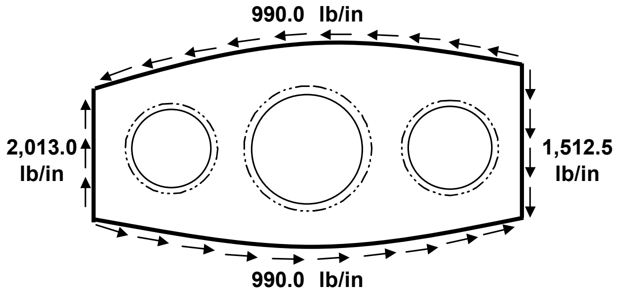
Correcting Shear Flows, WS90 to WS120



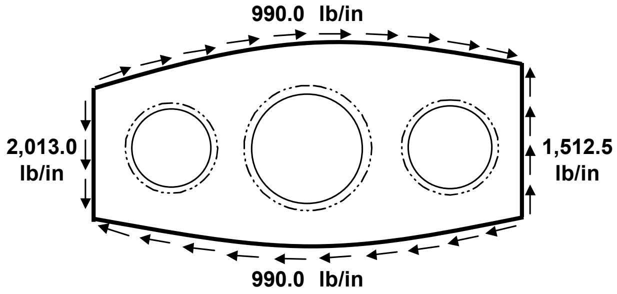
Final Corrected Shear Flows, WS90 to WS120



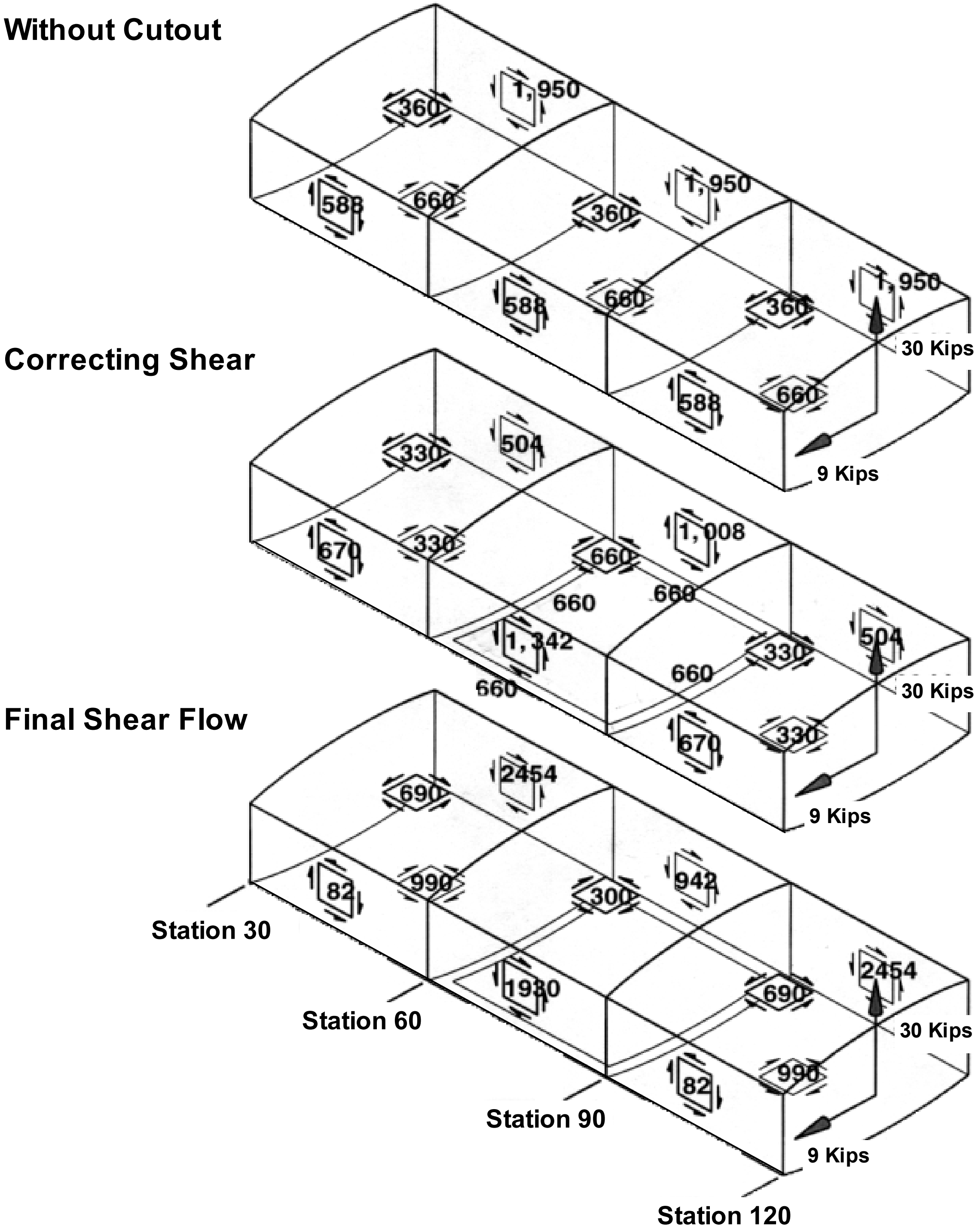
Rib at WS60



Rib at WS90



Cutouts in Semi-Monocoque Structures



Lug Shear-Out

David Peery *Aircraft Structures* First Edition, page 296

Use forty degree angles to define the length of the shear area.

Equation for the Length of Shear-Out Area

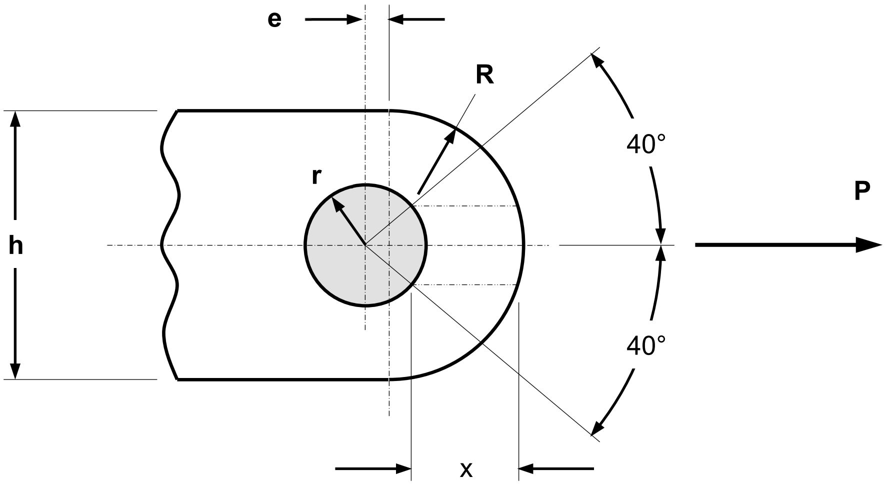
$$x = e + R \left[\sqrt{\left(1 - \frac{r^2}{R^2} \sin^2 40° \right)} - \frac{r}{R} \cos 40° \right]$$

Example

R = 0.70 inch r = 0.3125 inch t = 0.5625 inch e = 0.125 inch

P = 12,000 lb Factor of Safety = 1.50 Fitting Factor = 1.20 d = 2 r

Design Load = 12,000 lb (1.50) 1.20 = 21,600 lb h = 2 R



Length of Shear-Out Area

$$x = 0.125 + 0.70 \left[\sqrt{\left(1 - \frac{0.3125^2}{0.70^2} \sin^2 40° \right)} - \frac{0.3125}{0.70} \cos 40° \right] = 0.556\ inch$$

Shear Stress

$$f_s = \frac{Design\ Load}{2\ x\ t} = \frac{21{,}600\ lb}{2\ Shear\ Areas\ (\ 0.556\ inch\ )\ 0.5625\ inch} = 34.5\ ksi$$

Tension at Net Section

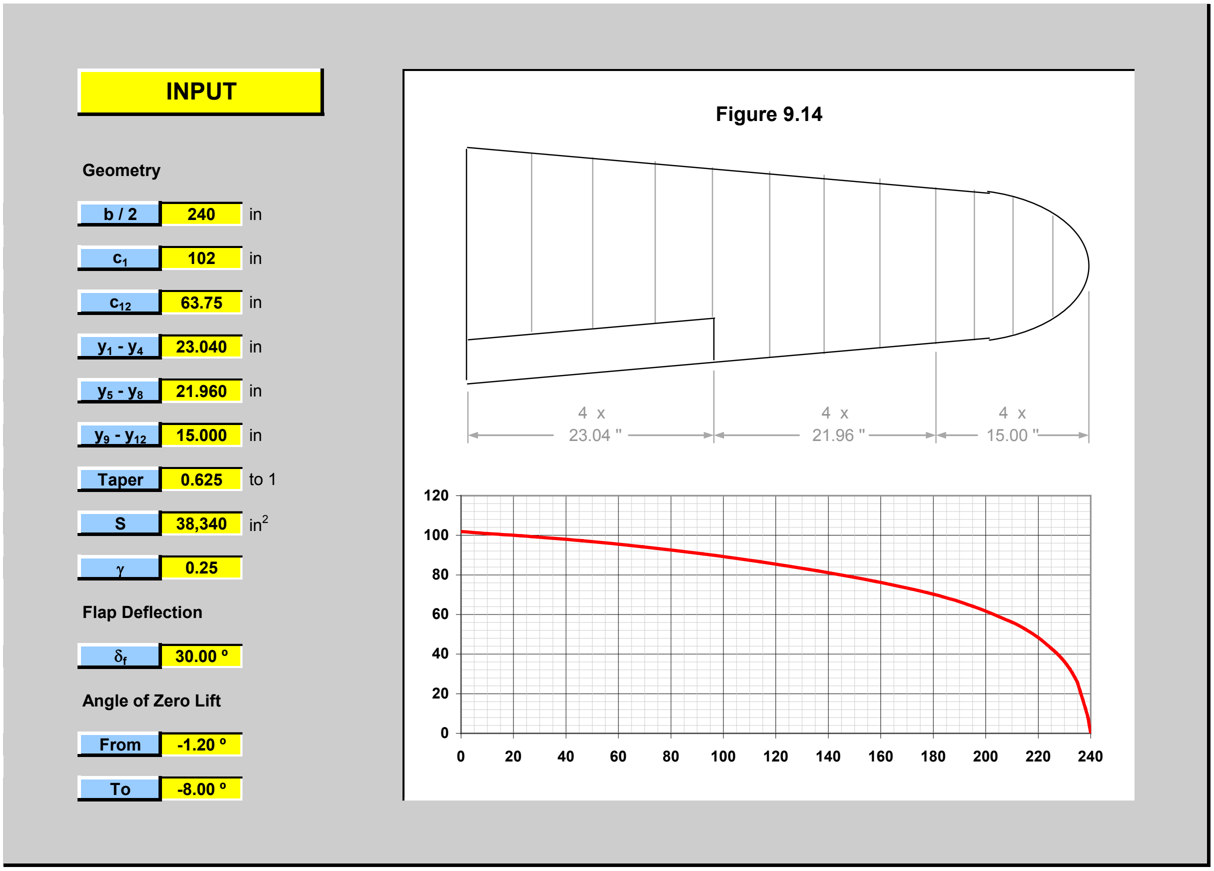
$$f_t = \frac{Design\ Load}{(\ h - d\ )\ t} = \frac{21{,}600\ lb}{(\ 1.40\ inch - 0.625\ inch\ diameter\ )\ 0.5625\ inch} = 49.5\ ksi$$

Schrenk's Method of Spanwise Air-Load Distribution

David J. Peery *Aircraft Structures* First Edition, pages 228-232

NACA TN-948 *A Simple Approximation Method for Obtaining the Spanwise Lift Distribution*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094469_1993094469.pdf .... O. Schrenk


INPUT

Geometry

b / 2	240	in
c_1	102	in
c_{12}	63.75	in
y_1 - y_4	23.040	in
y_5 - y_8	21.960	in
y_9 - y_{12}	15.000	in
Taper	0.625	to 1
S	38,340	in^2
γ	0.25	

Flap Deflection

δ_f	30.00 °

Angle of Zero Lift

From	-1.20 °
To	-8.00 °

Figure 9.14

Table 9.1

Span y (in)	2 y / b	Chord c (in)	$\sqrt{1-\left(\frac{2y}{b}\right)^2}$	$\frac{4S}{\pi b}$ (Col 4)	cc_l	c_l
0	0	102.0	1	101.7	101.85	0.999
10.00	0.042	100.0	0.999	101.6	100.81	1.008
23.04	0.096	98.3	0.995	101.2	99.77	1.015
46.08	0.192	94.7	0.981	99.8	97.25	1.027
69.12	0.288	91.0	0.958	97.4	94.20	1.035
92.16	0.384	87.4	0.923	93.9	90.65	1.037
114.12	0.476	83.8	0.880	89.5	86.63	1.034
136.08	0.567	80.3	0.824	83.8	82.04	1.022
158.04	0.659	76.8	0.753	76.5	76.67	0.998
180.00	0.750	73.3	0.661	67.3	70.28	0.959
195.00	0.813	69.2	0.583	59.3	64.24	0.928
210.00	0.875	63.0	0.484	49.2	56.12	0.891
215.00	0.896	60.0	0.444	45.2	52.60	0.877
220.00	0.917	55.8	0.400	40.6	48.22	0.864
225.00	0.938	49.5	0.348	35.4	42.45	0.857
240.00	1.000	0	0	0	0	0

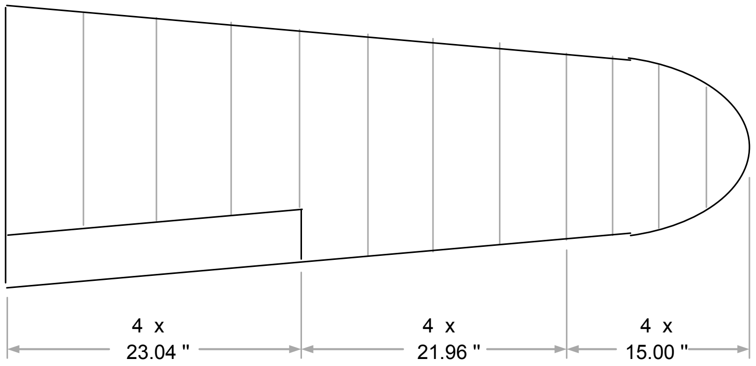
DATA

Simpson's Rule

$\Sigma\, \alpha_{aR}\, c$	59,335	
S / 2	19,170	in^2
α_{w0}	3.10 °	
m_0	0.10	per degree
m_0 / 2	0.050	per degree
A	6.01	to 1
δ_f	30.00 °	

$$\alpha_{w0} = \frac{\int_0^{b/2} m_0\, \alpha_{aR}\, c\, dy}{\int_0^{b/2} m_0\, c\, dy}$$

Figure 9.14



c c_{lb} vs. Span Position

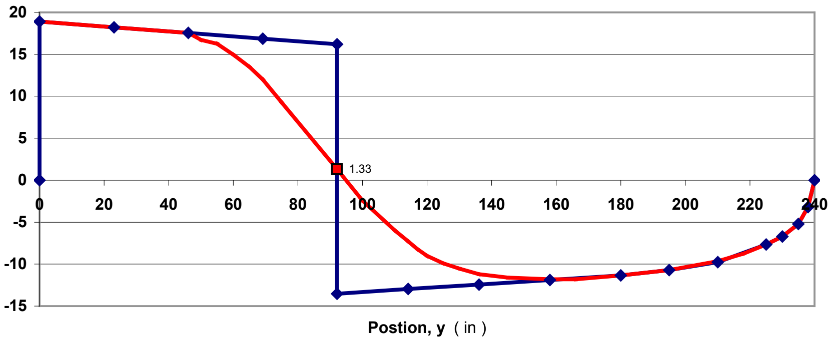


Table 9.2

Span y	Chord c (in)	α_{aR} from ref. line	α_{aR} c	α zero lift	c_{lb}	c c_{lb} (in)	c c_{lb} faired	c_{lb} / c
0	102.0	6.80	693.7	3.71 °	0.185	18.90	18.90	0.185
23.04	98.3	6.80	668.5	3.71 °	0.185	18.21	18.21	0.185
46.08	94.7	6.80	644.1	3.71 °	0.185	17.55	***17.49***	0.185
69.12	91.0	6.80	618.9	3.71 °	0.185	16.86	***11.85***	0.130
92.16	87.4	6.80	594.4	3.71 °	0.185	16.19	***1.29***	0.015
92.16	87.4	0	0	-3.10 °	-0.155	-13.53	***1.29***	0.015
114.12	83.8	0	0	-3.10 °	-0.155	-12.97	***-7.32***	-0.087
136.08	80.3	0	0	-3.10 °	-0.155	-12.43	***-11.08***	-0.138
158.04	76.8	0	0	-3.10 °	-0.155	-11.89	***-11.80***	-0.154
180.00	73.3	0	0	-3.10 °	-0.155	-11.34	-11.34	-0.155
195.00	69.2	0	0	-3.10 °	-0.155	-10.71	-10.72	-0.155
210.00	63.0	0	0	-3.10 °	-0.155	-9.75	-9.65	-0.153
225.00	49.5	0	0	-3.10 °	-0.155	-7.66	-7.67	-0.155
240.00	0.0	0	0	-3.10 °	-0.155	0	0	0

Figure 9.16

C_L = 1.72

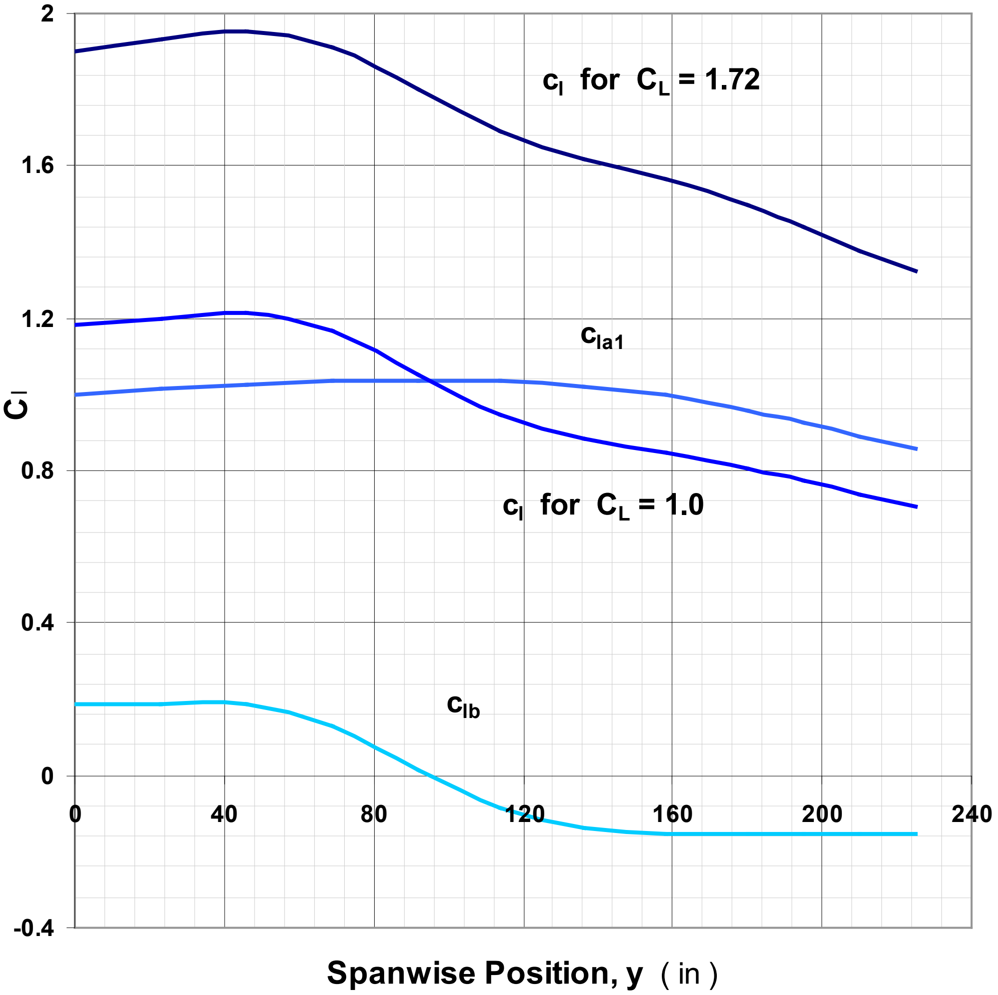


Table 9.4

θ (deg.)	θ (radians)	2 y / b	m / m	c (in)	c_s / c	sin q	c_i	y
0.0	0.00	1.000	1	0	0	0	0	y_4
22.5	0.39	0.924	1	53.0	1.9245	0.383	0.736	y_3
45.0	0.79	0.707	1	75.0	1.3600	0.707	0.962	y_2
67.5	1.18	0.383	1	87.4	1.1670	0.924	1.078	y_1
90.0	1.57	0	1	102.0	1.0000	1.000	1.000	y_0

Spanwise Distribution of Induced Drag

David J. Peery *Aircraft Structures* First Edition, pages 242-246

INPUT

Geometry

b / 2	**240**	in
c_1	**102**	in
c_{12}	**63.75**	in
y_1 - y_4	**23.040**	in
y_5 - y_8	**21.960**	in
y_9 - y_{12}	**15.000**	in
Taper	**0.625**	to 1
S	**38,340**	in^2

DATA

Assume Angle of Attack

α_a	**1.00**	radian

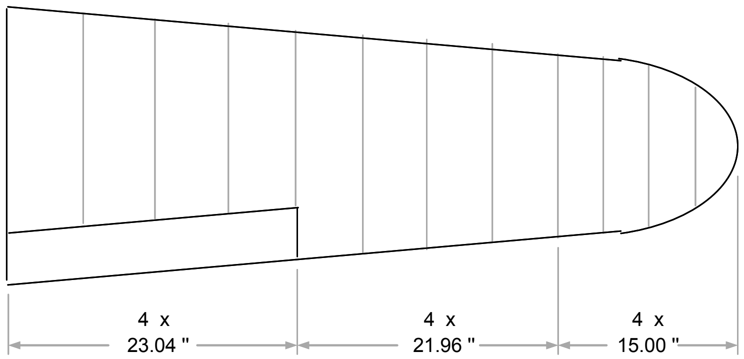
Aspect Ratio

A	**6.01**	to 1

Slope of Section Lift Curves

m_s	**0.1000**	per degree

Figure 9.14



A_1	0.7664	**C_0**	0.819	**C_2 - C_4**	-0.1400
A_3	0.0313	**C_2**	-0.371	**C_2 - C_6**	-0.2416
A_5	0.0185	**C_4**	-0.231	**C_2 - C_8**	-0.2826
		C_6	-0.129	**C_4 - C_6**	-0.1017
		C_8	-0.088		
B_1	1 = α_a	**C_{10}**	0	**P_1**	1.309
B_3	0			**P_3**	1.797
B_5	0	**C_l**	0.10	**P_5**	2.341

180 / π 57.30 ° / radian m_s 5.730 / radian $\frac{c_s\ m_s}{4\ b}$ 0.3044

Solve for A

$$\begin{vmatrix} 2.6178 & -0.1400 & -0.1017 \\ -0.1400 & 3.5937 & -0.2826 \\ -0.1017 & -0.2826 & 4.6820 \end{vmatrix} \begin{matrix} A_1 \\ A_3 \\ A_5 \end{matrix} = \begin{vmatrix} 2 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} B_1 \\ B_3 \\ B_5 \end{matrix}$$

$$\begin{vmatrix} 0.383 & 0.016 & 0.009 \\ 0.016 & 0.280 & 0.017 \\ 0.009 & 0.017 & 0.215 \end{vmatrix} \begin{matrix} 2 \\ 0 \\ 0 \end{matrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} 0.7664 \\ 0.0313 \\ 0.0185 \end{vmatrix}$$

Coefficient of Lift for the Entire Wing

$$C_L = \frac{\pi\ c_s\ m_s\ b\ A_1}{4\ S} \quad \boxed{4.404}$$

A_1 / C_L	0.1740
A_2 / C_L	0.0071
A_3 / C_L	0.0042

Table 9.4

θ (deg.)	θ (radians)	2 y / b	m / m	c (in)	c_s / c	sin q	c_i	y
0.0	0.00	1.000	1	0	0	0	0	y_4
22.5	0.39	0.924	1	53.0	1.9245	0.383	0.736	y_3
45.0	0.79	0.707	1	75.0	1.3600	0.707	0.962	y_2
67.5	1.18	0.383	1	87.4	1.1670	0.924	1.078	y_1
90.0	1.57	0	1	102.0	1.0000	1.000	1.000	y_0

Table 9.5

	Table 9.5					
1	**θ**	90 °	72 °	54 °	36 °	18 °
2	**$y = b / 2 \cos \theta$**	0	74.2	141.1	194.2	228.3
3	**$\sin \theta$**	1	0.951	0.809	0.588	0.309
4	**$\sin 3\theta$**	-1	-0.588	0.309	0.951	0.809
5	**$\sin 5\theta$**	1	0	-1	0	1
6	**$0.1740 \sin \theta$**	0.1740	0.1655	0.1408	0.1023	0.0538
7	**$0.0071 \sin 3\theta$**	-0.0071	-0.0042	0.0022	0.0068	0.0058
8	**$0.0042 \sin 5\theta$**	0.0042	0	-0.0042	0	0.0042
9	**$\Sigma A_n \sin n\theta$**	0.1711	0.1613	0.1388	0.1090	0.0637
10	**$c\, c_l$**	100.00	94.28	81.10	63.73	37.25

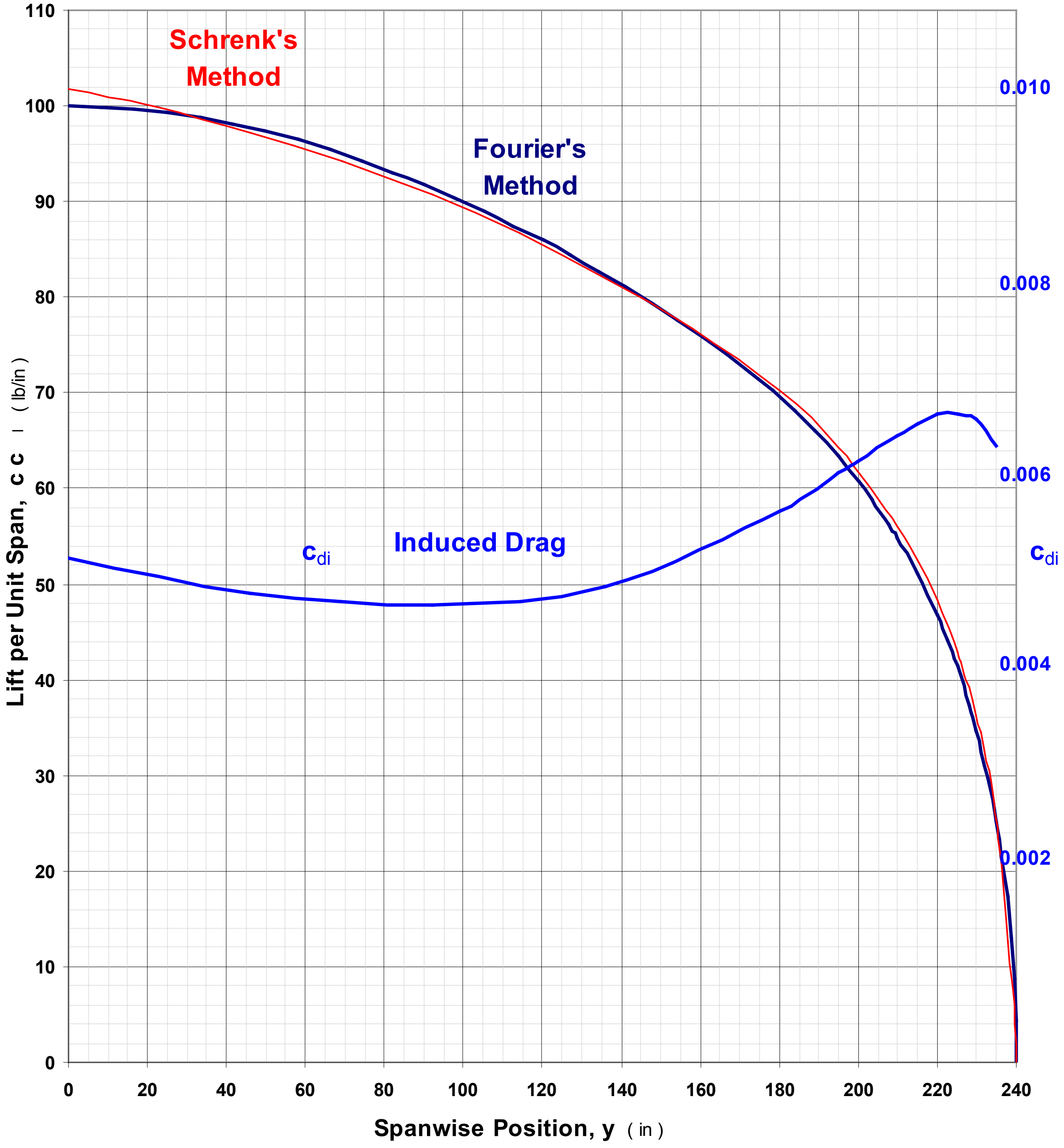
Table 9.6

	Table 9.6										
1	**y**	0	46.1	92.2	136.1	180	195	210	215	220	225
2	**cc_l**	101.85	97.3	90.7	82.0	70.0	63.3	54.8	51.1	46.7	41.4
3	**Chord Length, c**	102	94.7	87.4	80.3	73.3	69.2	*63.0*	60.0	55.8	49.5
4	**c_l**	0.999	1.027	1.037	1.022	0.955	0.915	0.870	0.852	0.838	0.837
5	**$\alpha_a = 1 / 4.41$**	0.227	0.227	0.227	0.227	0.227	0.227	0.227	0.227	0.227	0.227
6	**$c_l / m_o = c_l / 5.73$**	0.174	0.179	0.181	0.178	0.167	0.160	0.152	0.149	0.146	0.146
7	**$\alpha_a = \alpha_a - c_l / m_o$**	0.053	0.048	0.046	0.049	0.060	0.067	0.075	0.078	0.081	0.081
8	**$c_{di} = c_l\, \alpha_i$**	0.053	0.049	0.048	0.050	0.058	0.062	0.065	0.067	0.068	0.068

Plot

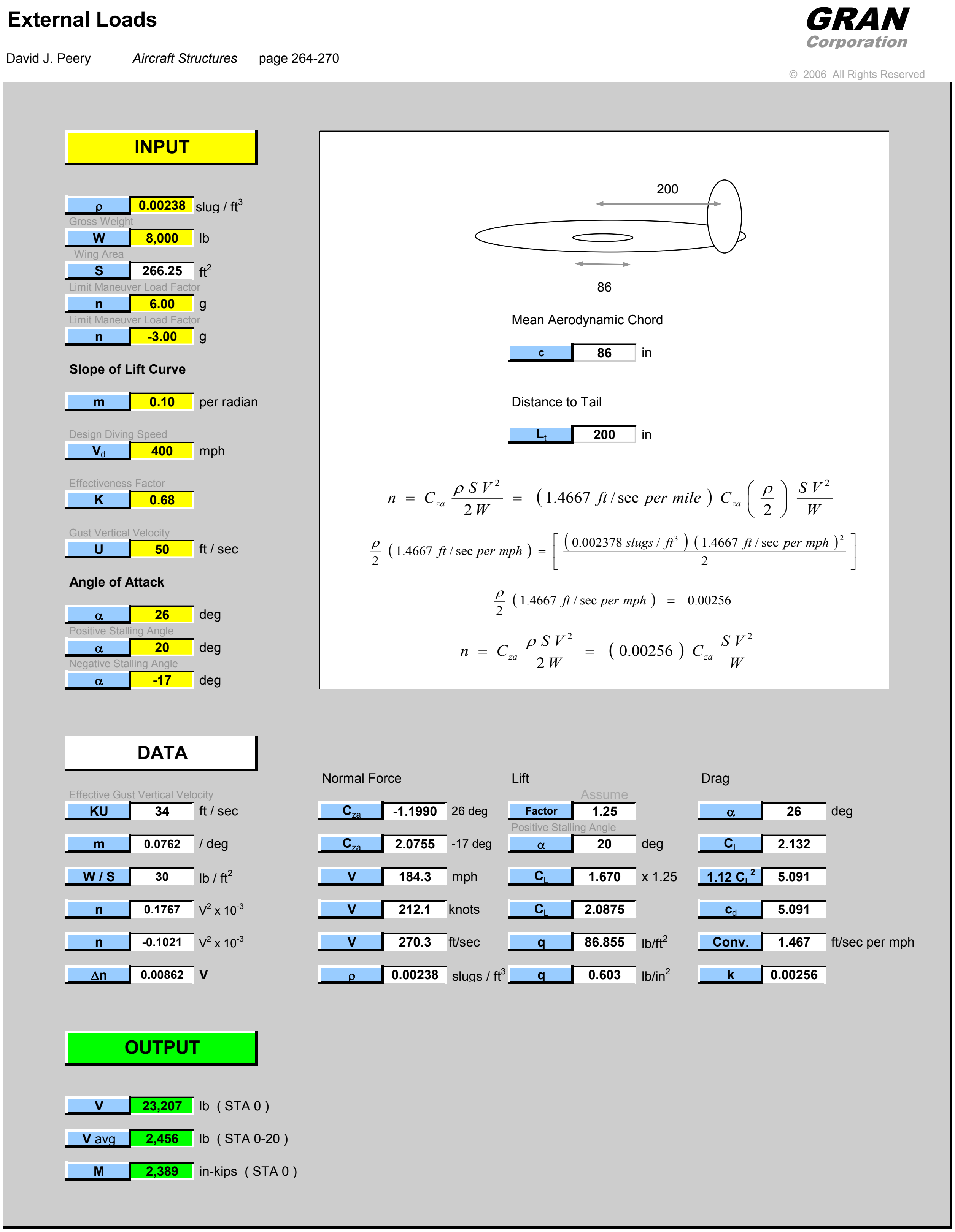
y	**1,000 c**	**c**
0	52.7	0.053
46.08	49.1	0.049
92.16	47.7	0.048
136.08	49.8	0.050
180	57.7	0.058
195	61.6	0.062
200	62.9	0.063
210	65.4	0.065
215	66.8	0.067
220	67.7	0.068
225	67.8	0.068
230	67.2	0.067
235	64.5	0.064

Graph


Schrenk's Method
Fourier's Method
Induced Drag
c_{di}
c_{di}
110
100
90
80
70
60
50
40
30
20
10
0
0.010
0.008
0.006
0.004
0.002
0
20
40
60
80
100
120
140
160
180
200
220
240
Lift per Unit Span, c c_l (lb/in)
Spanwise Position, y (in)

External Loads

David J. Peery *Aircraft Structures* First Edition, pages 264-270



External Loads

David J. Peery *Aircraft Structures* page 264-270

GRAN Corporation

INPUT

	ρ	0.00238	slug / ft^3
Gross Weight	W	8,000	lb
Wing Area	S	266.25	ft^2
Limit Maneuver Load Factor	n	6.00	g
Limit Maneuver Load Factor	n	-3.00	g

Slope of Lift Curve

	m	0.10	per radian
Design Diving Speed	V_d	400	mph
Effectiveness Factor	K	0.68	
Gust Vertical Velocity	U	50	ft / sec

Angle of Attack

	α	26	deg
Positive Stalling Angle	α	20	deg
Negative Stalling Angle	α	-17	deg

Mean Aerodynamic Chord

c	86	in

Distance to Tail

L_t	200	in

$$n = C_{za} \frac{\rho S V^2}{2W} = (1.4667 \ ft/\sec \ per \ mile) \ C_{za} \left(\frac{\rho}{2} \right) \frac{S V^2}{W}$$

$$\frac{\rho}{2} (1.4667 \ ft/\sec \ per \ mph) = \left[\frac{(0.002378 \ slugs/ft^3)(1.4667 \ ft/\sec \ per \ mph)^2}{2} \right]$$

$$\frac{\rho}{2} (1.4667 \ ft/\sec \ per \ mph) = 0.00256$$

$$n = C_{za} \frac{\rho S V^2}{2W} = (0.00256) \ C_{za} \frac{S V^2}{W}$$

DATA

Effective Gust Vertical Velocity	KU	34	ft / sec
	m	0.0762	/ deg
	W / S	30	lb / ft^2
	n	0.1767	V^2 x 10^{-3}
	n	-0.1021	V^2 x 10^{-3}
	Δn	0.00862	V

Normal Force

C_{za}	-1.1990	26 deg
C_{za}	2.0755	-17 deg
V	184.3	mph
V	212.1	knots
V	270.3	ft/sec
ρ	0.00238	slugs / ft^3

Lift

	Factor	1.25 (Assume)	
Positive Stalling Angle	α	20	deg
	C_L	1.670	x 1.25
	C_L	2.0875	
	q	86.855	lb/ft^2
	q	0.603	lb/in^2

Drag

α	26	deg
C_L	2.132	
1.12 C_L^2	5.091	
c_d	5.091	
Conv.	1.467	ft/sec per mph
k	0.00256	

OUTPUT

V	23,207	lb (STA 0)
V avg	2,456	lb (STA 0-20)
M	2,389	in-kips (STA 0)

Tables 10.1 and 10.2

Table 10.1

α	C_L	C_D	C_M
26	**2.132**	0.324	0.0400
20	1.670	0.207	0.0350
15	1.285	0.131	0.0280
10	0.900	0.076	0.0185
5	0.515	0.040	0.0070
0	0.130	0.023	-0.0105
-5	-0.255	0.026	-0.0316
-10	-0.640	0.049	-0.0525
-15	-1.025	0.092	-0.0770
-17	-1.180	0.115	-0.0860

Table 10.2

α	$C_D \sin\theta$ Drag	$C_L \cos\theta$ Lift	C_t Tail	C_{za} Airplane
26	0.1420	1.9162	0.0172	**2.0755**
20	0.0708	1.5693	0.0151	1.6551
15	0.0339	1.2412	0.0120	1.2872
10	0.0132	0.8863	0.0080	0.9075
5	0.0035	0.5130	0.0030	0.5195
0	0.0000	0.1300	-0.0045	0.1255
-5	-0.0023	-0.2540	-0.0136	-0.2699
-10	-0.0085	-0.6303	-0.0226	-0.6614
-15	-0.0238	-0.9901	-0.0331	-1.0470
-17	-0.0336	-1.1284	-0.0370	**-1.1990**

Figure 10.9

Figure 10.9

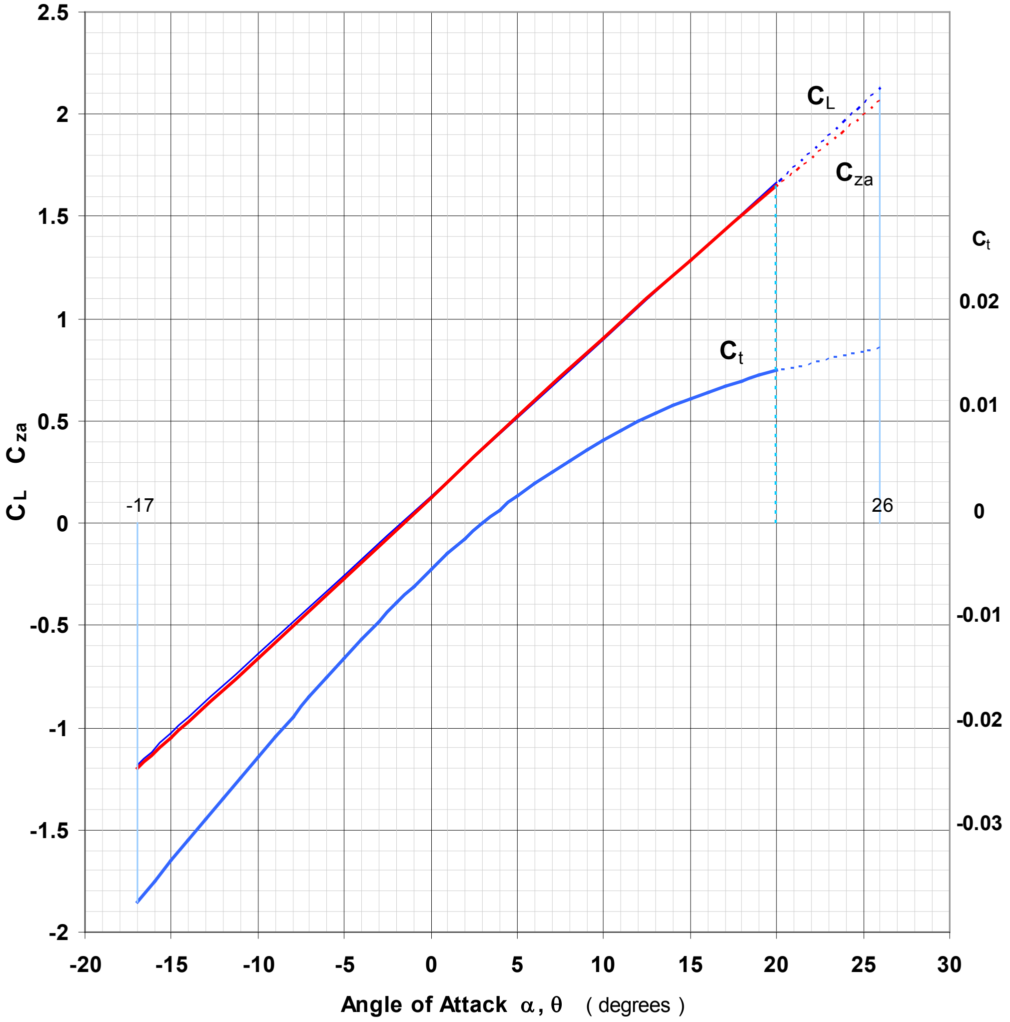


Table 10.3

Station	c_{di1}	$1.12\ C_L^2\ c_{di1}$	c_d	$c_d \sin\alpha$	c_{la1}	$c_l \cos\alpha$	c_n
	Fig 9.20		0.01 + (3)	(4) sin α	Fig 9.16	(6) cos α	(5) + (7)
220	0.068	0.345	0.355	0.156	0.880	1.687	1.843
200	0.063	0.320	0.330	0.145	0.924	1.771	1.916
180	0.058	0.294	0.304	0.133	0.964	1.847	1.981
160	0.053	0.271	0.281	0.123	0.996	1.909	2.032
140	0.050	0.252	0.262	0.115	1.019	1.952	2.067
120	0.047	0.240	0.250	0.110	1.032	1.978	2.088
100	0.046	0.235	0.245	0.107	1.038	1.988	2.096
80	0.046	0.235	0.245	0.107	1.037	1.987	2.094
60	0.047	0.241	0.251	0.110	1.032	1.977	2.087
40	0.049	0.251	0.261	0.114	1.024	1.962	2.077
20	0.052	0.264	0.274	0.120	1.013	1.942	2.062
0	0.055	0.279	0.289	0.127	0.998	1.913	2.040

Figure 9.16

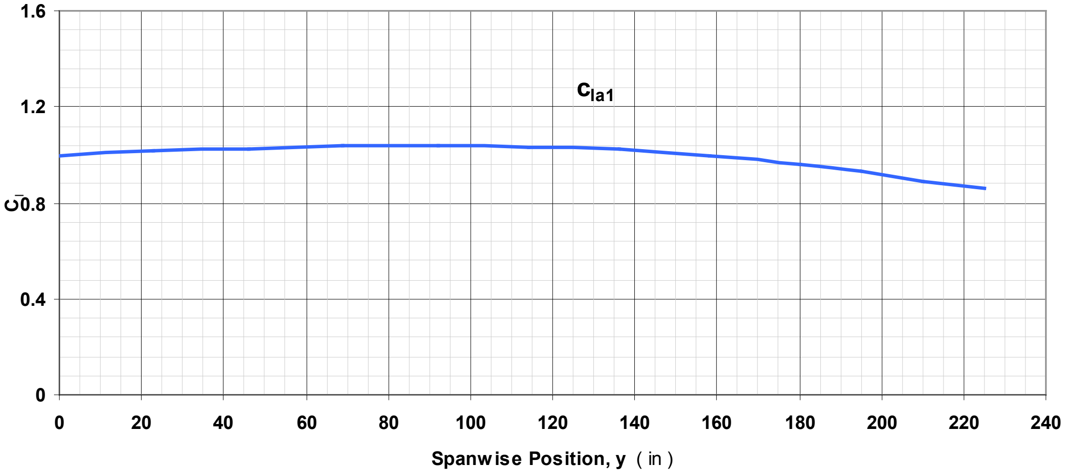
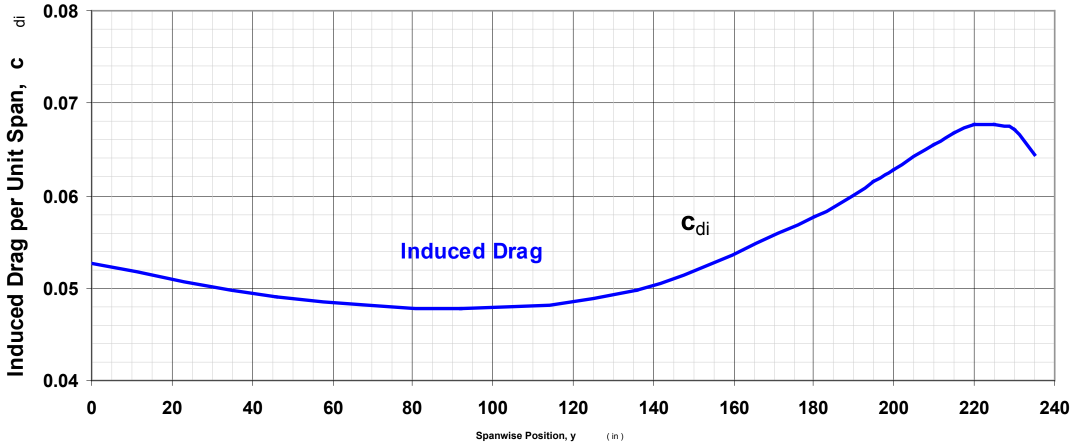


Figure 9.20



Data

V	184.3	mph
q	86.9	lb / ft^2
q	0.603	lb / in^2

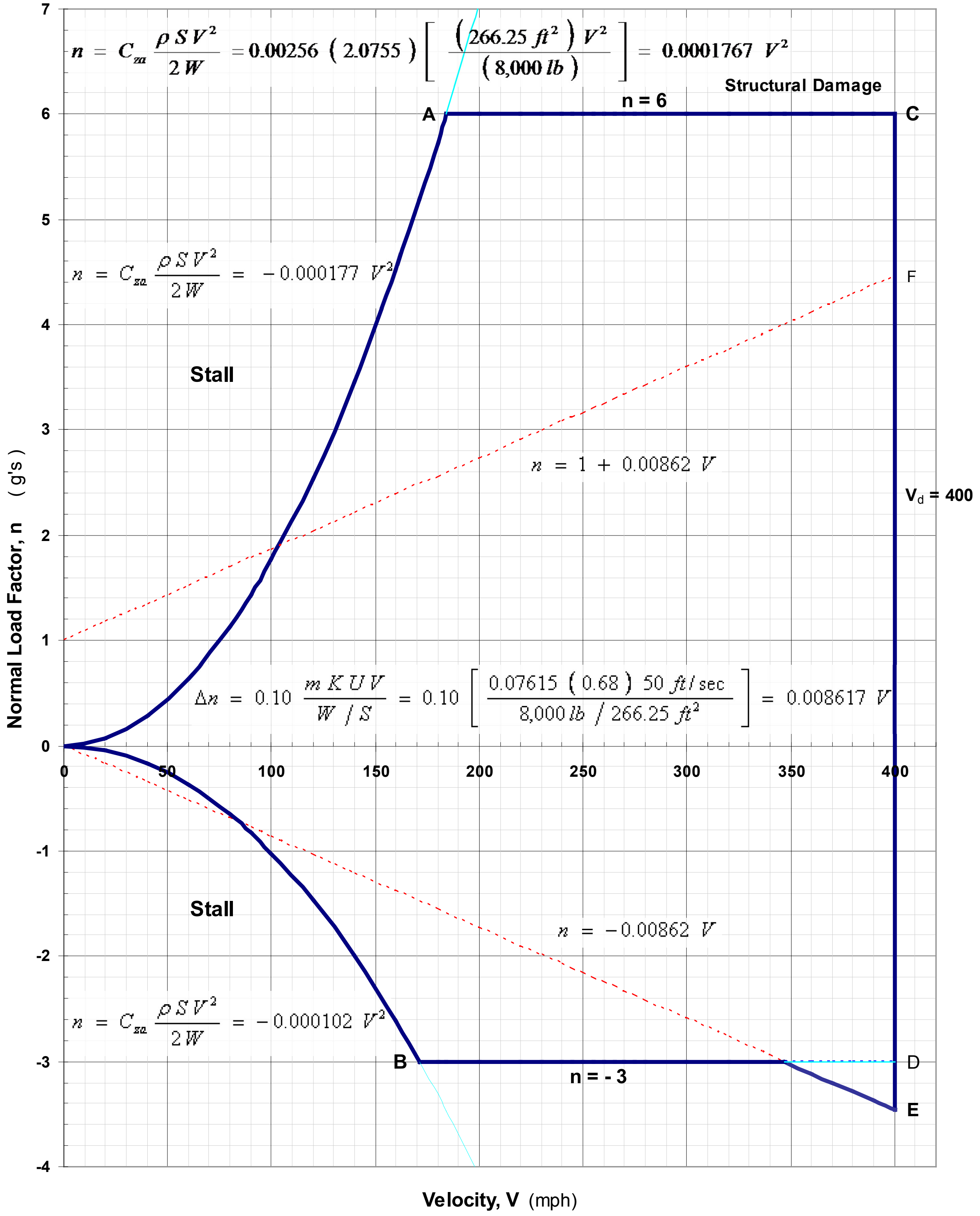
Table 10.4

					Shear Increment	Shear	Average Shear	Moment Increment	Bending Increment
Station (in)	**c** (in)	**c_n**	**$c\ c_n$** (in)	**$c\ c_n\ q/144$** (lb / in)	**ΔV** (lb)	**V** (lb)	**V_{avg}** (lb)	**ΔM /1000** (in-kip)	**M/1000** (in-kip)
240						0			0
					555.7		277.8	5.6	
220	*50.0*	1.843	92.1	55.6		555.7			5.6
					1,318.2		936.9	24.3	
200	*66.0*	1.916	126.4	76.3		1,873.9			29.9
					1,638.4		1,478.3	53.9	
180	73.3	1.981	145.2	87.6		3,512.3			83.7
					1,813.4		1,725.9	88.4	
160	76.5	2.032	155.4	93.8		5,325.7			172.1
					1,931.3		1,872.4	125.8	
140	79.7	2.067	164.8	99.4		7,257.0			297.9
					2,037.4		1,984.4	165.5	
120	82.9	2.088	173.0	104.4		9,294.4			463.4
					2,131.6		2,084.5	207.2	
100	86.1	2.096	180.4	108.8		11,426.0			670.6
					2,215.5		2,173.6	250.7	
80	89.3	2.094	187.0	112.8		13,641.5			921.3
					2,291.6		2,253.6	295.7	
60	92.4	2.087	193.0	116.4		15,933.1			1,217.1
					2,362.0		2,326.8	342.3	
40	95.6	2.077	198.6	119.8		18,295.1			1,559.3
					2,427.2		2,394.6	390.2	
20	98.8	2.062	203.8	122.9		20,722.3			1,949.5
					2,484.5		2,455.8	439.3	
0	102.0	2.040	208.1	125.5		23,206.8			2,388.8

Station	**c** (in)	**c_n**	**$c\ c_n$** (in)	**$c\ c_n\ q/144$** (lb / in)	**ΔV** (lb)	**V** (lb)	**V_{avg}** (lb)	**ΔM /1000** (in-kip)	**M/1000** (in-kip)
240						0			0
220	50.00	1.843	92.1	55.57	556	556	0	5.6	6
200	66.00	1.916	126.4	76.26	1,318	1,874	937	24.3	30
180	73.31	1.981	145.2	87.58	1,638	3,512	1,478	53.9	84
160	76.50	2.032	155.4	93.76	1,813	5,326	1,726	88.4	172
140	79.69	2.067	164.8	99.37	1,931	7,257	1,872	125.8	298
120	82.88	2.088	173.0	104.37	2,037	9,294	1,984	165.5	463
100	86.07	2.096	180.4	108.79	2,132	11,426	2,084	207.2	671
80	89.26	2.094	187.0	112.76	2,216	13,642	2,174	250.7	921
60	92.45	2.087	193.0	116.40	2,292	15,933	2,254	295.7	1,217
40	95.64	2.077	198.6	119.80	2,362	18,295	2,327	342.3	1,559
20	98.83	2.062	203.8	122.92	2,427	20,722	2,395	390.2	1,950
0	102.02	2.040	208.1	125.53	2,485	23,207	2,456	439.3	2,389

V-n Diagram

Figure 10.10



Shear and Bending Moment Diagrams

Chapter 5, pages 100-112

Beam Shear Stresses

Section 6.2, page 115   Example 1, page 117

Shear Flow in Thin Webs

Section 6.4, page 123   Example 1, page 126   Example 2, page 128   Example 3, page 129

Shear Flow Distribution in Box Beams

Section 6.7, page 133   Example 1, page 136

Tapered Beams

Section 6.8, page 141   Example 1, page 143

Beams With Variable Flange Areas

Section 6.9, page 147   Example 1, page 149

Beams With Unsymmetrical Cross Sections

Section 7.2, page 156   The “K” Method

Example 1 page 159, Example 2, page 161

Unsymmetrical Beams Supported Laterally

Section 7.3, page 162   Example 1, page 159   Example 2 page 161

Shear Flow in Unsymmetrical Beams

Section 7.5, page 169   Example, page 170

Beams with Varying Cross Sections

Section 7.5, page 169   Example, page 170

Correction of Wing Bending Moments for Sweepback

Section 7.5, page 177   Example, page 178

Distribution of Concentrated Loads to Thin Webs

Section 8.1, page 181   Example 1, Figure 8.3, page 184

Loads on Fuselage Bulkheads

Section 8.2, page 186   Example 1, page 188

Analysis of Wing Ribs

Section 8.3, Example 1, page 191-193   Example 2, page 193-194

Shear Flow in Tapered Webs

Section 8.4, page 197-202

Differential Bending

Section 8.5, page 203

Cutouts in Semi-Monocoque Structures

Section 8.5, pages 202-210

Trusses with Single Redundancy

Section 17.2, page 455   Examples 1 and 2, pages 457-459

Trusses with Multiple Redundancy

Section 17.4, page 464 Example 1, page 466

Circular Fuselage Rings

Section 17.7, page 482

Irregular Fuselage Rings

Section 17.8, page 485

Torsion of Multi-Cell Box Beams

Section 17.10, page 491

Beam Shear in Multi-Cell Structures

Section 17.11, page 493 Example, page 494

Analysis of Practical Multi-Cell Structures

Section 17.12, page 497 Example, page 498

Shear Lag

Section 17.13, page 502 Numerical Example, Section 17.15, page 506

See also Section A19.18 beginning on page A19.24 … *Analysis and Design of Flight Vehicle Structures*

Spanwise Variation of Warping Deformation

Section 17.14, page 503

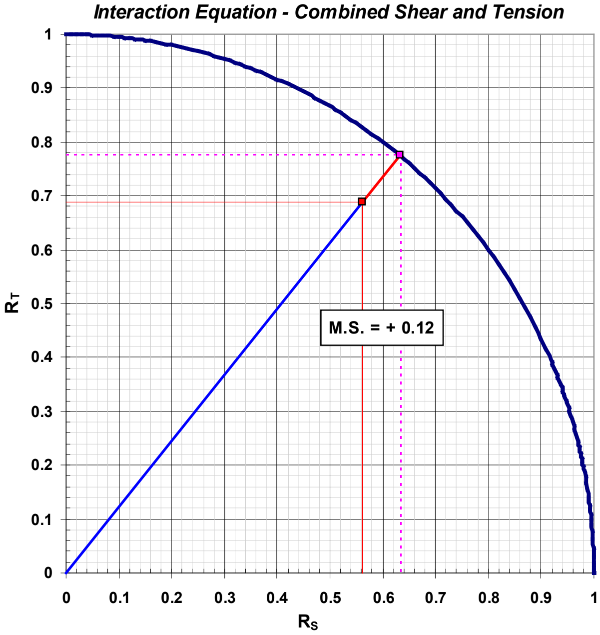
3.0 Charts and Graphs

Fastener Shear and Tension Interaction

MMPDS-01

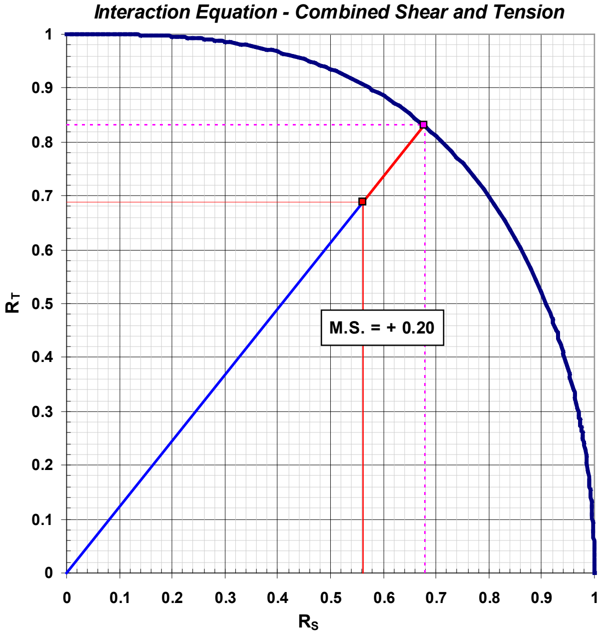
General Case

$$R_t^{\,2} + R_s^{\,2} = 1$$



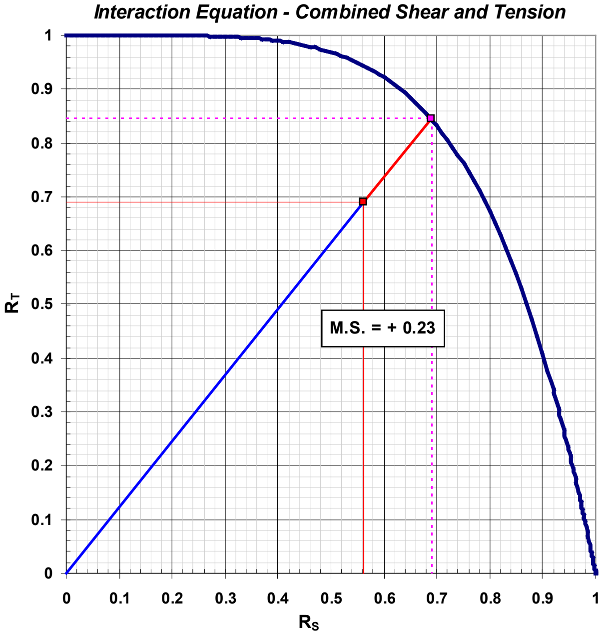
AN3 Series Bolts see MMPDS-01, page 8-125

$$R_t^{\,2} + R_s^{\,3} = 1$$



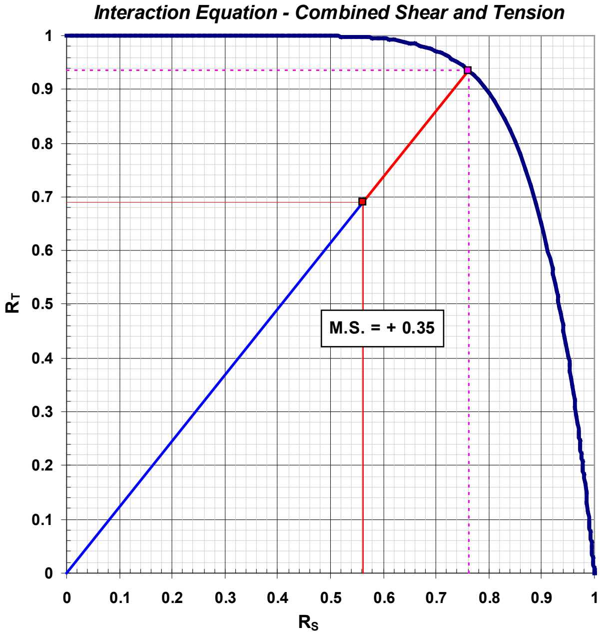
7075-T6 Lockbolts see MMPDS-01, page 8-110

$$R_t^{\,1} + R_s^{\,5} = 1$$



Steel Lockbolts see MMPDS-01, page 8-110

$$R_t^{\,1} + R_s^{\,10} = 1$$



AN Steel Bolts

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures*

Figure D1.4 and Figure D1.5, page D1.4

Figure D1.5

Figure D1.5 Shear and Tension on AN Steel Bolts

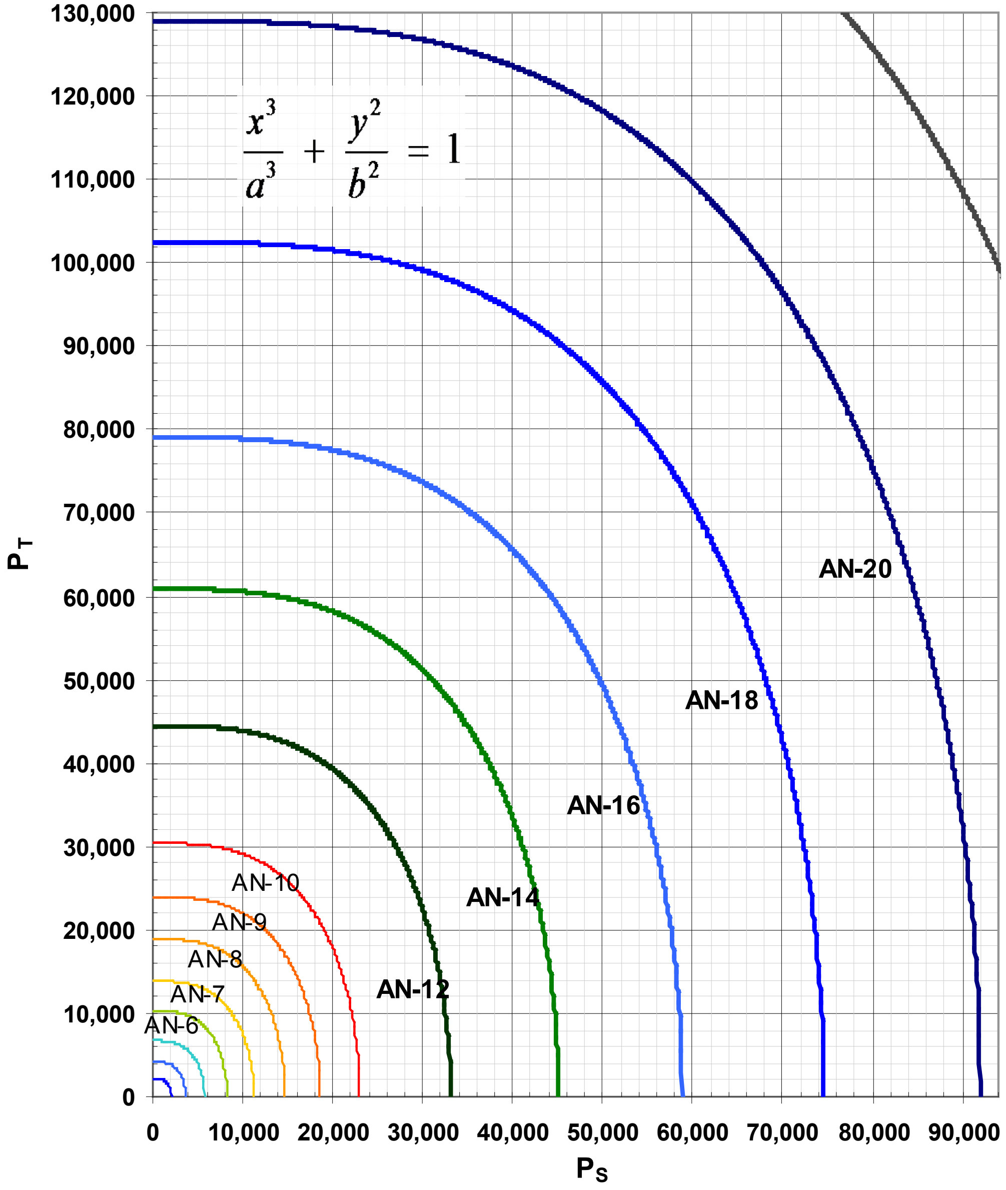


Figure D1.4

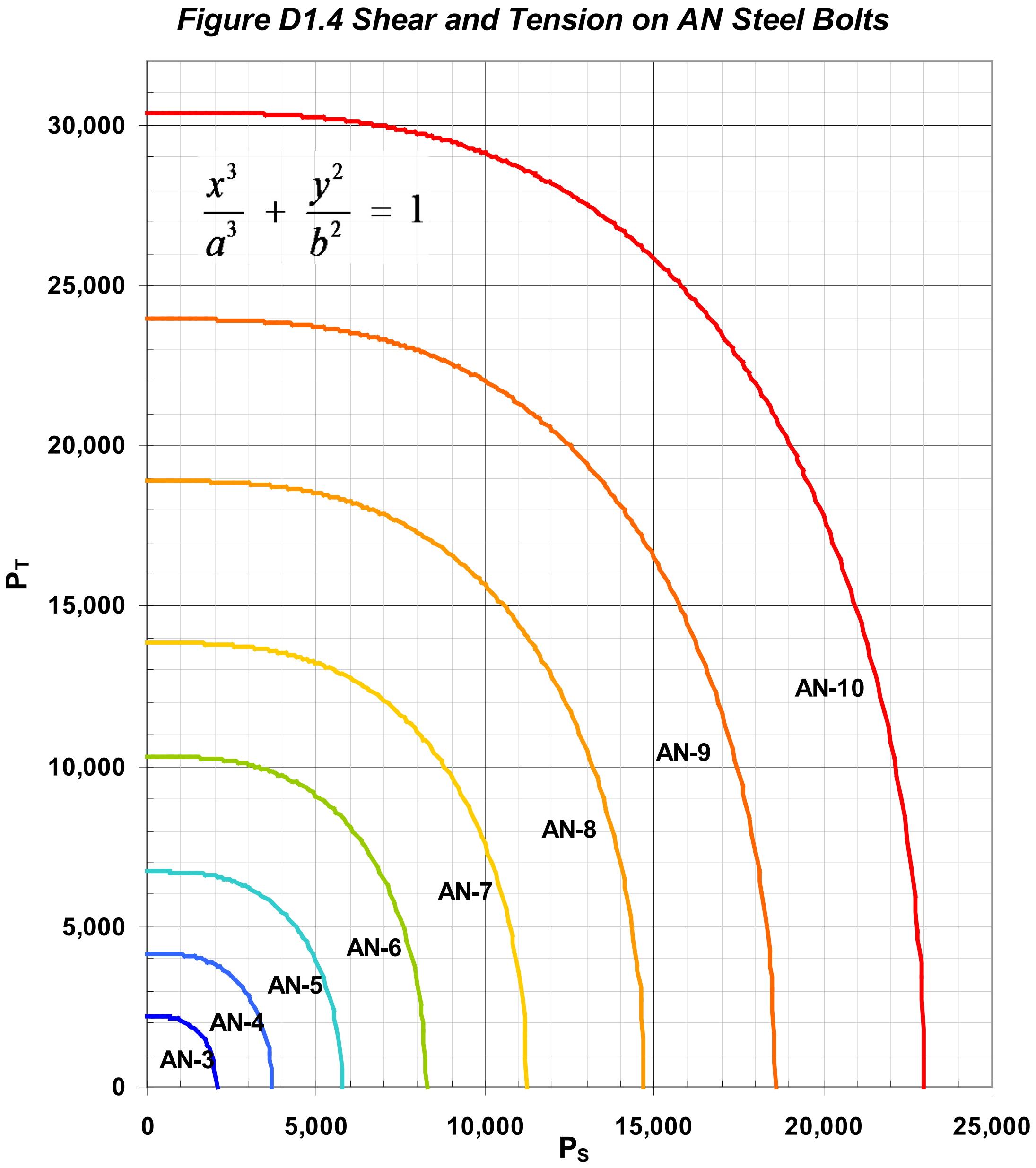


Figure D1.4 Shear and Tension on AN Steel Bolts
$\frac{x^3}{a^3} + \frac{y^2}{b^2} = 1$
P_T
P_S
30,000
25,000
20,000
15,000
10,000
5,000
0
0
5,000
10,000
15,000
20,000
25,000
AN-10
AN-9
AN-8
AN-7
AN-6
AN-5
AN-4
AN-3

ANC-5 Strength of Metal Aircraft Elements

ANC-5 *Strength of Metal Aircraft Elements* Revised March 1955

Department of the Air Force
Air Research and Development Command

Department of the Navy
Bureau of Aeronautics

Department of Commerce
Civil Aeronautics Administration

Section 1.535 Failure Under Combined Loadings, pages 11-13.

(For ANC-5, June 1951 see Section 1.535, pages 11-12)

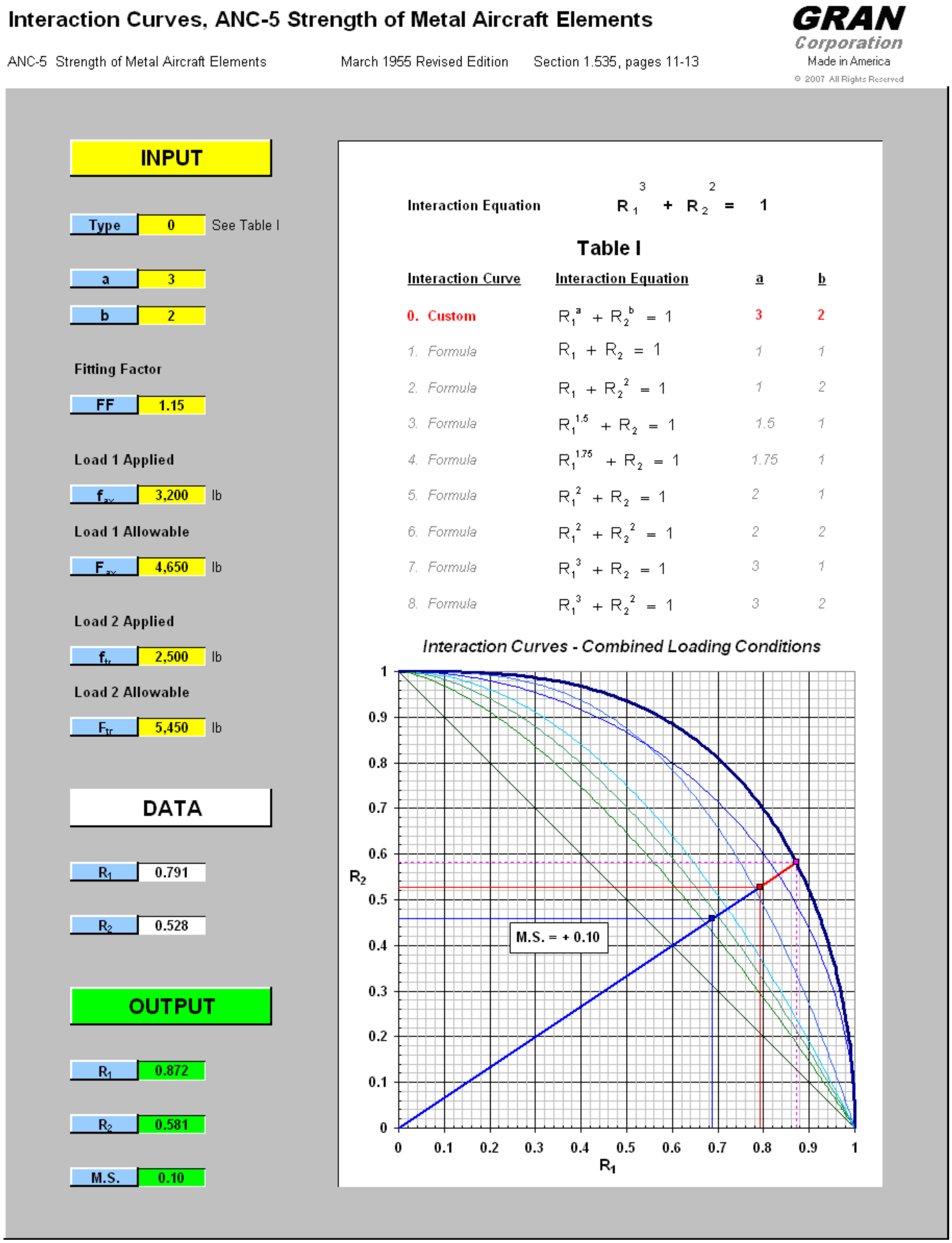
Interaction Formula	**Margin of Safety**
$R_1 + R_2 = 1.0$	$MS = \dfrac{1}{R_1 + R_2} - 1$
$R_1 + R_2^2 = 1.0$	$MS = \dfrac{2}{R_1 + \sqrt{R_1^2 + 4R_2^2}} - 1$
$R_1^2 + R_2 = 1.0$	$MS = \dfrac{2}{R_2 + \sqrt{R_2^2 + 4R_1^2}} - 1$
$R_1^2 + R_2^2 = 1.0$	$MS = \dfrac{1}{\sqrt{R_1^2 + R_2^2}} - 1$
$R_1 + R_2 + R_3 = 1.0$	$MS = \dfrac{1}{R_1 + R_2 + R_3} - 1$
$R_1 + R_2 + R_3^2 = 1.0$	$MS = \dfrac{2}{R_1 + R_2 + \sqrt{\left(R_1 + R_2\right)^2 + 4R_3^2}} - 1$
$R_1 + R_2^2 + R_3^2 = 1.0$	$MS = \dfrac{2}{R_1 + \sqrt{R_1^2 + 4\left(R_2^2 + R_3^2\right)}} - 1$
$R_1^2 + R_2^2 + R_3^2 = 1.0$	$MS = \dfrac{1}{\sqrt{R_1^2 + R_2^2 + R_3^2}} - 1$

Interaction Curves

ANC-5 *Strength of Metal Aircraft Elements* Revised March 1955

Figure 1.535 Typical Interaction Curves for Combined Loading Conditions, page 13.

(For ANC-5, June 1951 see Figure 1.535, page 12)



Interaction Curves, ANC-5 Strength of Metal Aircraft Elements

ANC-5 Strength of Metal Aircraft Elements March 1955 Revised Edition Section 1.535, pages 11-13

GRAN Corporation
Made in America

INPUT

Type: 0 See Table I

a: 3

b: 2

Fitting Factor

FF: 1.15

Load 1 Applied

f_{ax}: 3,200 lb

Load 1 Allowable

F_{ax}: 4,650 lb

Load 2 Applied

f_{tr}: 2,500 lb

Load 2 Allowable

F_{tr}: 5,450 lb

DATA

R_1: 0.791

R_2: 0.528

OUTPUT

R_1: 0.872

R_2: 0.581

M.S.: 0.10

Interaction Equation $R_1^{3} + R_2^{2} = 1$

Table I

Interaction Curve	Interaction Equation	a	b
0. Custom	$R_1^{a} + R_2^{b} = 1$	3	2
1. Formula	$R_1 + R_2 = 1$	1	1
2. Formula	$R_1 + R_2^{2} = 1$	1	2
3. Formula	$R_1^{1.5} + R_2 = 1$	1.5	1
4. Formula	$R_1^{1.75} + R_2 = 1$	1.75	1
5. Formula	$R_1^{2} + R_2 = 1$	2	1
6. Formula	$R_1^{2} + R_2^{2} = 1$	2	2
7. Formula	$R_1^{3} + R_2 = 1$	3	1
8. Formula	$R_1^{3} + R_2^{2} = 1$	3	2

Interaction Curves - Combined Loading Conditions

Interaction Surfaces

Interaction Formula

$$R_1 + R_2 + R_3 = 1$$

Margin of Safety

$$M\,S = \frac{1}{R_1 + R_2 + R_3} - 1$$

Interaction Curves, ANC-5 Strength of Metal Aircraft Elements

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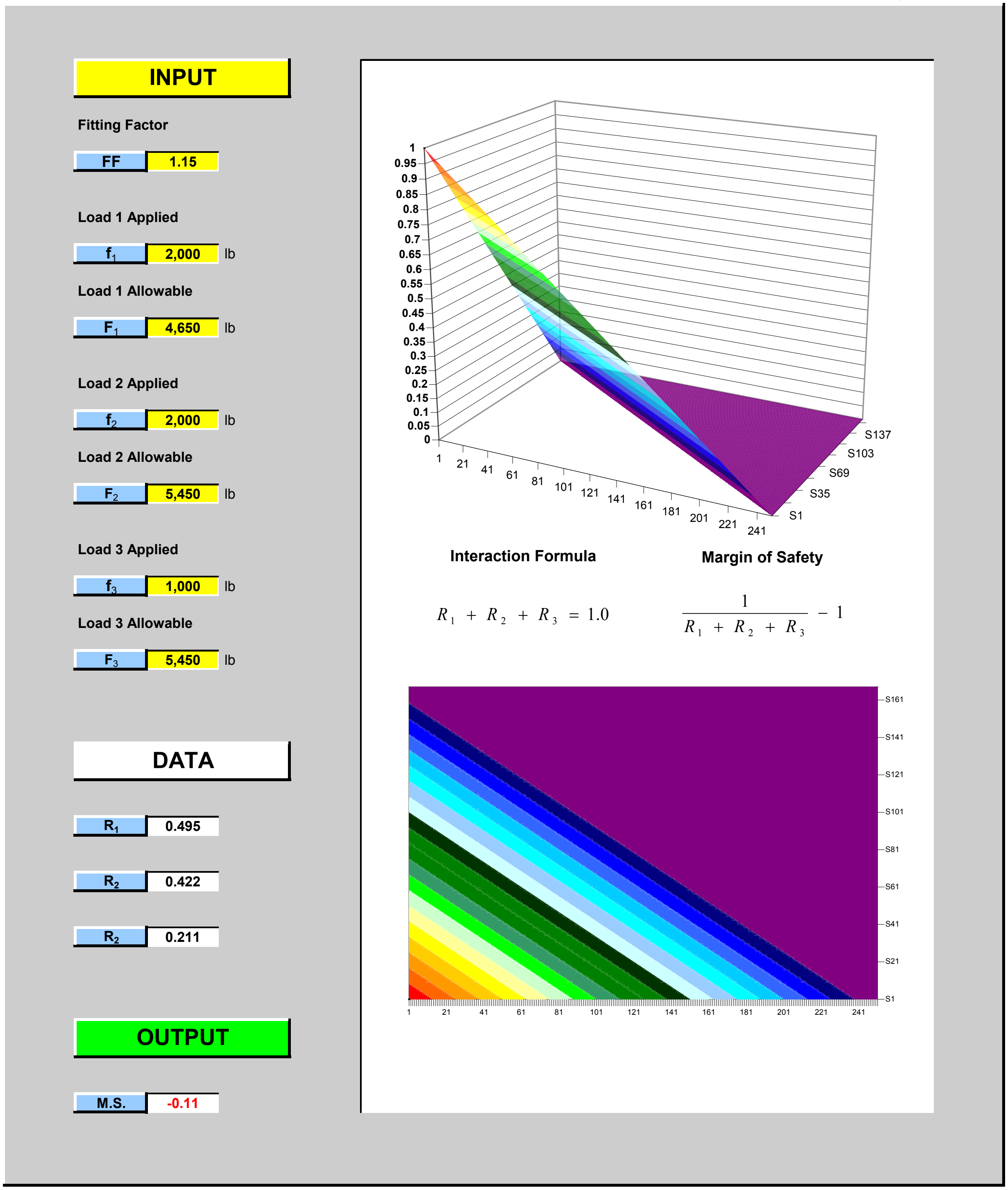
ANC-5 Strength of Metal Aircraft Elements March 1955 Revised Edition Section 1.535, pages 11-13



INPUT

Fitting Factor

FF	1.15

Load 1 Applied

f_1 2,000 lb

Load 1 Allowable

F_1 4,650 lb

Load 2 Applied

f_2 2,000 lb

Load 2 Allowable

F_2 5,450 lb

Load 3 Applied

f_3 1,000 lb

Load 3 Allowable

F_3 5,450 lb

DATA

R_1 0.495

R_2 0.422

R_2 0.211

OUTPUT

M.S. -0.11

Interaction Formula

$$R_1 + R_2 + R_3 = 1.0$$

Margin of Safety

$$\frac{1}{R_1 + R_2 + R_3} - 1$$

Interaction Formula

$$R_1 + R_2 + R_3^{\,2} = 1$$

Margin of Safety

$$M\,S = \frac{2}{R_1 + R_2 + \sqrt{(R_1 + R_2)^2 + 4\,R_3^{\,2}}} - 1$$

Interaction Curves, ANC-5 Strength of Metal Aircraft Elements

ANC-5 Strength of Metal Aircraft Elements   March 1955 Revised Edition   Section 1.535, pages 11-13

GRAN
Corporation
Made in America

INPUT

Fitting Factor

FF	1.15

Load 1 Applied

f_1	2,000	lb

Load 1 Allowable

F_1	4,650	lb

Load 2 Applied

f_2	2,000	lb

Load 2 Allowable

F_2	5,450	lb

Load 3 Applied

f_3	1,000	lb

Load 3 Allowable

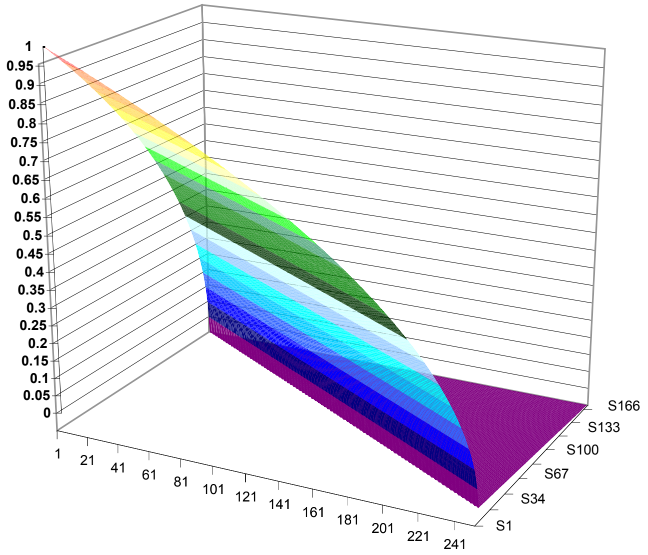
F_3	5,450	lb

DATA

R_1	0.495
R_2	0.422
R_2	0.211

OUTPUT

M.S.	0.04

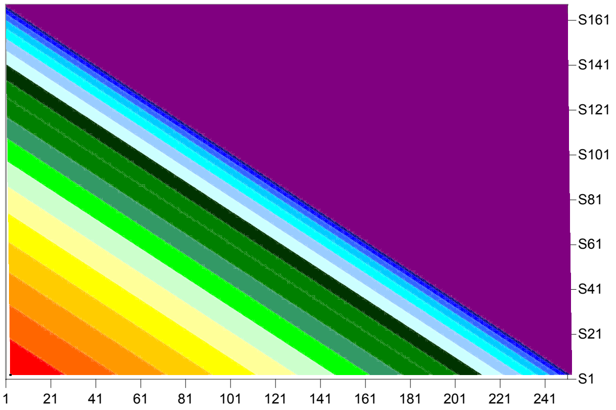


Interaction Formula

$$R_1 + R_2 + R_3^{\,2} = 1.0$$

Margin of Safety

$$\frac{2}{R_1 + R_2 + \sqrt{(R_1 + R_2)^2 + 4\,R_3^{\,2}}} - 1$$



Interaction Formula

$$R_1 + R_2^{\,2} + R_3^{\,2} = 1$$

Margin of Safety

$$M\,S = \frac{2}{R_1 + \sqrt{R_1^{\,2} + 4\left(R_2^{\,2} + R_3^{\,2}\right)}} - 1$$

Interaction Curves, ANC-5 Strength of Metal Aircraft Elements

ANC-5 Strength of Metal Aircraft Elements March 1955 Revised Edition Section 1.535, pages 11-13

GRAN
Corporation
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INPUT

Fitting Factor

FF	1.15	

Load 1 Applied

f_1	2,000	lb

Load 1 Allowable

F_1	4,650	lb

Load 2 Applied

f_2	2,000	lb

Load 2 Allowable

F_2	5,450	lb

Load 3 Applied

f_3	1,000	lb

Load 3 Allowable

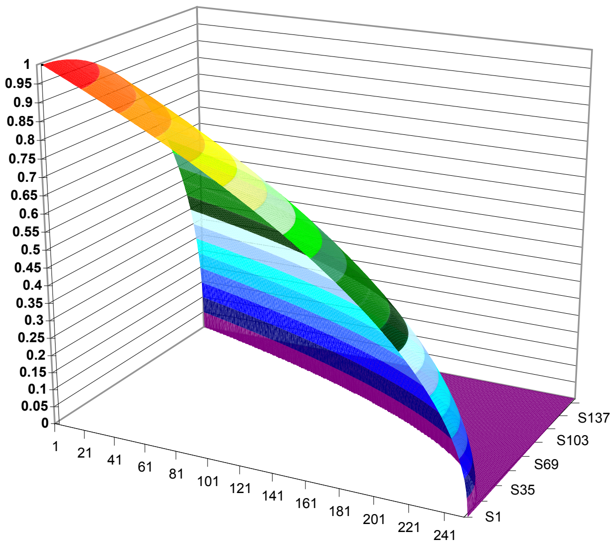
F_3	5,450	lb

DATA

R_1	0.495
R_2	0.422
R_2	0.211

OUTPUT

M.S.	0.28

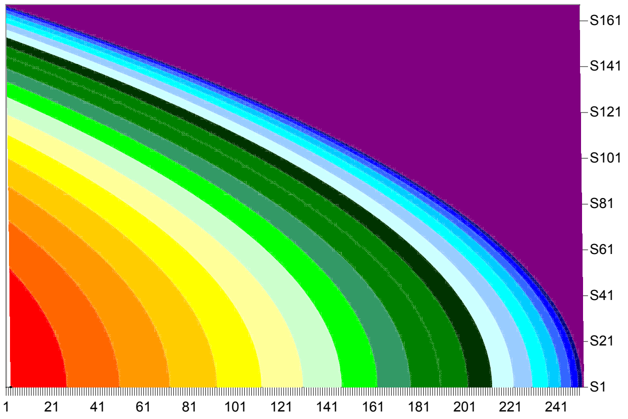


Interaction Formula

$$R_1 + R_2^{\,2} + R_3^{\,2} = 1.0$$

Margin of Safety

$$\frac{2}{R_1 + \sqrt{R_1^{\,2} + 4\left(R_2^{\,2} + R_3^{\,2}\right)}} - 1$$



Interaction Formula

$$R_1^2 + R_2^2 + R_3^2 = 1$$

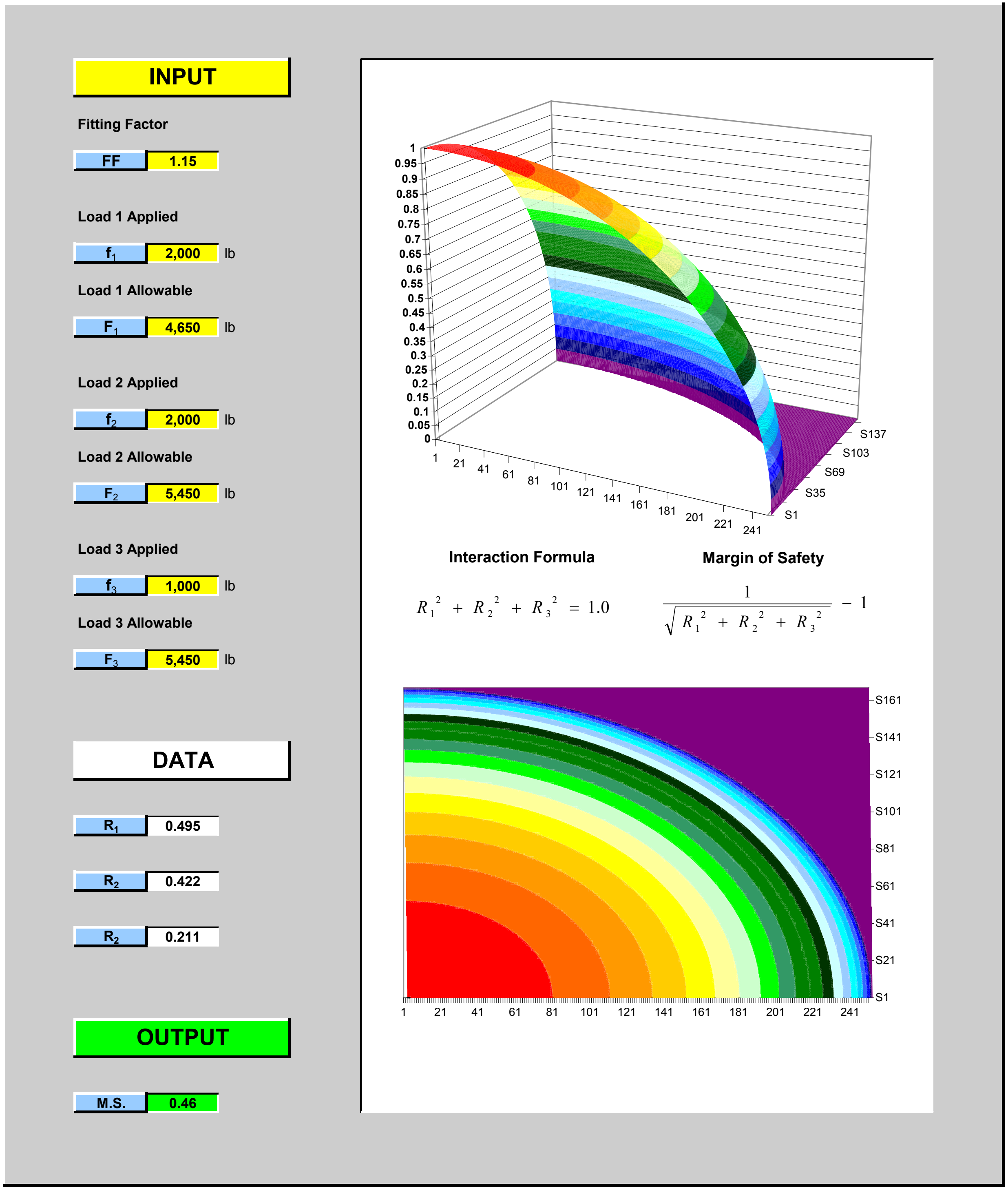
Margin of Safety

$$MS = \frac{1}{\sqrt{R_1^2 + R_2^2 + R_3^2}} - 1$$

Interaction Curves, ANC-5 Strength of Metal Aircraft Elements

ANC-5 Strength of Metal Aircraft Elements March 1955 Revised Edition Section 1.535, pages 11-13

GRAN Corporation
Made in America



INPUT

Fitting Factor

FF	1.15	
Load 1 Applied		
f_1	2,000	lb
Load 1 Allowable		
F_1	4,650	lb
Load 2 Applied		
f_2	2,000	lb
Load 2 Allowable		
F_2	5,450	lb
Load 3 Applied		
f_3	1,000	lb
Load 3 Allowable		
F_3	5,450	lb

DATA

R_1	0.495
R_2	0.422
R_2	0.211

OUTPUT

M.S.	0.46

Interaction Formula

$$R_1^2 + R_2^2 + R_3^2 = 1.0$$

Margin of Safety

$$\frac{1}{\sqrt{R_1^2 + R_2^2 + R_3^2}} - 1$$

Lug Loads - Axial and Transverse Interaction

Lug Interaction Equation

$$R_{ax}^{1.6} + R_{tr}^{1.6} = 1$$

$$M.S. = \frac{1}{\left(R_{ax}^{1.6} + R_{tr}^{1.6} \right)^{0.625}} - 1$$

Bruhn page D1.8

Product Engineering

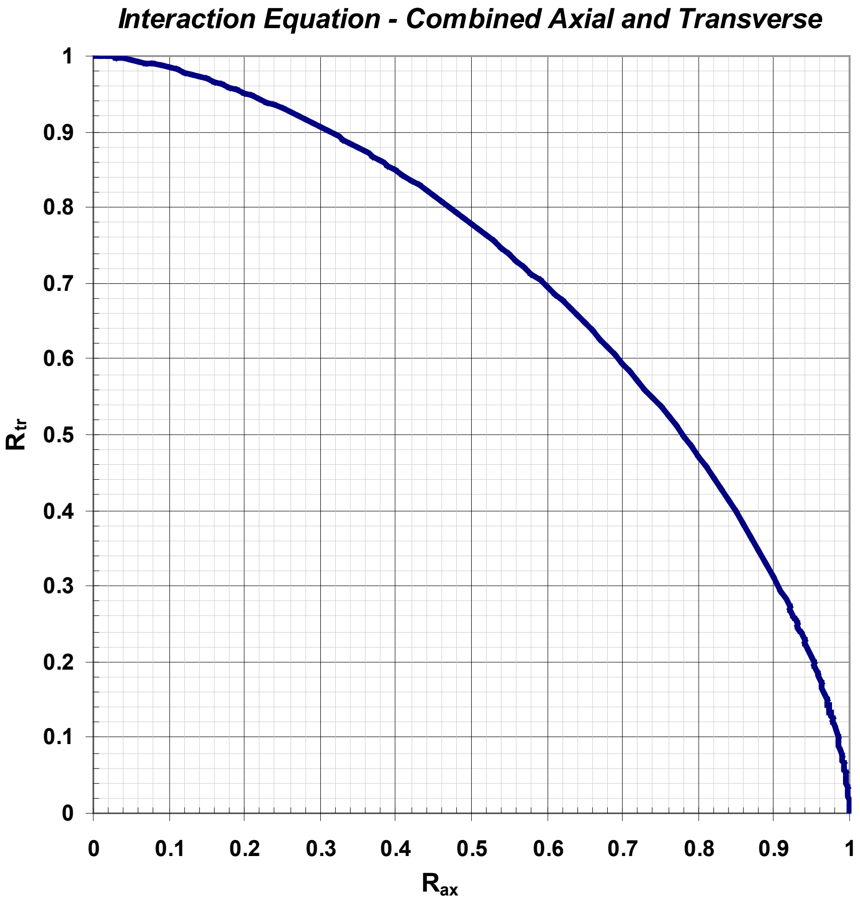
Analysis of Lugs and Shear Pins Made of Aluminum and Steel Alloys

F.P.Cozzone, M.A. Melcon and F.M.Hoblit

Product Engineering, Volume 21, Number 5, pages 113-117, May 1950

Developments in the Analysis of Lugs and Shear Pins, M.A. Melcon and F.M.Hoblit

Product Engineering, Volume 24, Number 6, pages 160-170, June 1953



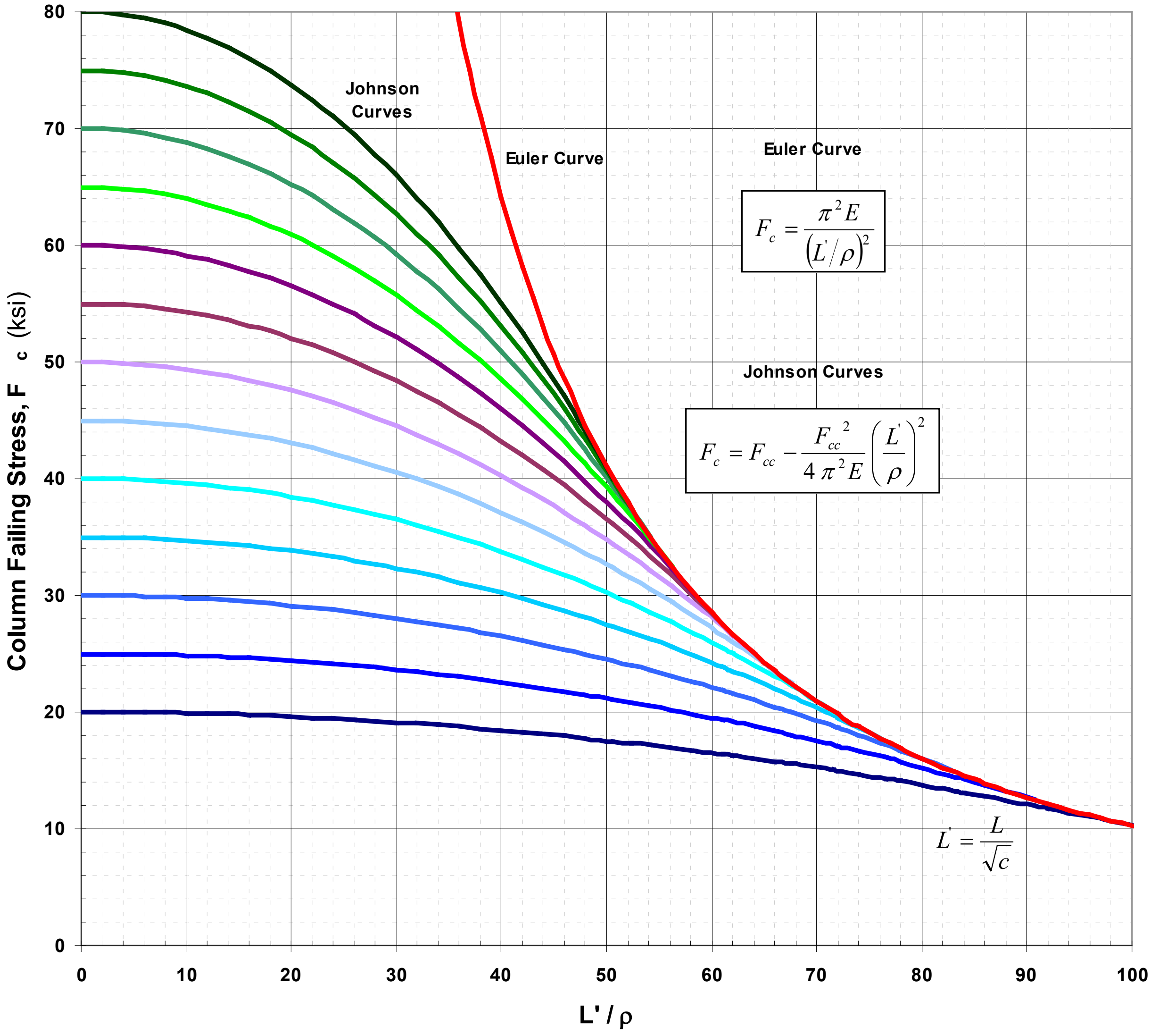
Johnson-Euler Column Curves

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures*, Figure C7.33, page C7.23

Aluminum Alloy

For E = 10.4 E6 psi

Figure C7.33 *Johnson and Euler Column Curves*

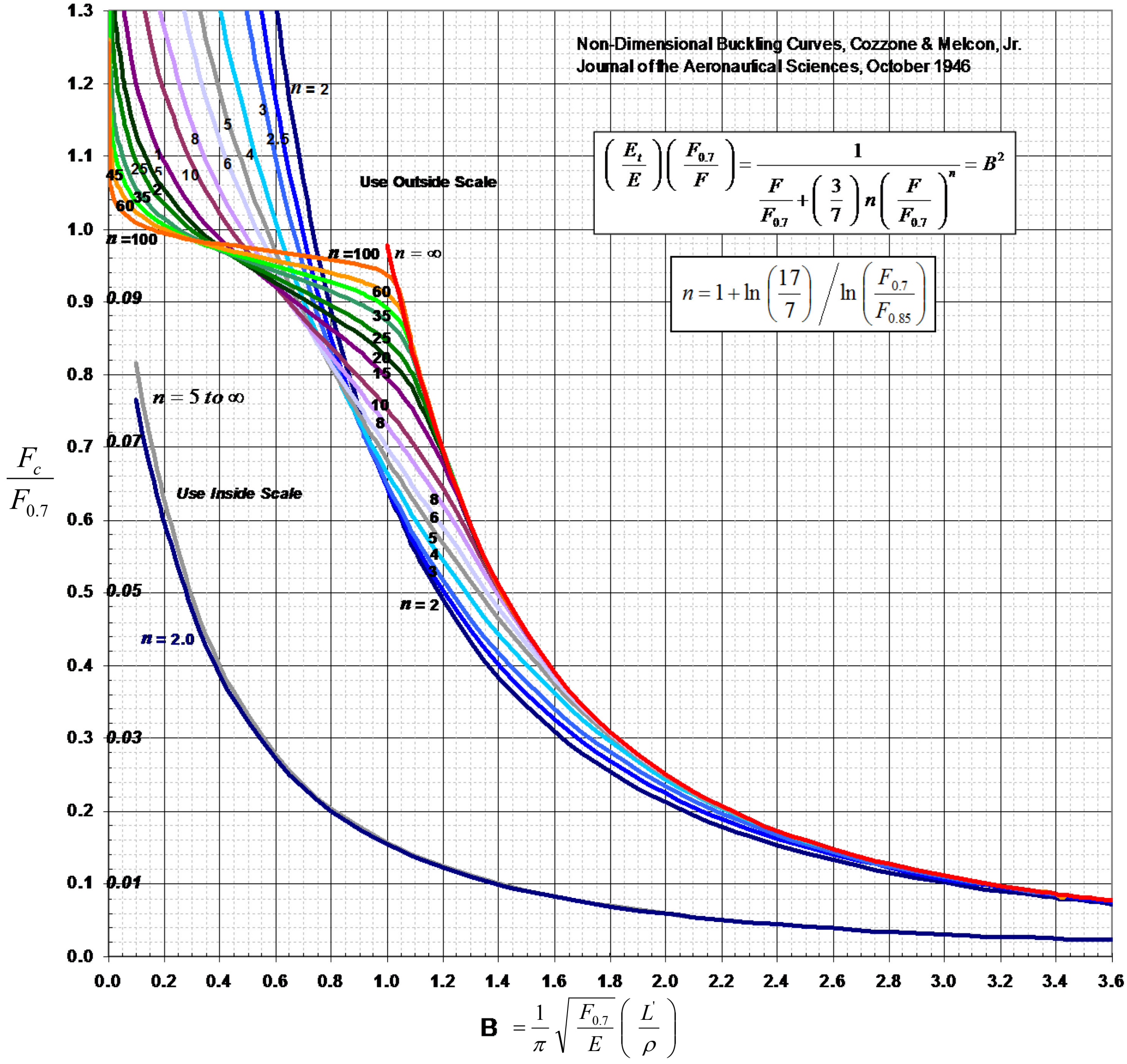


Cozzone-Melcon Non-Dimensional Column Buckling Curves

Cozzone & Melcon, Jr.	*Non-Dimensional Buckling Curves*
	Journal of the Aeronautical Sciences, October 1946
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*, Figure C2.17, page C2.7

Figure C2.17 ***Non-Dimensional Column Buckling Curves***

F_c / $F_{0.7}$ vs. B



Tangent Modulus from the Ramberg-Osgood Equation

Ramberg & Osgood	NACA TN-902 *Description of Stress-Strain Curves of Three Parameters*, Figure 8
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*, Figure C2.16, page C2.6

$$\frac{E_t}{E} = \frac{1}{1 + \frac{3}{7}\, n \left(\frac{F}{F_{0.7}} \right)^{n-1}} \qquad n = 1 + \frac{\ln\left(\frac{17}{7}\right)}{\ln\left(\frac{F_{0.7}}{F_{0.85}}\right)}$$

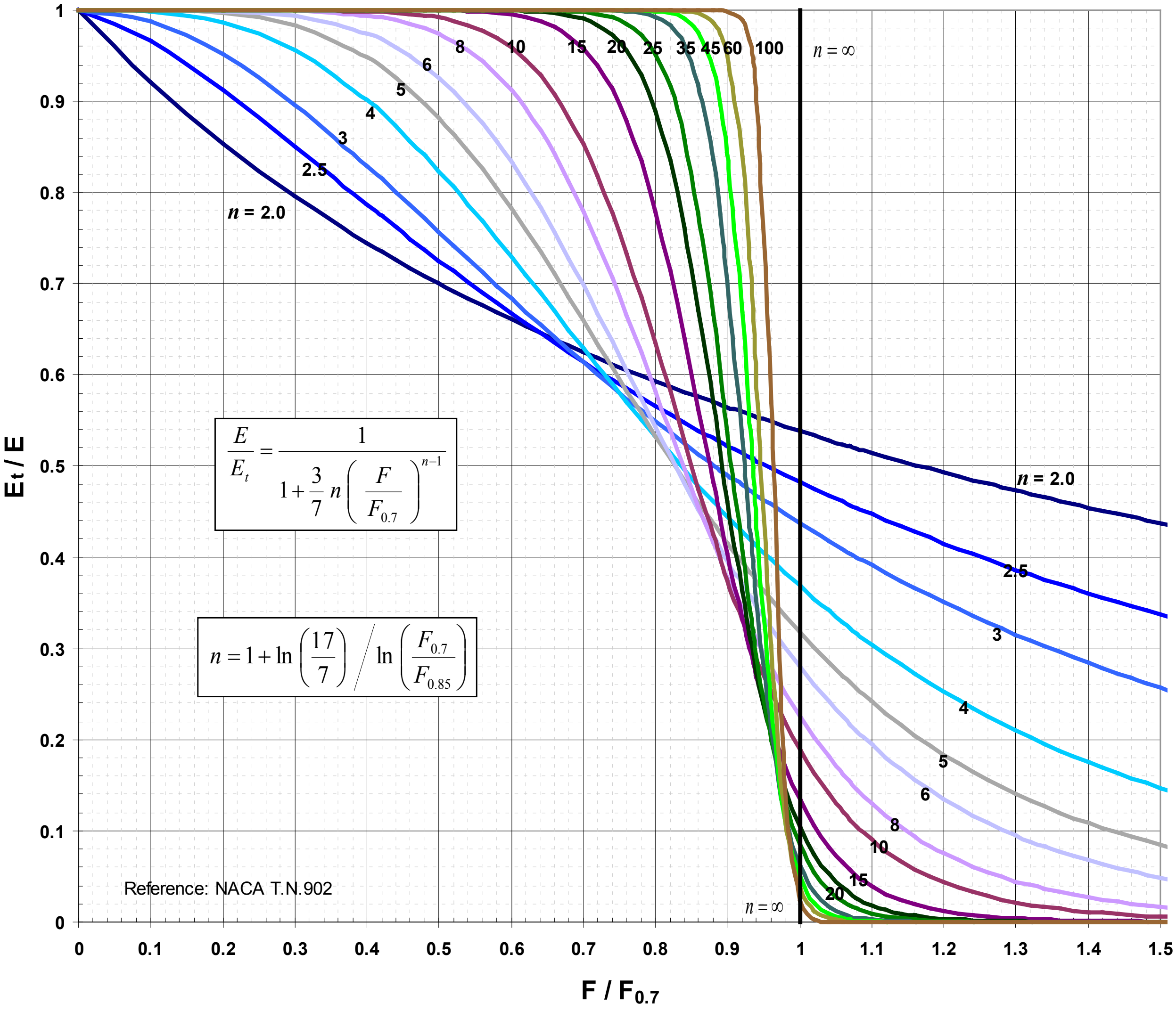


Figure C2.16 Dimensionless Tangent Modulus Stress Curves

E_t / E vs. F / $F_{0.7}$

Secant Modulus from the Ramberg-Osgood Equation

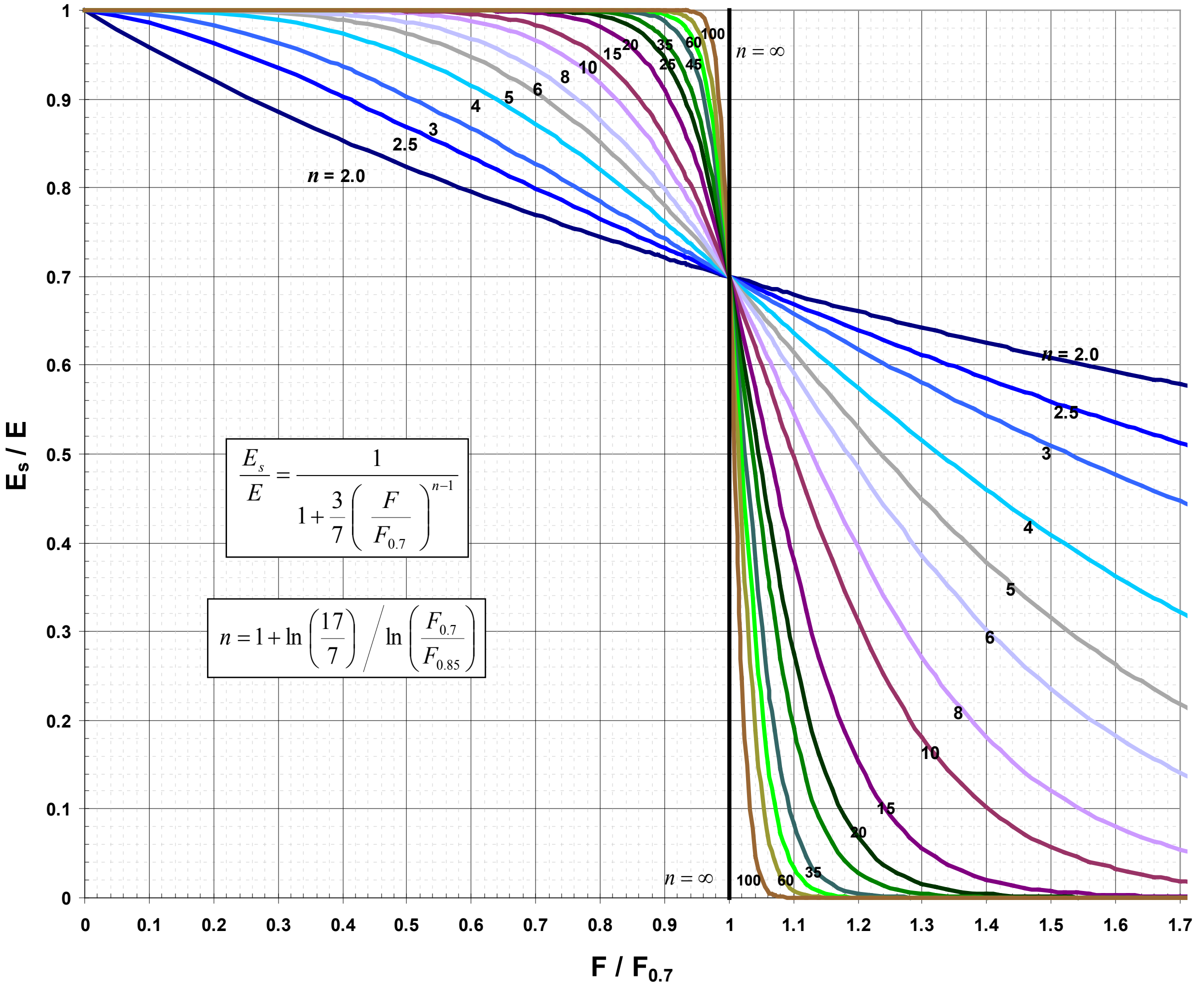
Ramberg & Osgood NACA TN-902 *Description of Stress-Strain Curves of Three Parameters*

Michael C. Y. Niu *Airframe Structural Analysis*, Figure 4.1.4, page 93

$$\frac{E_s}{E} = \frac{1}{1 + \frac{3}{7}\left(\frac{F}{F_{0.7}}\right)^{n-1}} \qquad n = 1 + \frac{\ln\left(\frac{17}{7}\right)}{\ln\left(\frac{F_{0.7}}{F_{0.85}}\right)}$$

Figure 4.1.4 Non-Dimensional Secant Modulus

E_s / E vs. F / $F_{0.7}$

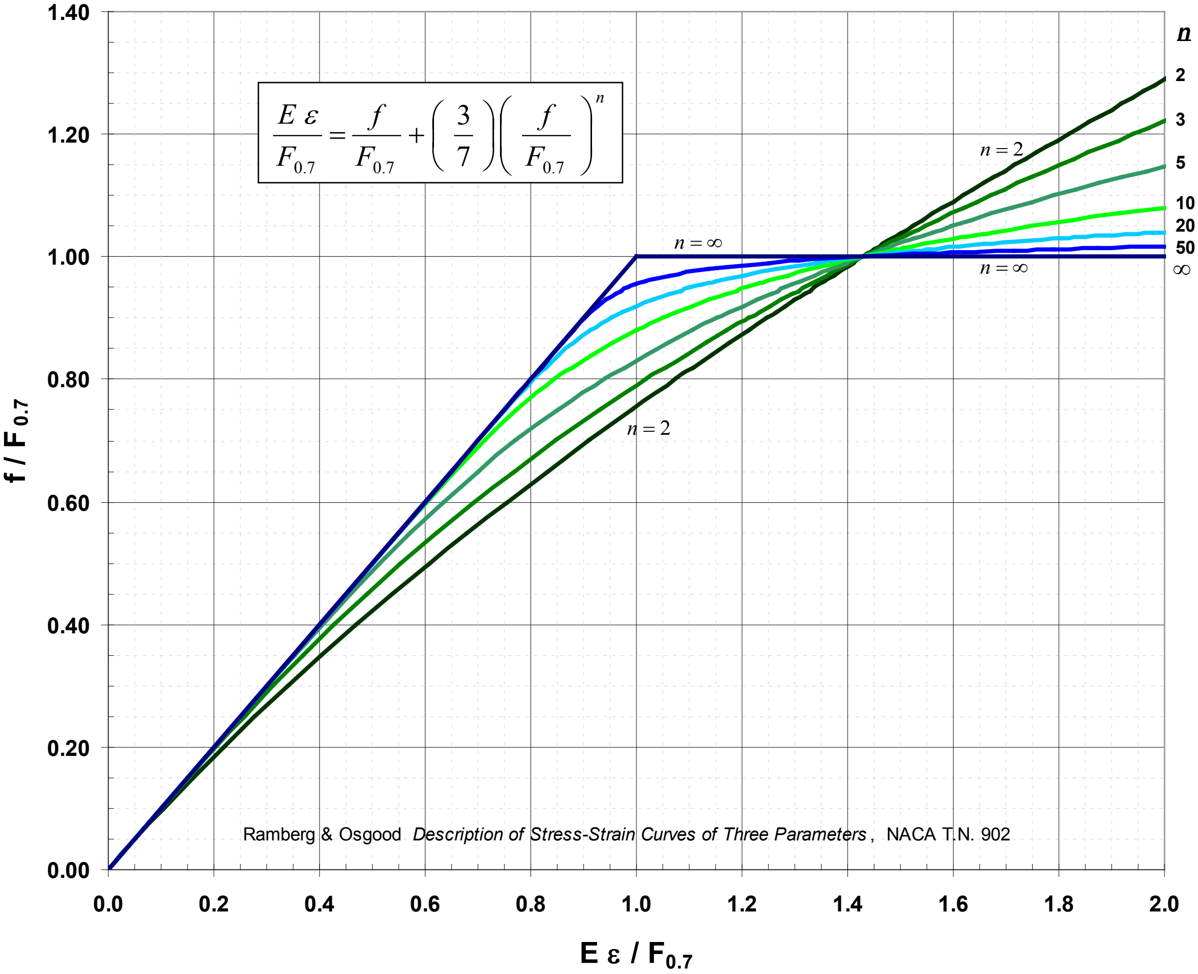


Ramberg-Osgood Stress-Strain Curve

Ramberg & Osgood	NACA TN-902 *Description of Stress-Strain Curves of Three Parameters*, Figure 6
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures* Figure B1.14, page B1.9

$$\frac{E\,\varepsilon}{F_{0.7}} = \frac{f}{F_{0.7}} + \frac{3}{7}\left(\frac{f}{F_{0.7}}\right)^n$$

Figure B1.14 *Stress-Strain in the Inelastic Range*



Ramberg & Osgood *Description of Stress-Strain Curves of Three Parameters*, NACA T.N. 902

Column Strength of Stiffener with Effective Sheet

Thesis by R.J.White *The Ultimate Compressive Strength of Thin Sheet Metal Panels*
Appendix A, C.I.T. 1935

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C7.36, page C7.26

F_{ST} Stiffener Column Failing Stress

A_o Area of Stiffener

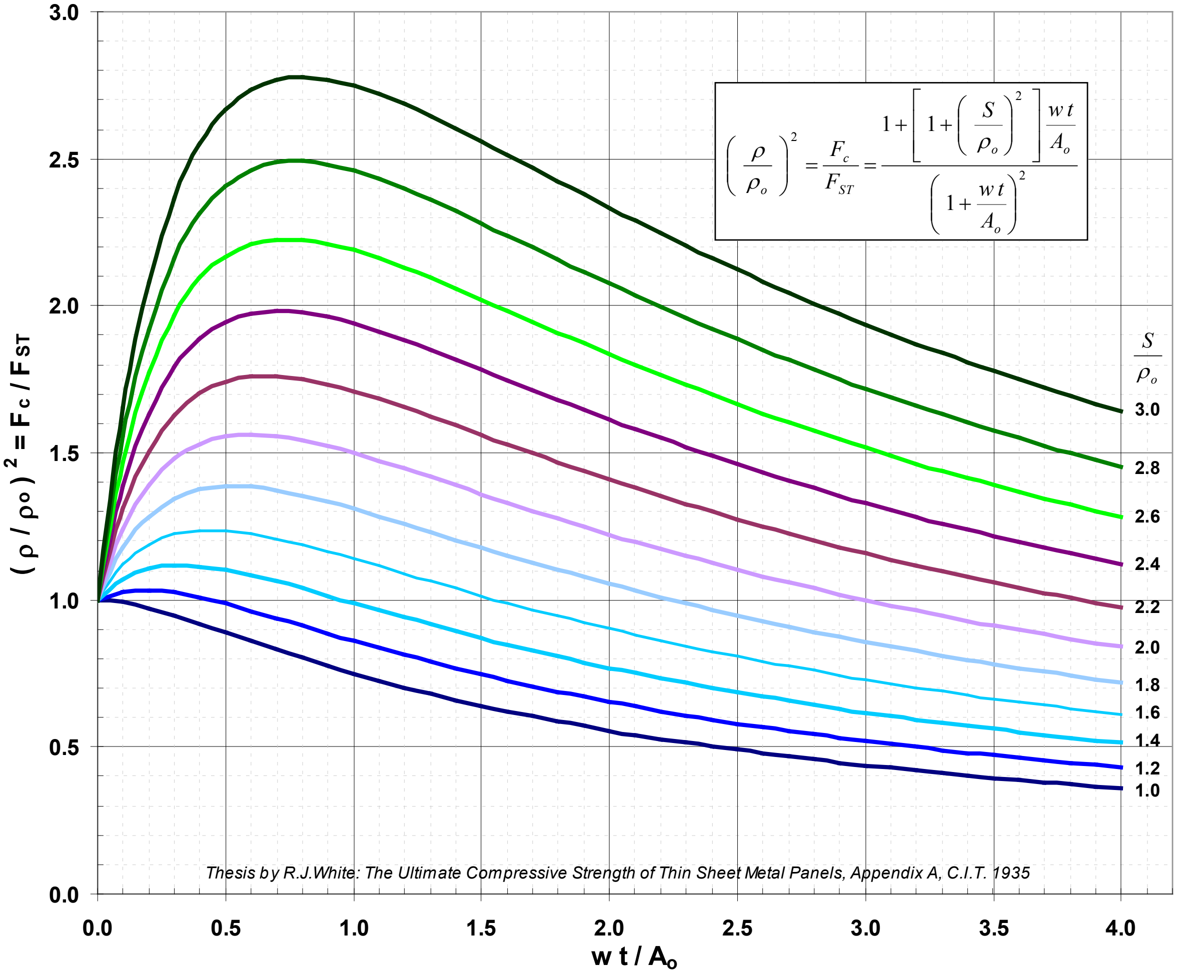
ρ_o Radius of Gyration – Stiffener Alone

S Distance from Sheet Centerline to Stiffener Neutral Axis

$$\left(\frac{\rho}{\rho_o}\right)^2 = \frac{F_c}{F_{ST}} = \frac{1 + \left[1 + \left(\frac{S}{\rho_o}\right)^2\right]\frac{w\ t}{A_o}}{\left(1 + \frac{w\ t}{A_o}\right)^2}$$

Figure C7.36

Curves for the Determination of $(\rho / \rho_o)^2 = F_c / F_{ST}$



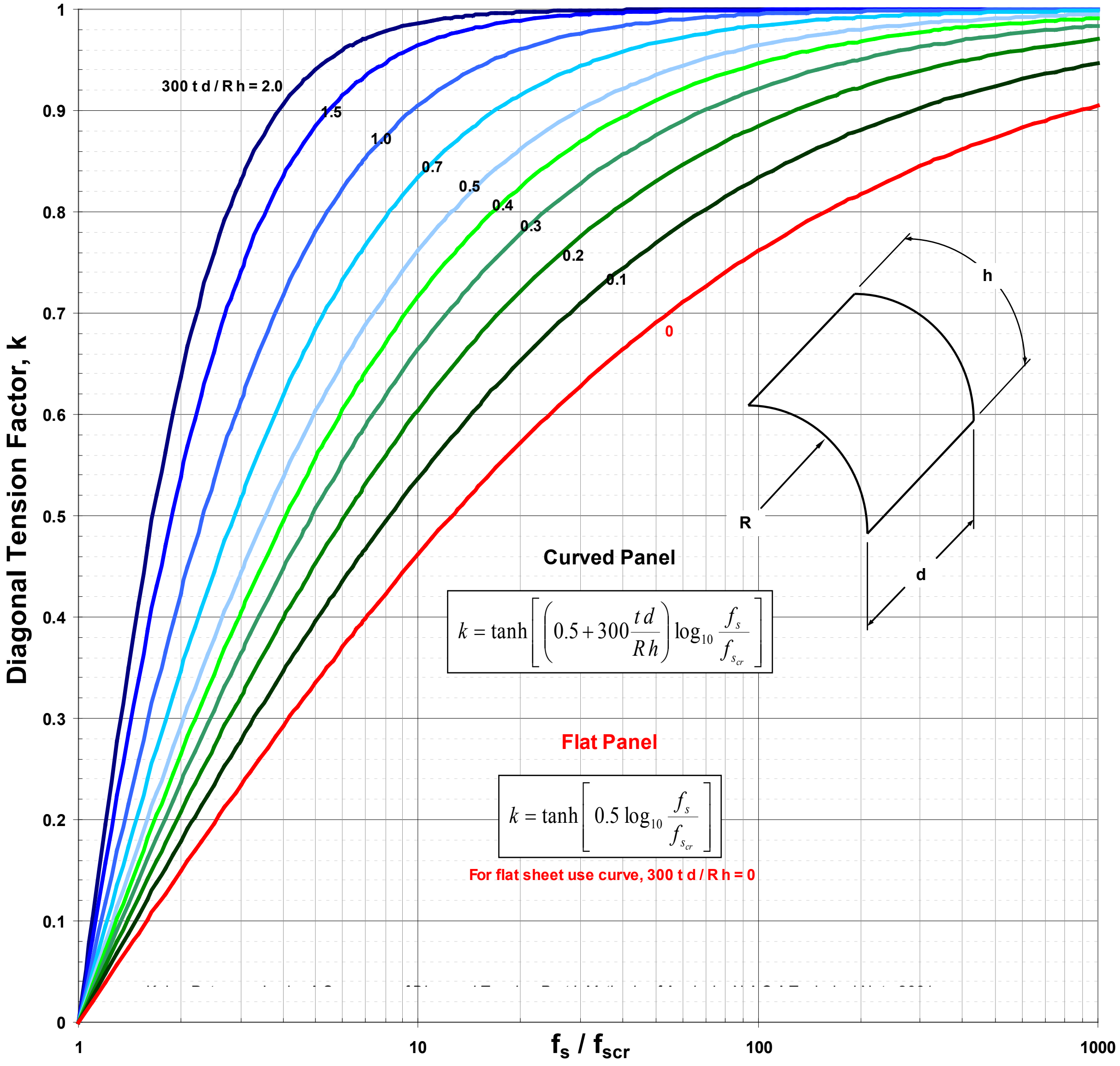
Diagonal Tension Factor

NACA TN-2661 *A Summary of Diagonal Tension. Part I: Methods of Analysis*

Paul Kuhn , James P. Peterson , L. Ross Levin Figure 13, page 108

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C11.19, page C11.26

Figure C11.19 *Diagonal Tension Factor, k*



Curved Panel

$$k = \tanh\left[\left(0.5 + 300\frac{t\,d}{R\,h}\right)\log_{10}\frac{f_s}{f_{s_{cr}}}\right]$$

Flat Panel

$$k = \tanh\left[0.5\log_{10}\frac{f_s}{f_{s_{cr}}}\right]$$

For flat sheet use curve, 300 t d / R h = 0

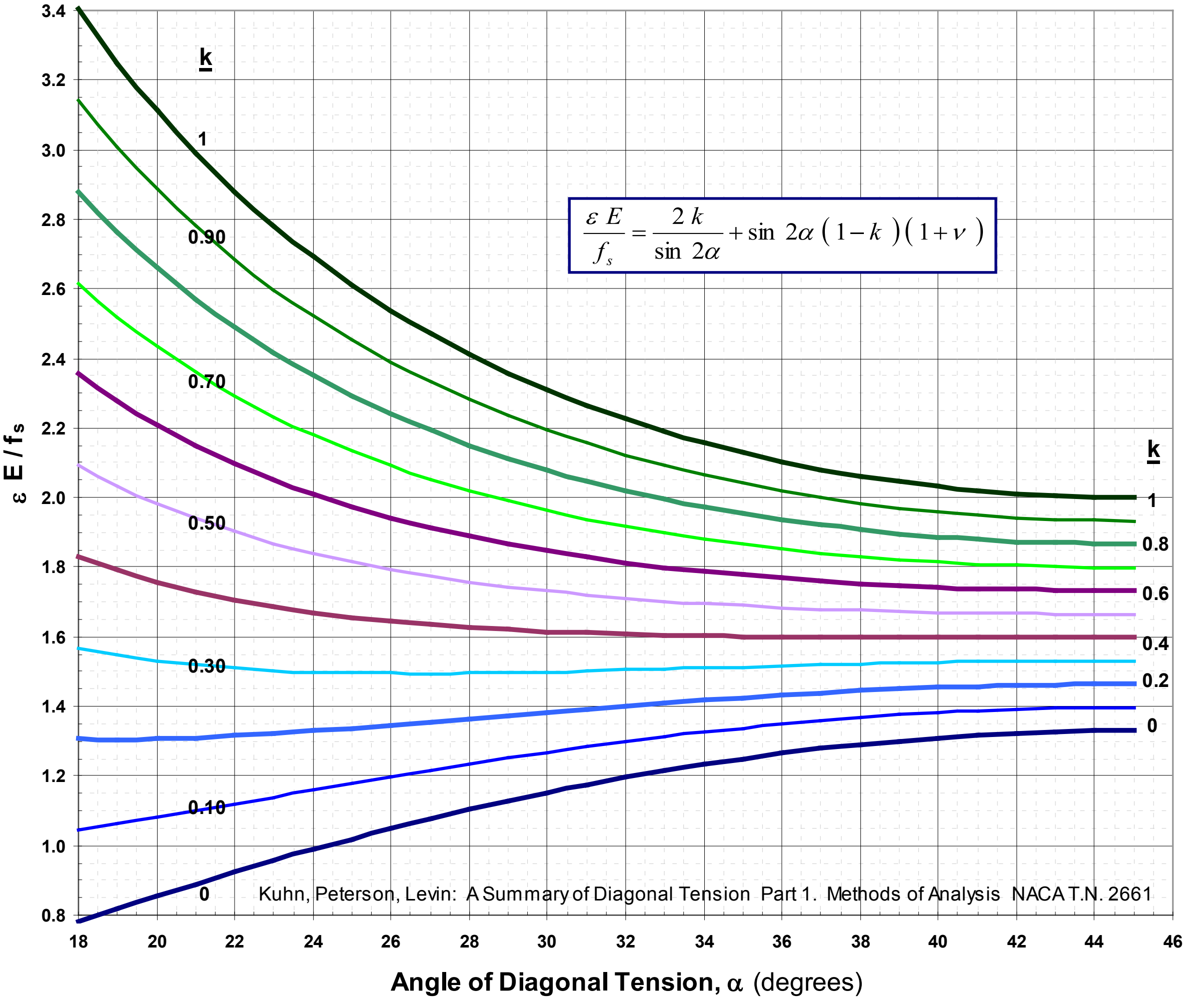
Graph for Calculating Web Strain

NACA TN-2661 *A Summary of Diagonal Tension. Part I: Methods of Analysis*

Paul Kuhn , James P. Peterson , L. Ross Levin Figure 31, page 127

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C11.36, page C11.50

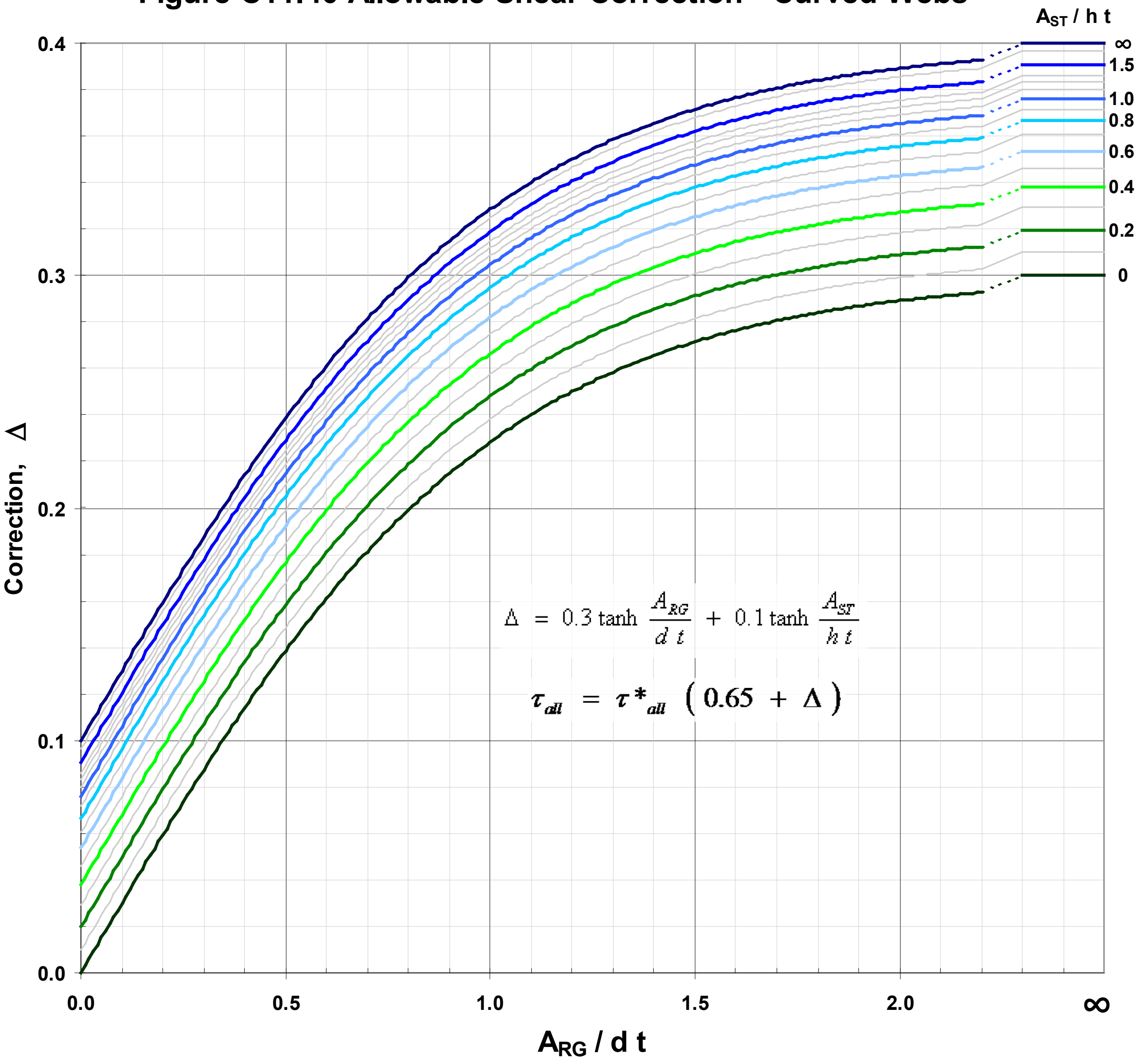
Figure C11.36 ***Web Strain***



Allowable Shear Correction – Curved Webs

NACA TN-2661	*A Summary of Diagonal Tension. Part I: Methods of Analysis*	
	Paul Kuhn , James P. Peterson , L. Ross Levin	Figure 33, page 128
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure C11.40, page C11.52

Figure C11.40 Allowable Shear Correction - Curved Webs



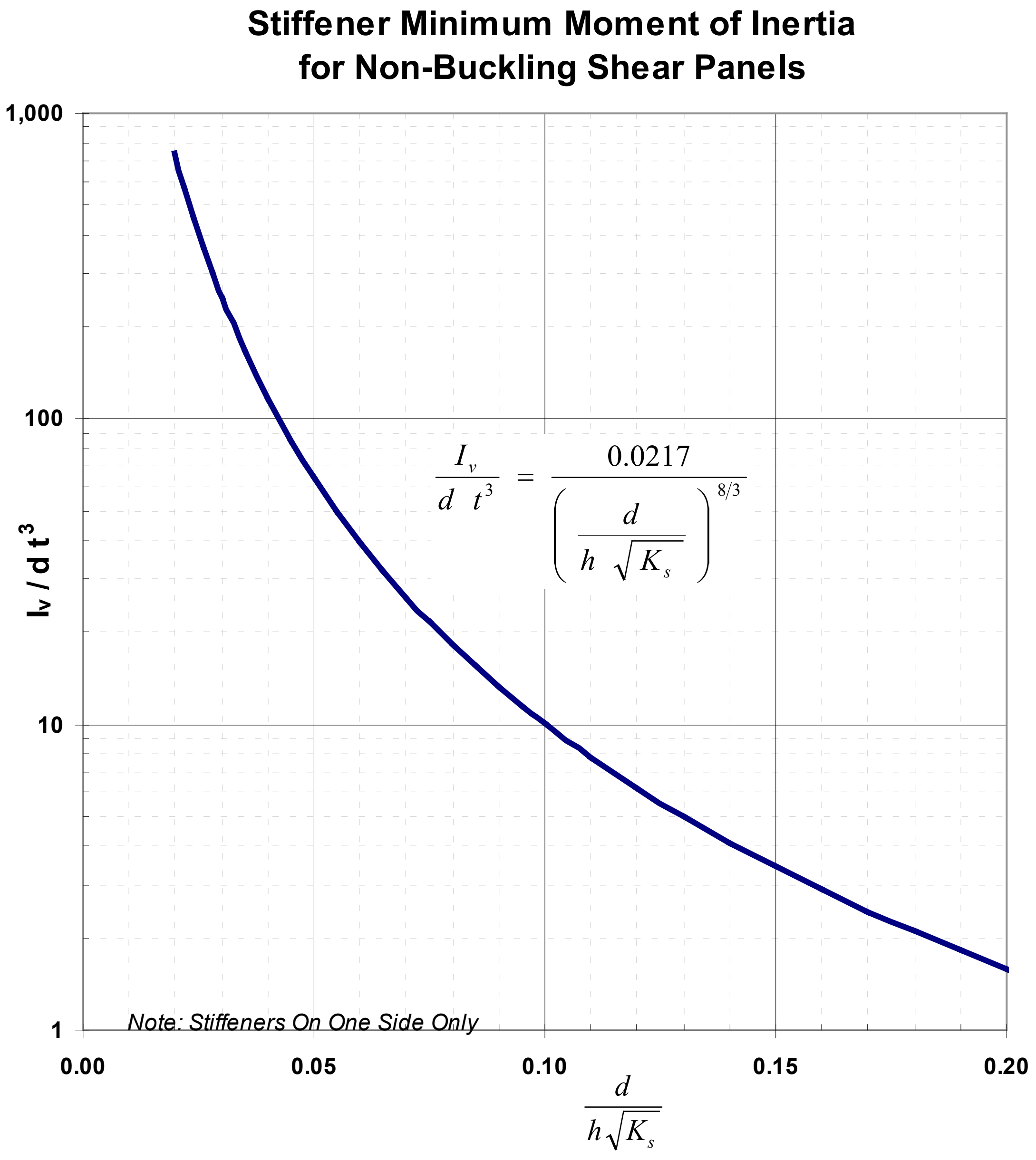
$$\Delta \;=\; 0.3\tanh\frac{A_{RG}}{d\,t} \;+\; 0.1\tanh\frac{A_{ST}}{h\,t}$$

$$\tau_{all} \;=\; \tau^*_{all}\;\left(0.65 \;+\; \Delta\right)$$

Stiffener Minimum Moment of Inertia

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C10.9, page C10.8

Figure C10.9 Minimum Moment of Inertia



1 This curve may be used for flat or curved panels
2 For elevated temperature designs multiply I_v / d t^3 by $E_{R.T.}$ / E_{temp}
3 This curve applies only when the stiffeners are attached at each end to the sub-structure (flanges)
4 Do NOT include any effective skin in stiffener moment of inertia
5 This curve applicable only when buckling is elastic and d is less than h
6 The gage of the stiffener should NOT be less than that of the skin or web except as noted in the table below:

Weg Gage	0.025	0.032	0.040	0.051	0.064	0.074	0.081	0.091	0.102	0.125
Minimum Stiffener Gage	0.035	0.032	0.040	0.051		0.064		0.072	0.081	

Buckling Coefficients

Shear Buckling Coefficients – Flat Plates

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C5.11 page C5.7

NACA TN-3781 *Handbook of Structural Stability Part I - Buckling of Flat Plates*

George Gerard and Herbert Becker Figure 22

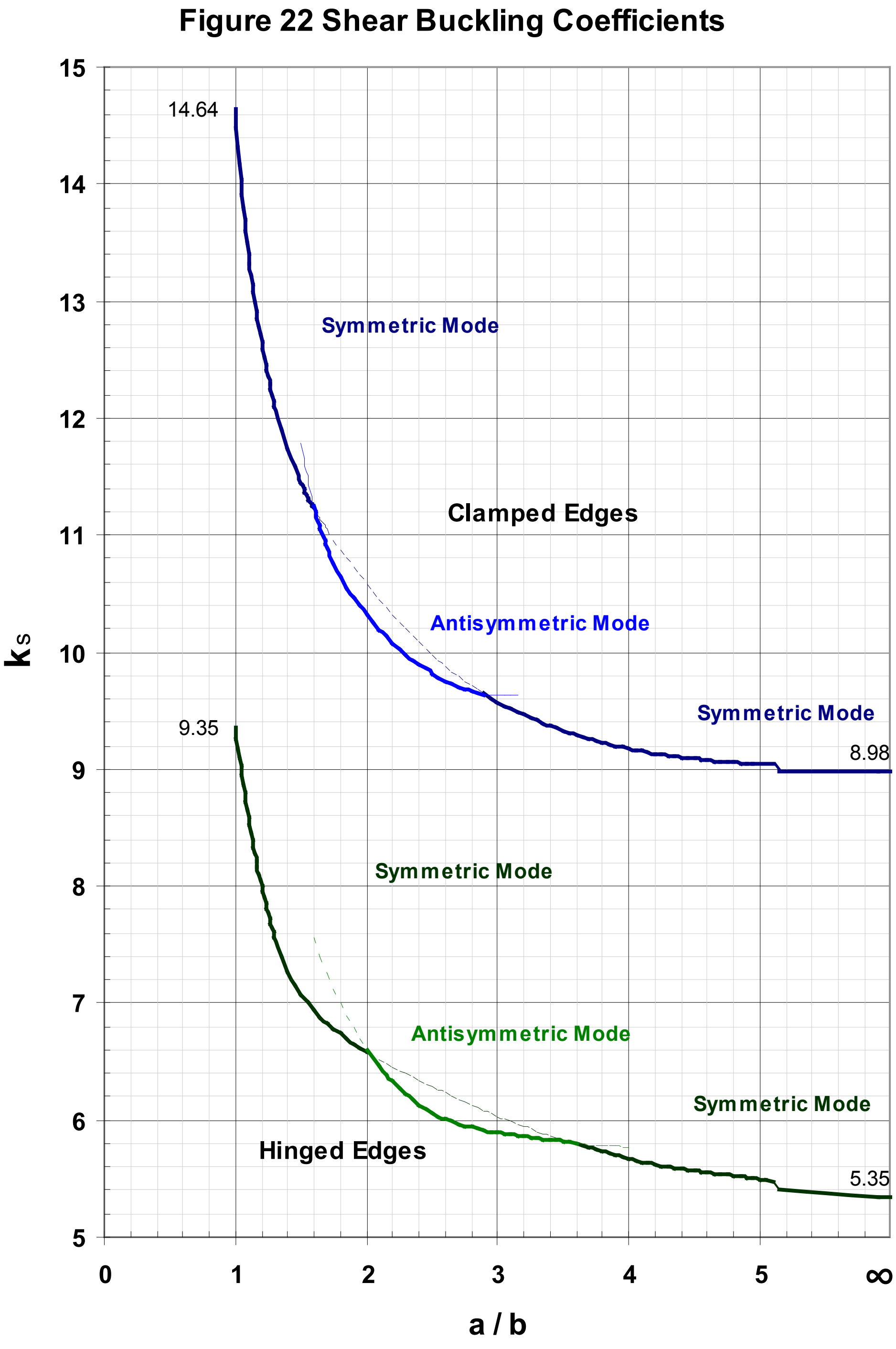


Figure C9.1 Axial Compressive Buckling Coefficient for Long Curved Plates

NACA TN-1928	*Critical Combinations of Shear and Direct Axial Stress for Curved Rectangular Panels*	
	Murry Schildcrout and Manuel Stein	Figures 4 & 5, pages 22-23
NACA TR-887	*Critical Stress of Thin Walled Cylinders in Axial Compression*	
	S. B. Batdorf, Murry Schildcrout, Manuel Stein	Figure 2
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure C9.1, page C9.2

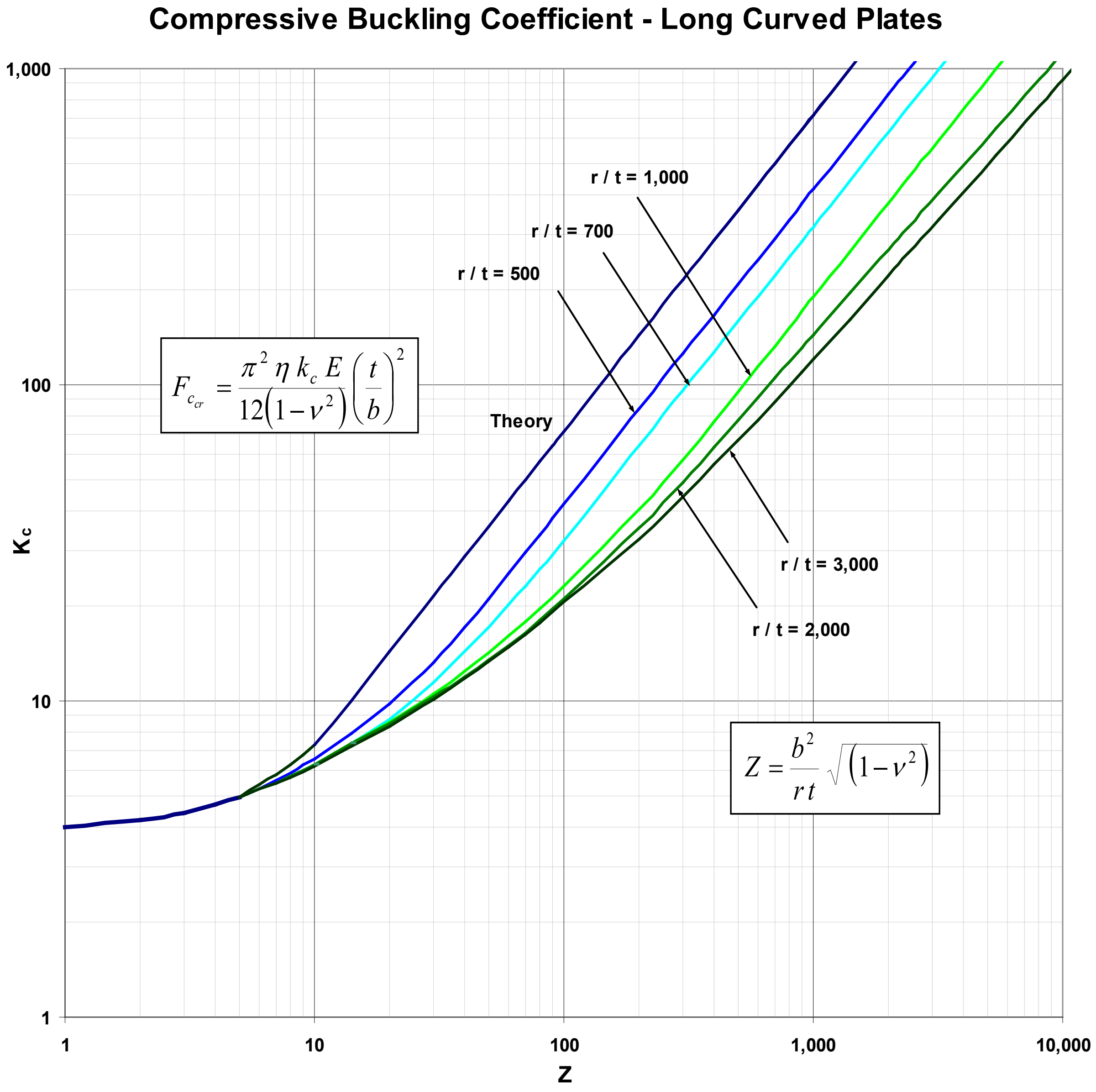


Figure C9.2 Shear Buckling Coefficient for Long, Clamped, Curved Plates

NACA TN-1348	Critical Shear Stress of Curved Rectangular Panels	Figure 7
	S. B. Batdorf, Manuel Stein, Murry Schildcrout	
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure C9.2, page C9.3

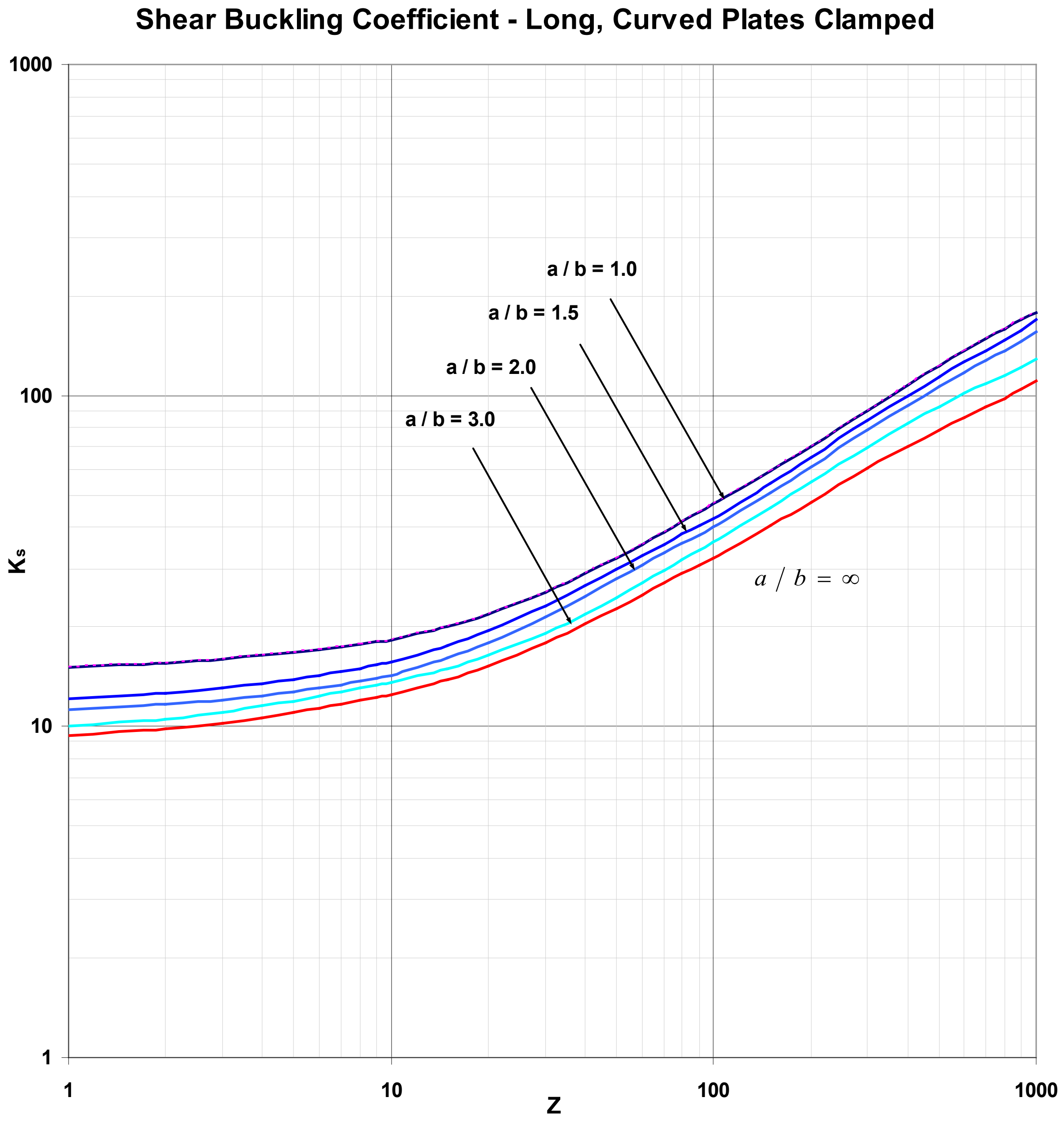


Figure C9.3 Shear Buckling Coefficient for Wide, Clamped, Curved Plates

NACA TN-1348	Critical Shear Stress of Curved Rectangular Panels	Figure 6
	S. B. Batdorf, Manuel Stein, Murry Schildcrout	
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure C9.3, page C9.4

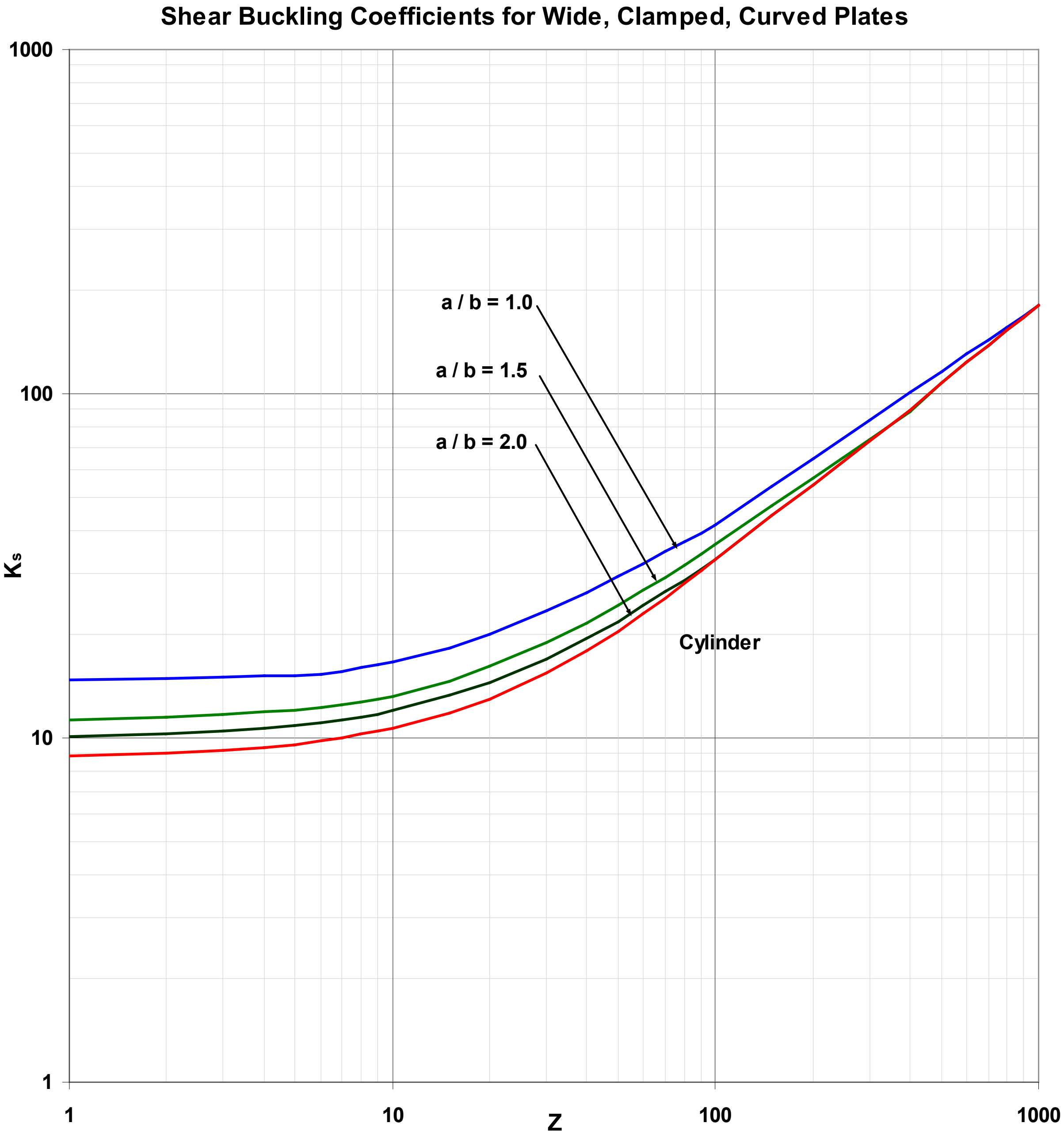


Figure C9.4 Shear Buckling Coefficient for Long, Simply Supported, Curved Plates

NACA TN-1348	Critical Shear Stress of Curved Rectangular Panels	Figure 2
	S. B. Batdorf, Manuel Stein, Murry Schildcrout	
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure C9.4, page C9.5

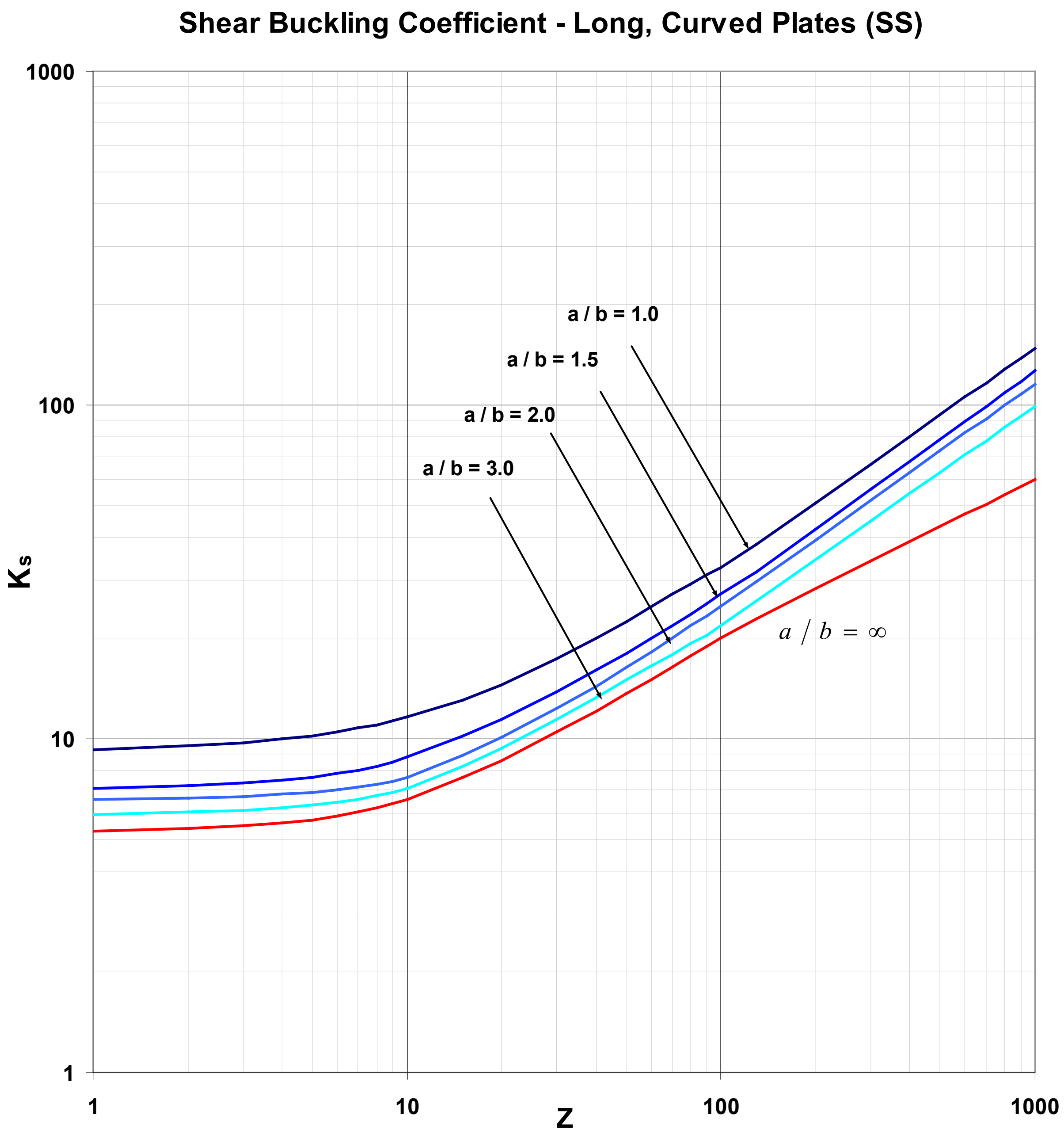
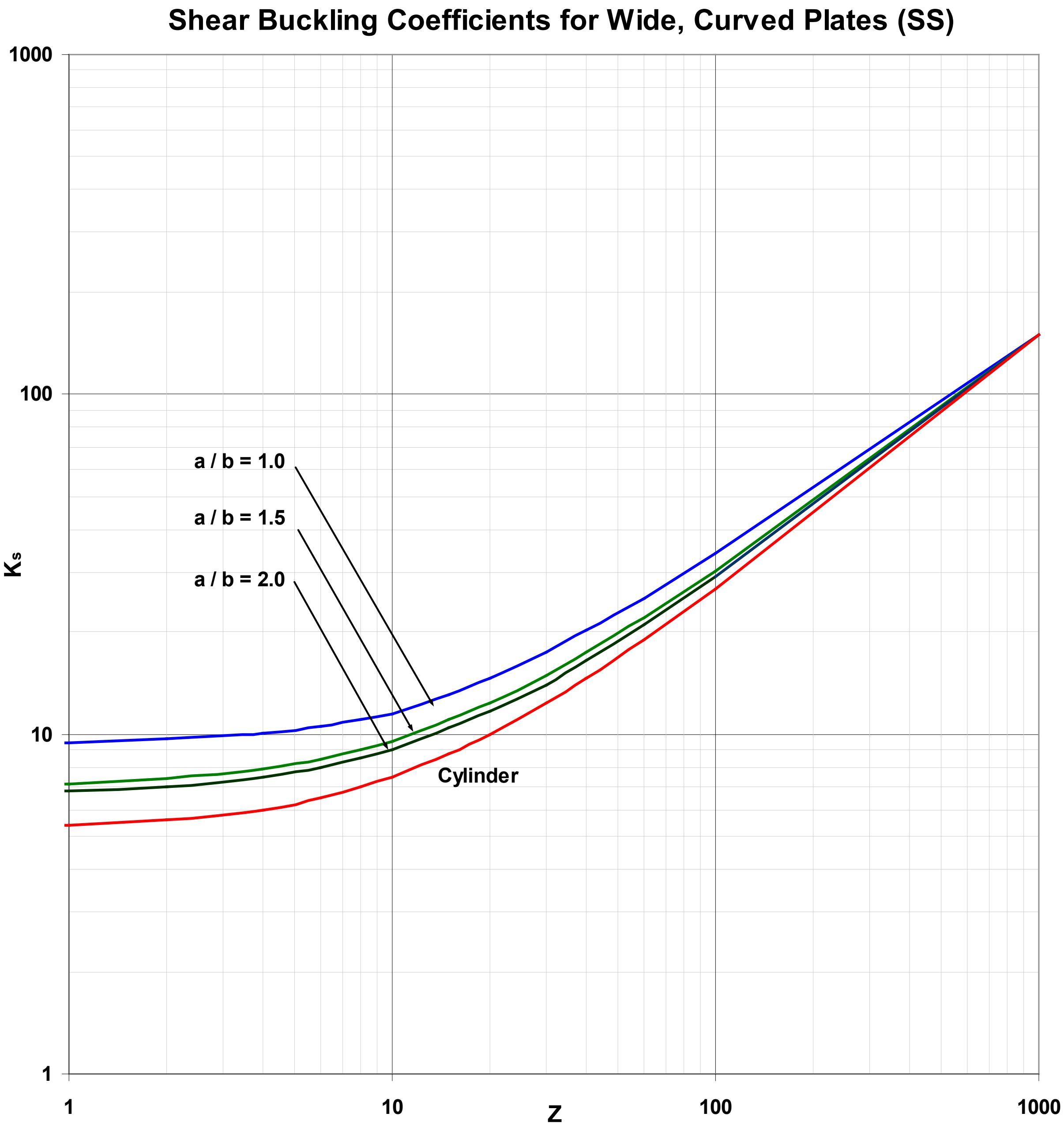


Figure C9.5 Shear Buckling Coefficient for Wide, Simply Supported, Curved Plates

NACA TN-1348	*Critical Shear Stress of Curved Rectangular Panels*	Figure 1
	S. B. Batdorf, Manuel Stein, Murry Schildcrout	
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure C9.5, page C9.5



Flat Sheet Buckling Interaction Curves

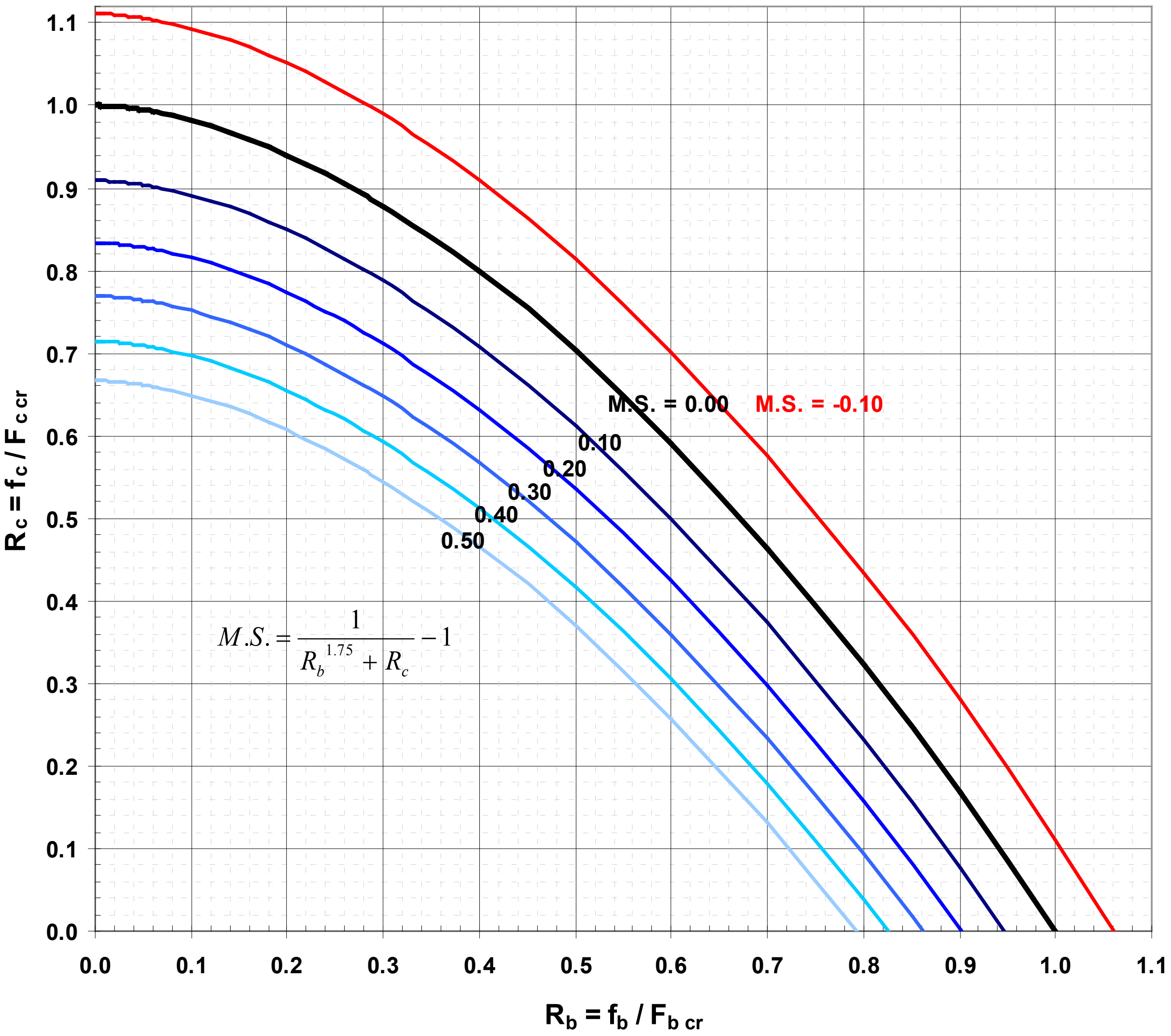
Combined Bending and Longitudinal Compression

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C5.15, page C5.8

Margin of Safety

$$M.S. = \frac{1}{R_b^{1.75} + R_s} - 1$$

Figure C5.15 *Combined Bending and Compression*



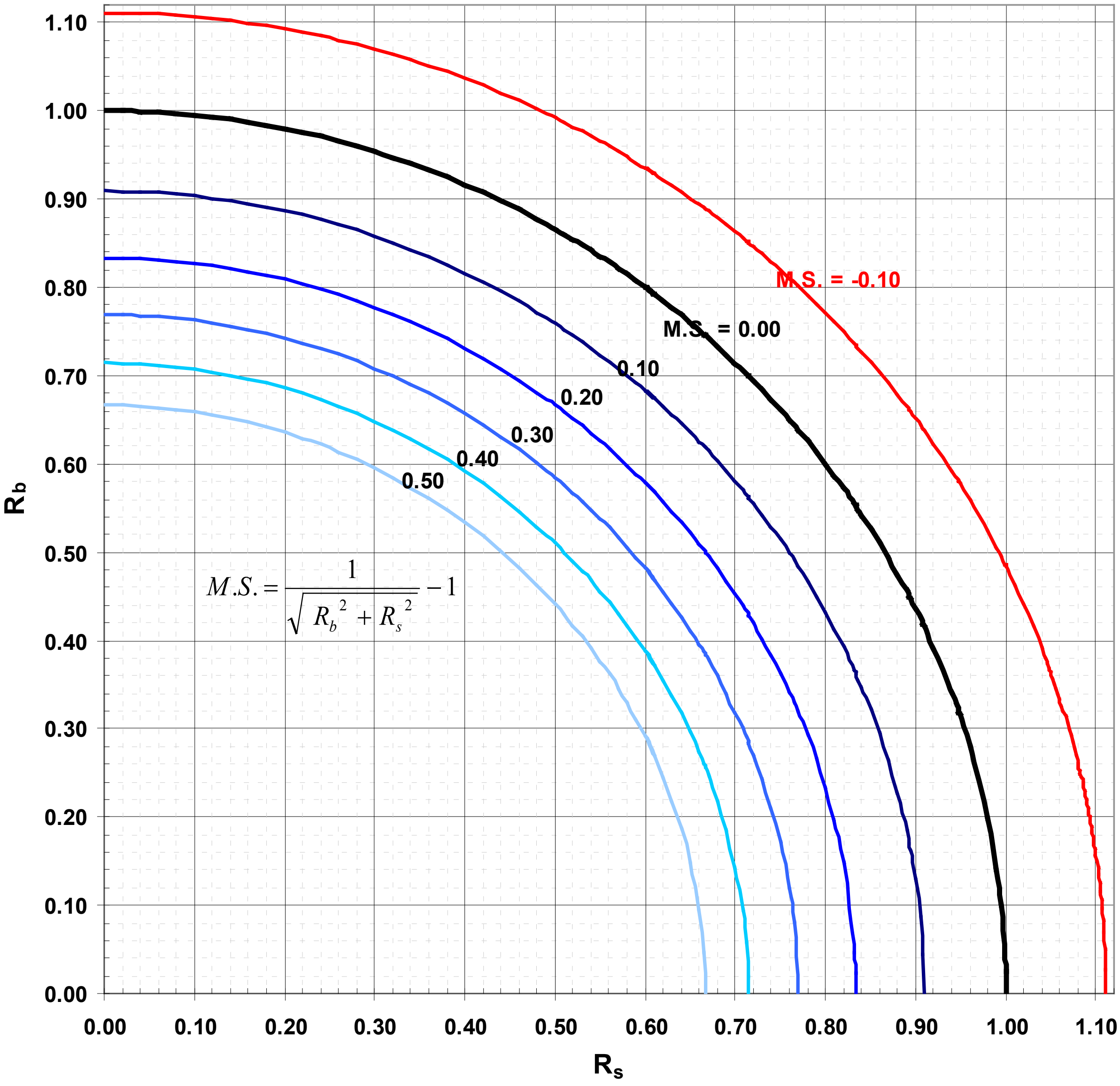
Combined Bending and Shear

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C5.16, page C5.8

Margin of Safety

$$M.S. = \frac{1}{\sqrt{R_b^2 + R_s^2}} - 1$$

Figure C5.16 *Combined Bending and Shear*



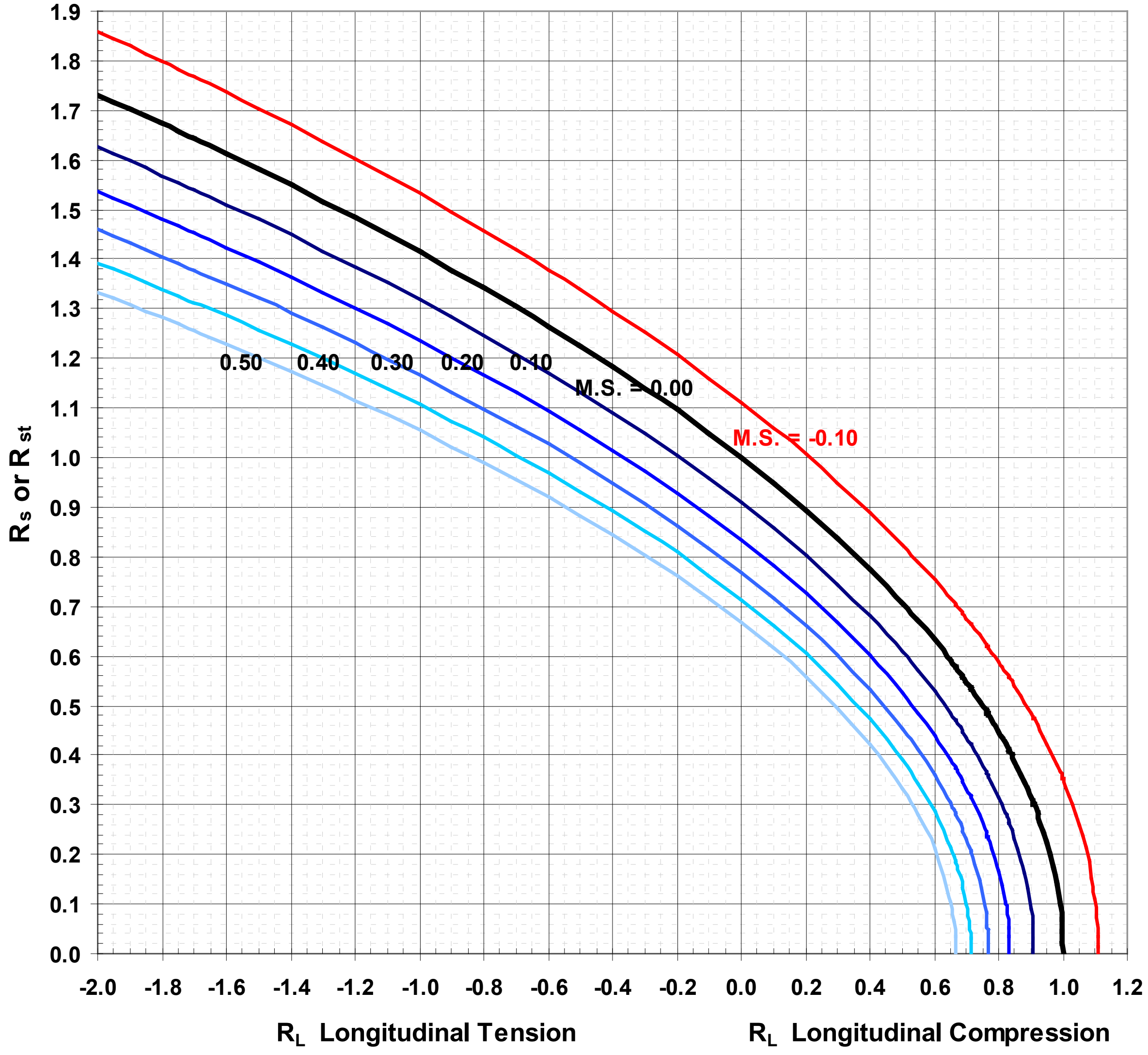
Combined Shear and Longitudinal Stress

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C5.17, page C5.9

Buckling Margin of Safety

$$M.S. = \frac{2}{R_L + \sqrt{R_L^2 + 4\ R_s^2}} - 1$$

Figure C5.17 *Combined Shear and Longitudinal Stress*

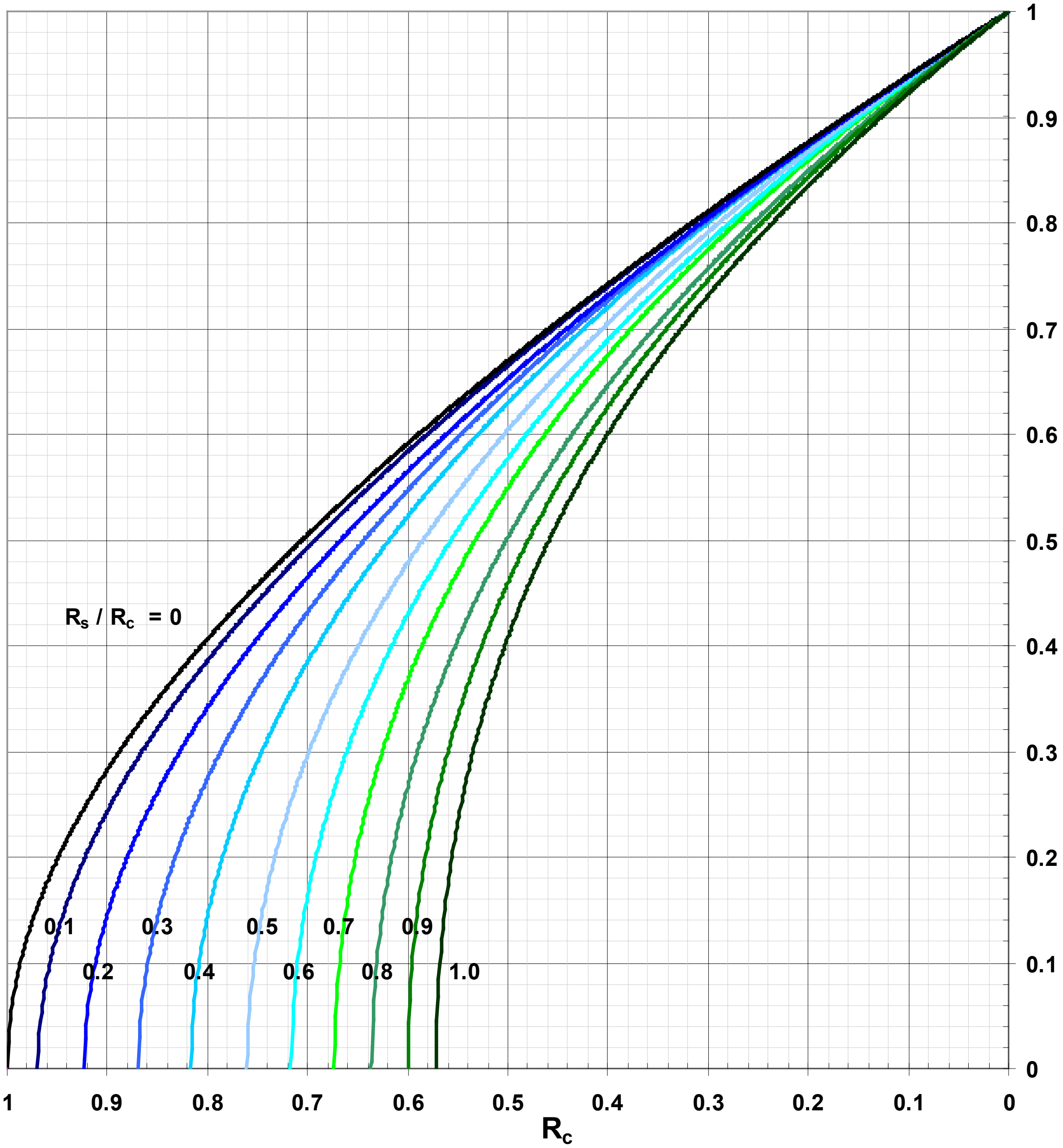


Compression, Bending and Shear $R_s < R_c$

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C5.19, page C5.9

For Shear Ratio less than the Compressive Ratio, $R_s < R_c$ Note: Curves are approximate.

Figure C5.19 ***Combined Compression, Bending and Shear***

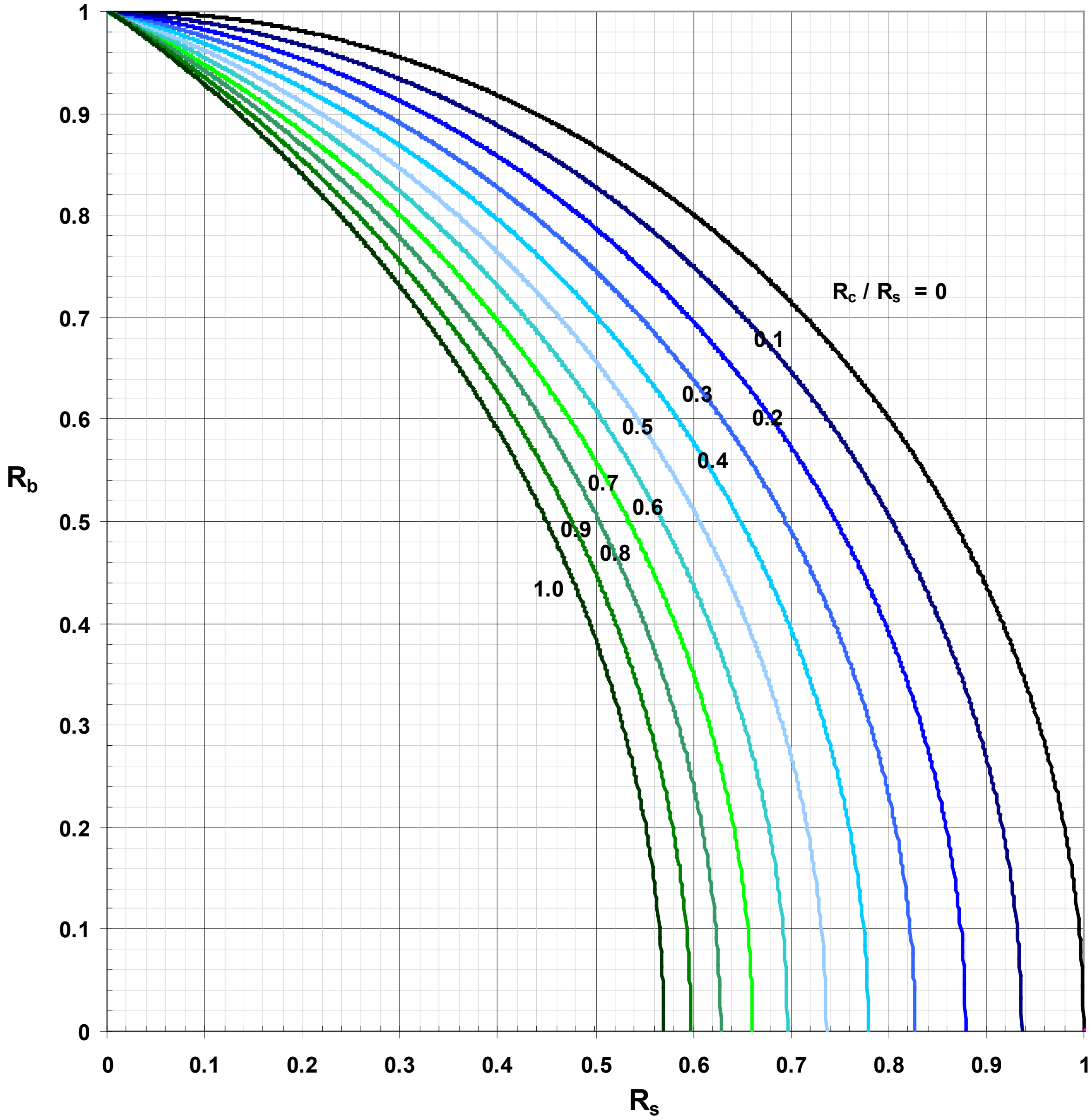


Compression, Bending and Shear, $R_c < R_s$

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C5.19, page C5.9

For Compressive Ratio less than the Shear Ratio, $R_c < R_s$

Figure C5.19 *Combined Compression, Bending and Shear*



The chart above was solved for the interaction equation below by setting MS = 0 and changing R_b with a VBA trial and error routine.

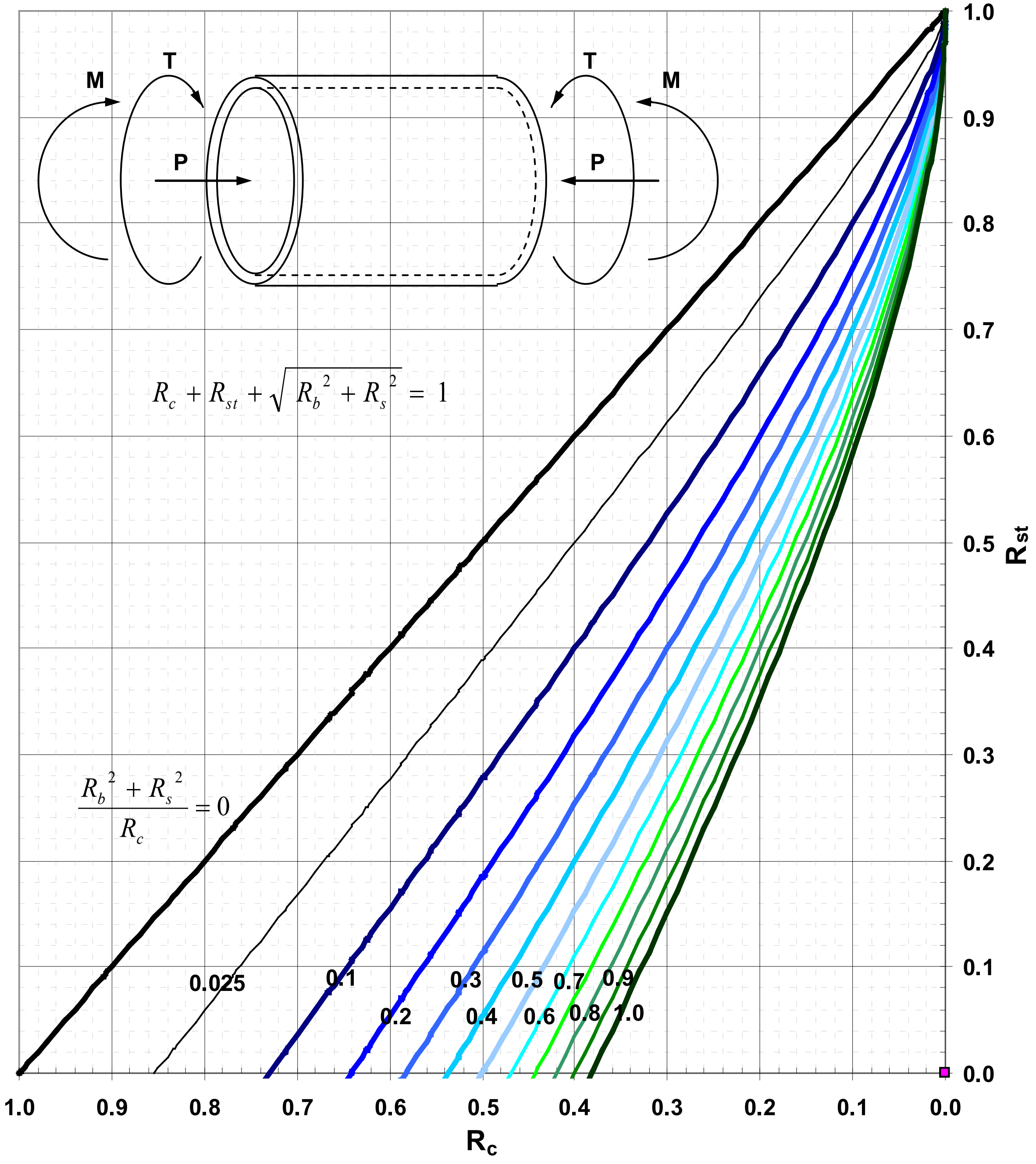
$$MS = \frac{1}{R_s \left(R_c / R_s \right) + \left(R_b^{\,2} + R_s^{\,2} \right)^{0.75}} - 1$$

Round Tubes – Interaction Curves

Compression, Bending, Shear and Torsion

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C4.40, page C4.26

Figure C4.40 ***Compression, Bending, Flexural Shear and Torsion***

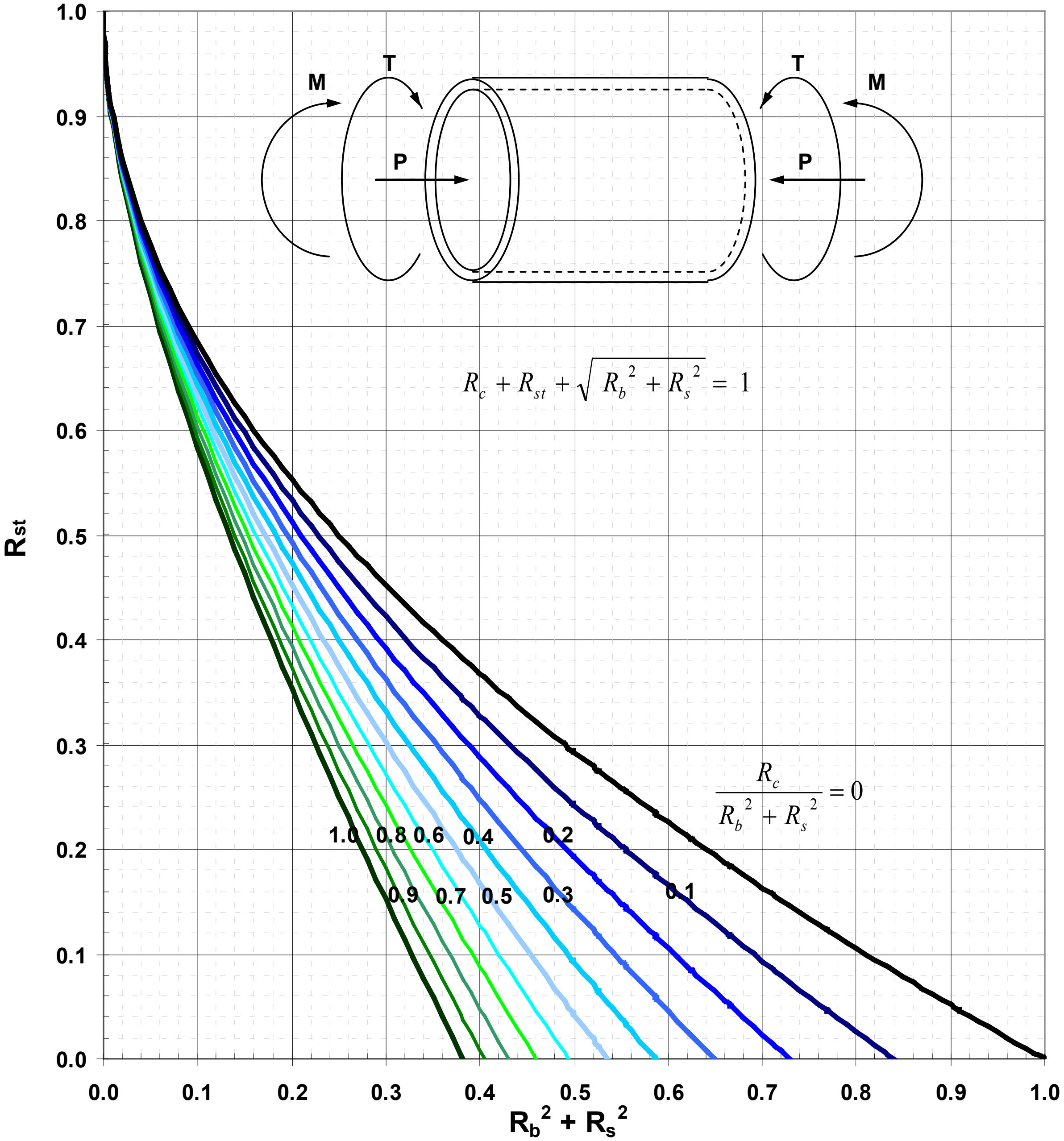


Compression, Bending, Shear and Torsion

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C4.40, page C4.26

R_s is the stress ratio for flexural shear … R_{st} for torsional shear … R_b for bending … R_c for compression.

Figure C4.40 ***Compression, Bending, Flexural Shear and Torsion***



Collapsing Shear Stress - Round Holes with Formed 45 Degree Flanges

NACA-WR-L-323 *The Strength and Stiffness Of Shear Webs With Round Lightening Holes Having 45 Degree Flanges*

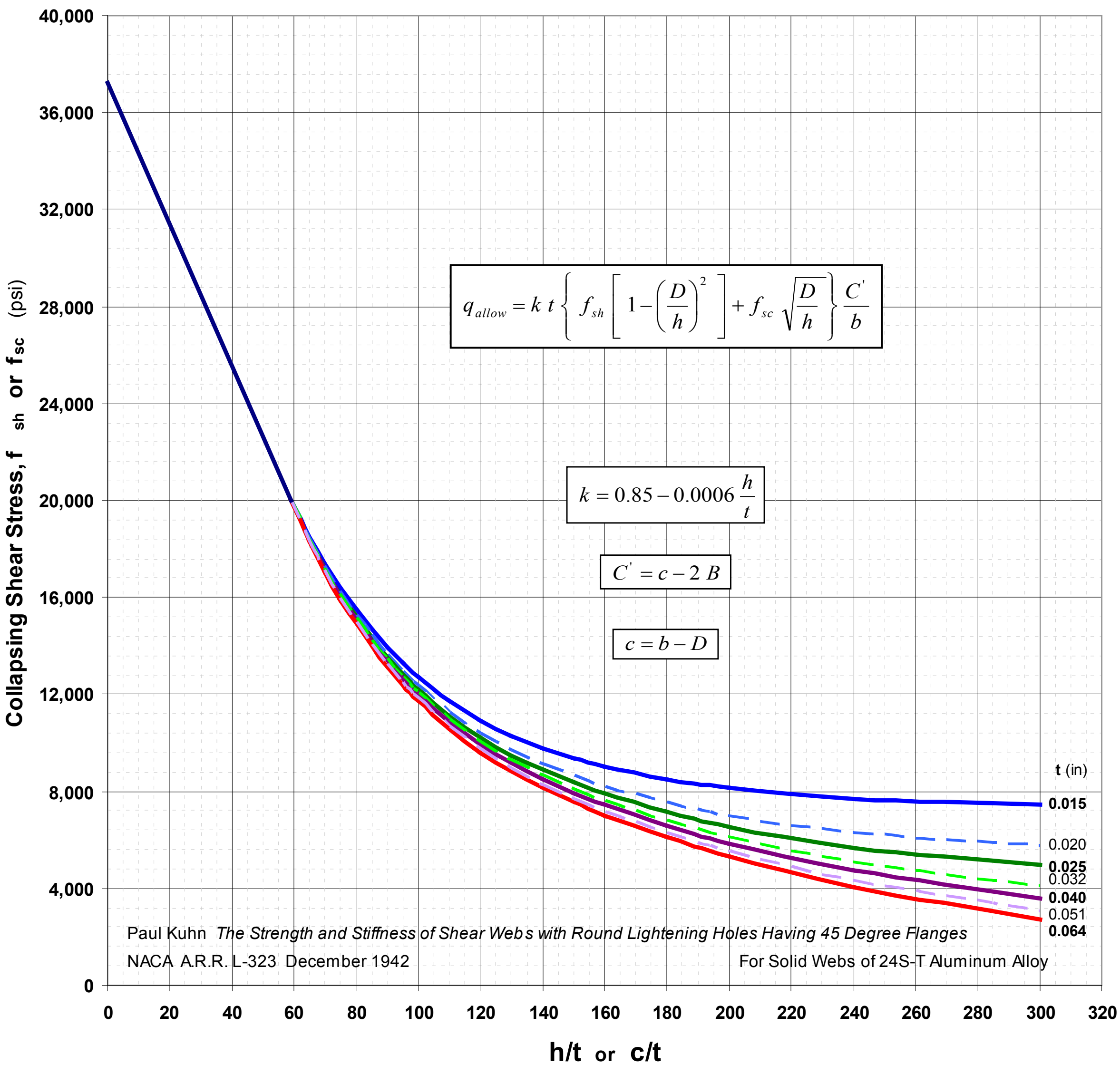
http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930093634_1993093634.pdf

.... Paul Kuhn

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C10.20, page C10.17

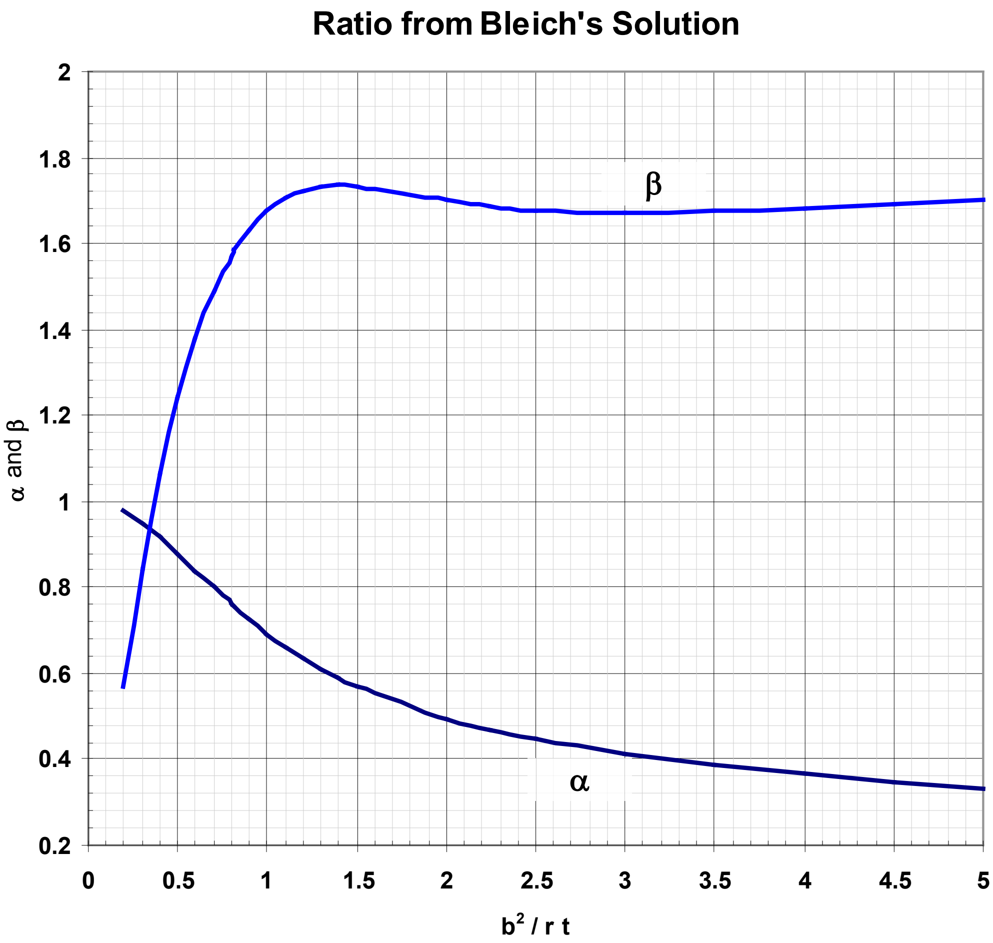
NACA A.R.R. L-323 December 1942

Figure C10.20 *Collapsing Shear Stress*



Curved Beams

Fred B. Seely James O. Smith *Advanced Mechanics of Materials* page 161



b’ = α b

b’ = reduced or effective projecting width of flange on each side

b = projecting width of the actual flange on each side

α = a ratio obtained from Bleich’s solution

$$\sigma' = \beta \, \sigma_G$$

σ’ = maximum lateral bending stress in the flange

σ_G = circumferential bending stress t / 2 from the extreme fiber, using the Winkler-Bach equation

β = a ratio obtained from Bleich’s solution

Lugs

Peaking Factor for Pin Bending

Analysis of Lugs and Shear Pins Made of Aluminum and Steel Alloys

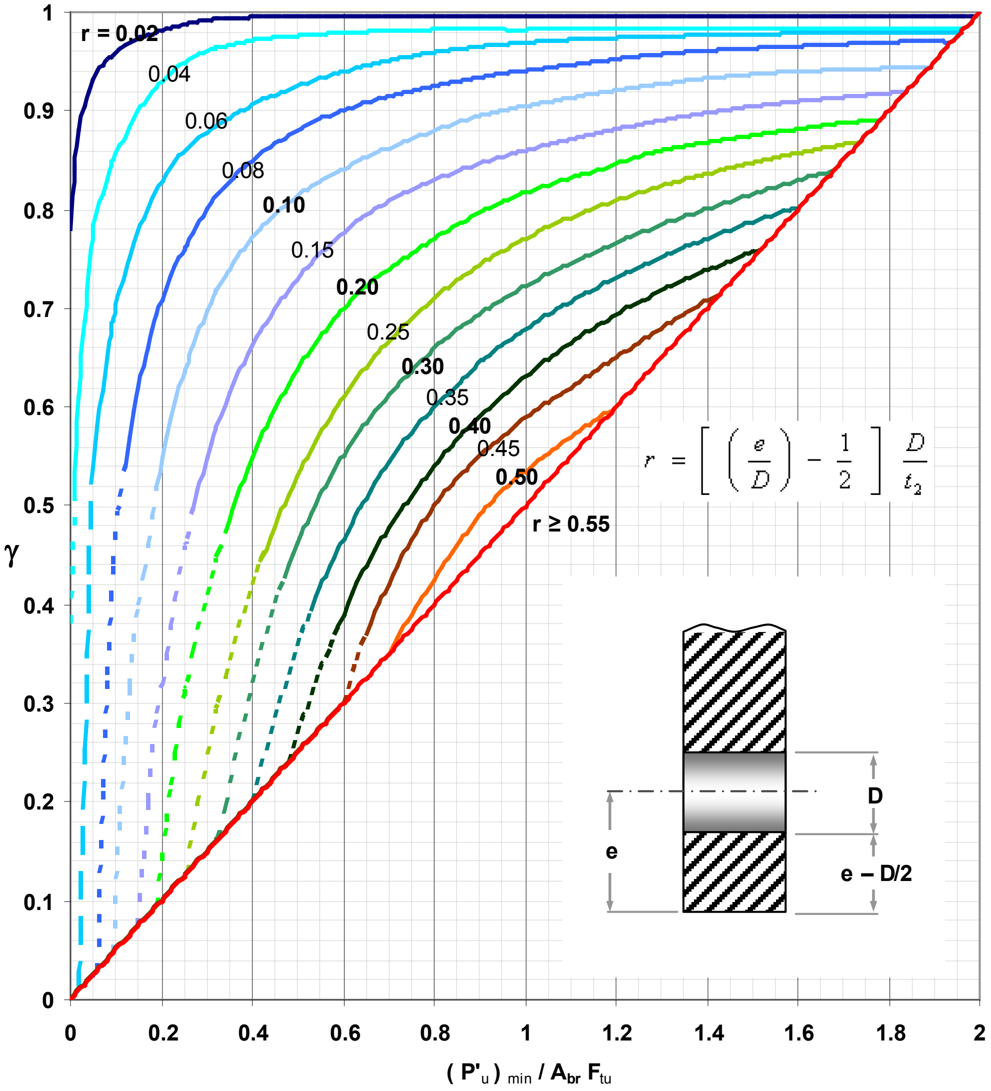
F.P.Cozzone, M.A. Melcon and F.M.Hoblit
Product Engineering, Volume 21, Number 5, pages 113-117, May 1950

Developments in the Analysis of Lugs and Shear Pins

M.A. Melcon and F.M.Hoblit
Product Engineering, Volume 24, Number 6, pages 160-170, June 1953

NASA/TMX-73305 *NASA Astronautics Structures Manual* Figure B2.1.0-6, page 13

Figure 4 Peaking Factor For Pin Bending



Dashed lines indicate region where these theoretical curves are not substantiated by test data.

Bolt Bending Strength

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure D1.16, page D1.9

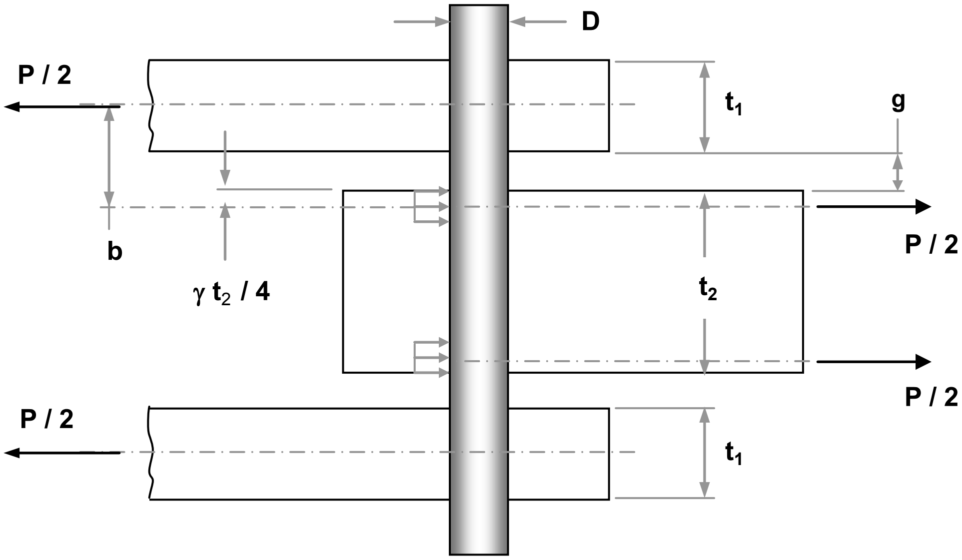
Developments in the Analysis of Lugs and Shear Pins

M.A. Melcon and F.M.Hoblit

Product Engineering, Volume 24, Number 6, Figure 5, page 162, June 1953

Figure 5

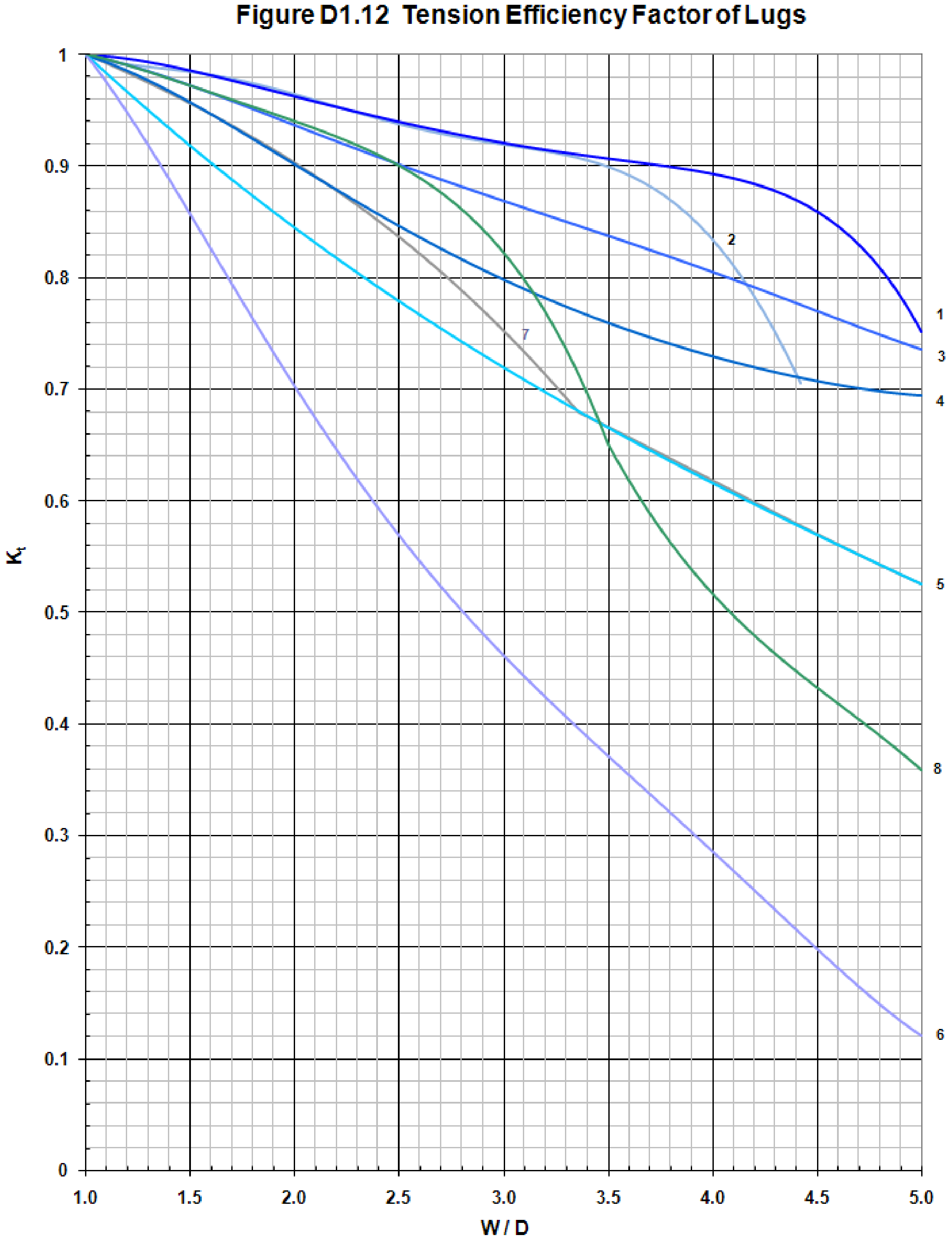
Method of Determining Moment Arm and Approximating Effective Lug



Tension Efficiency of Lugs

Developments in the Analysis of Lugs and Shear Pins, M.A. Melcon and F.M.Hoblit
Product Engineering, Volume 24, Number 6, pages 160-170, June 1953

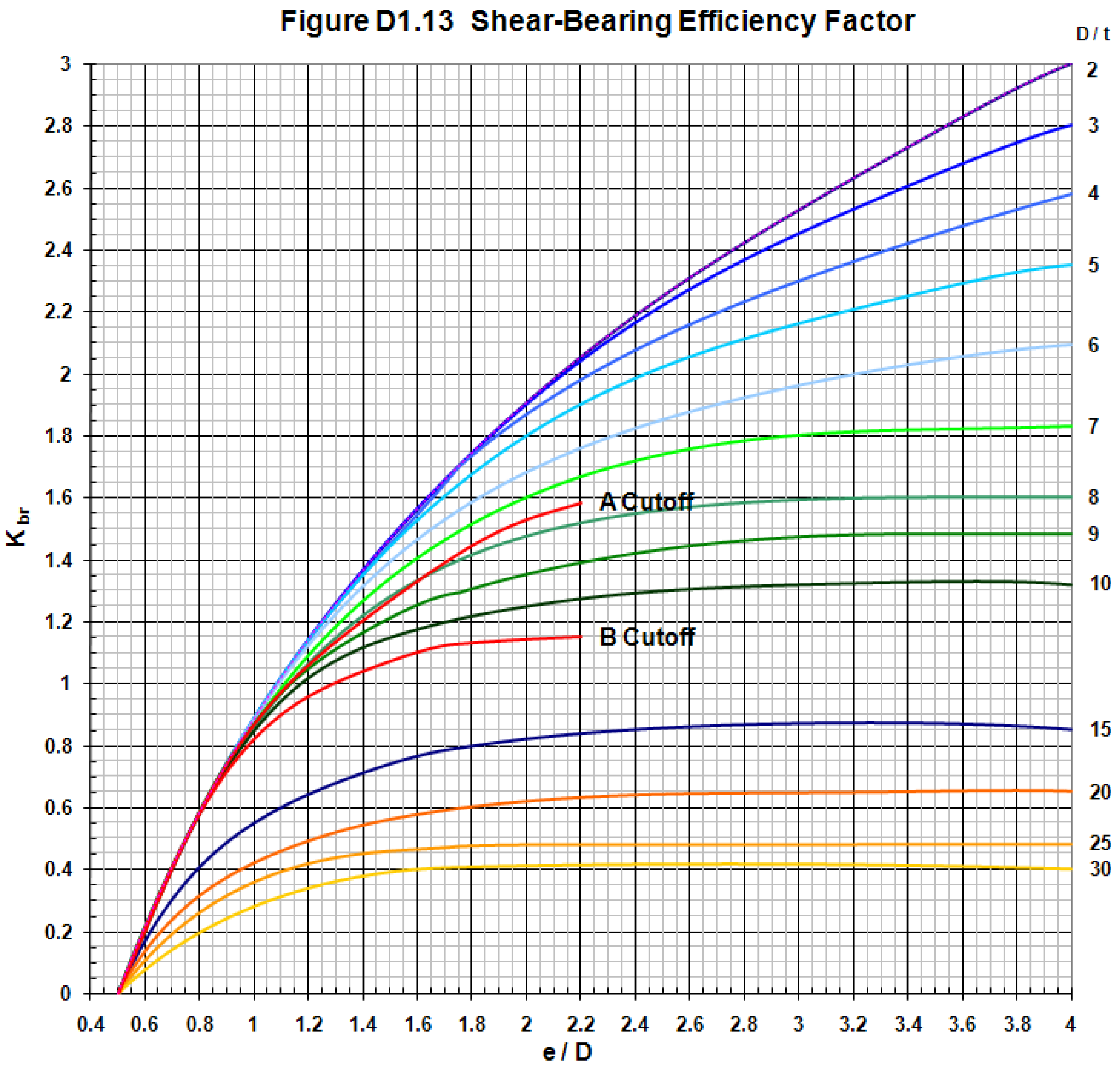
NASA/TMX-73305	NASA Astronautics Structures Manual	Figure B2.1.0-4, page 9
Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure D1.12, page D1.7



Shear-Bearing Efficiency Factor, Axially Loaded Lugs

Elmer F. Bruhn	*Analysis and Design of Flight Vehicle Structures*	Figure D1.13, page D1.7
NASA/TMX-73305	*NASA Astronautics Structures Manual*	Figure B2.1.0-3, page 7
AFFDL-TR-69-42	*Stress Analysis Manual*	Figure 9-3, page 9-5

Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base



Tension Efficiency Factors, Transverse Loads

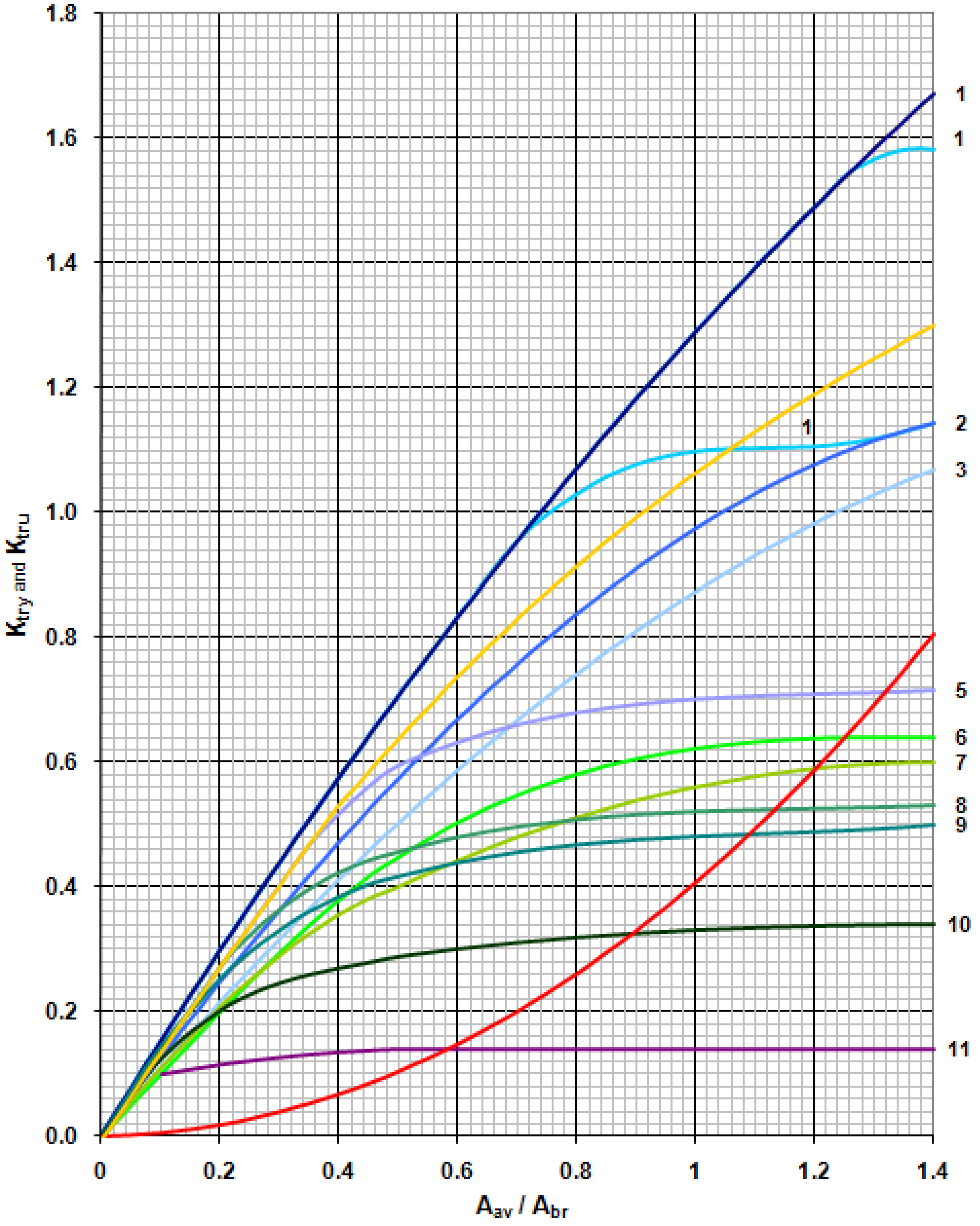
Analysis of Lugs and Shear Pins Made of Aluminum and Steel Alloys
F.P.Cozzone, M.A. Melcon and F.M.Hoblit
Product Engineering, Volume 21, Number 5, Figure 4, page 115, May 1950

Developments in the Analysis of Lugs and Shear Pins
M.A. Melcon and F.M.Hoblit
Product Engineering, Volume 24, Number 6, Figure 18, page 169, June 1953

NASA/TMX-73305 *NASA Astronautics Structures Manual* Figure B2.2.0-4, page 20

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure D1.15, page D1.8

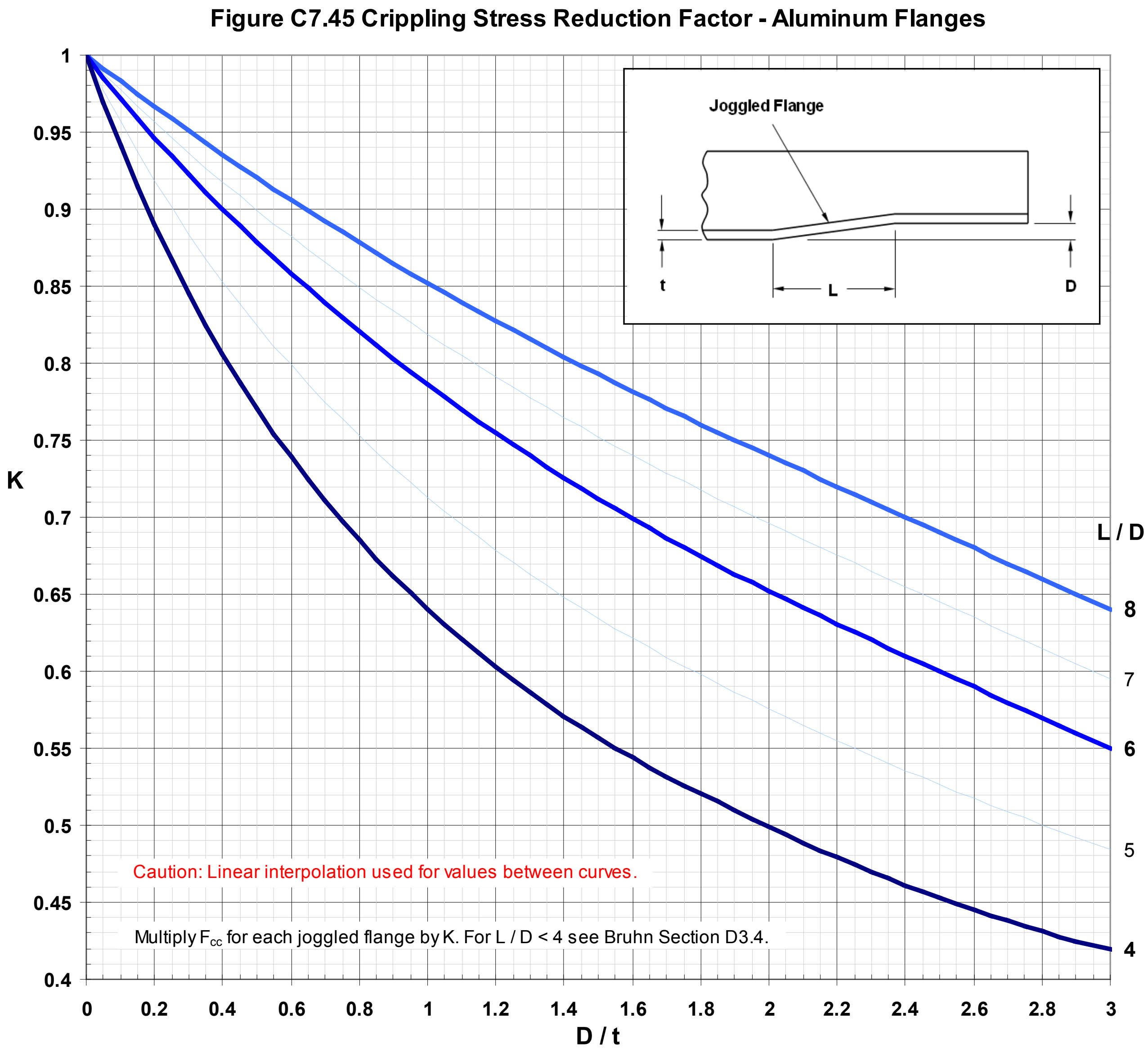
Fig. D1.15 Efficiency Factors for Transverse Loads



Crippling Reduction Factor – Joggles

William F. McCombs *Engineering Column Analysis – The Analysis of Compression Members*

Figure C7.45, page 11.9



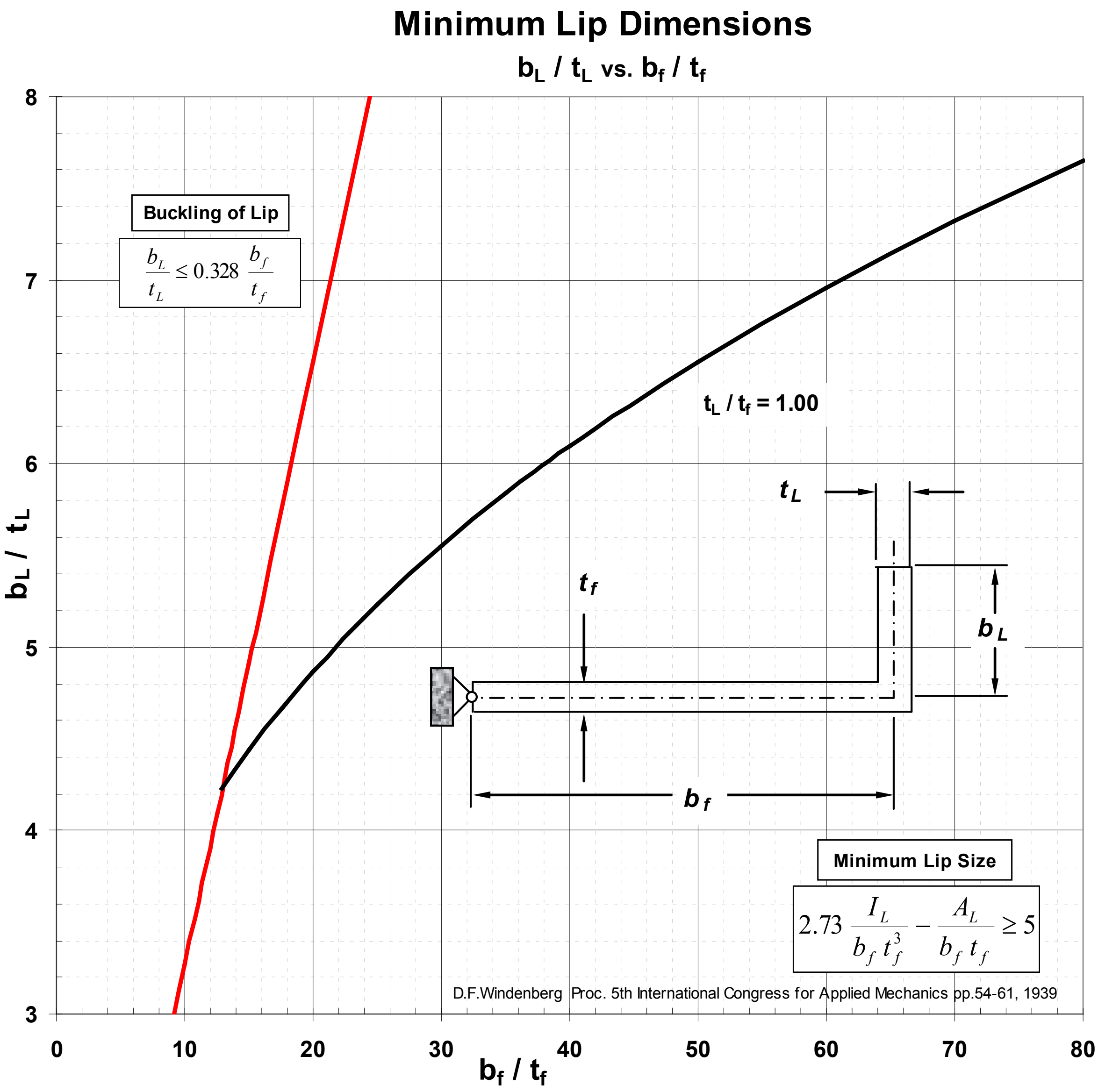
Minimum Lip Dimensions

D. F. Windenberg Proceedings. 5th International Congress for Applied Mechanics pages 54-61, 1939

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C7.11, page C7.6

Figure C7.11

Minimum Lip Dimensions Required for Flange to Buckle as a Simply Supported Plate



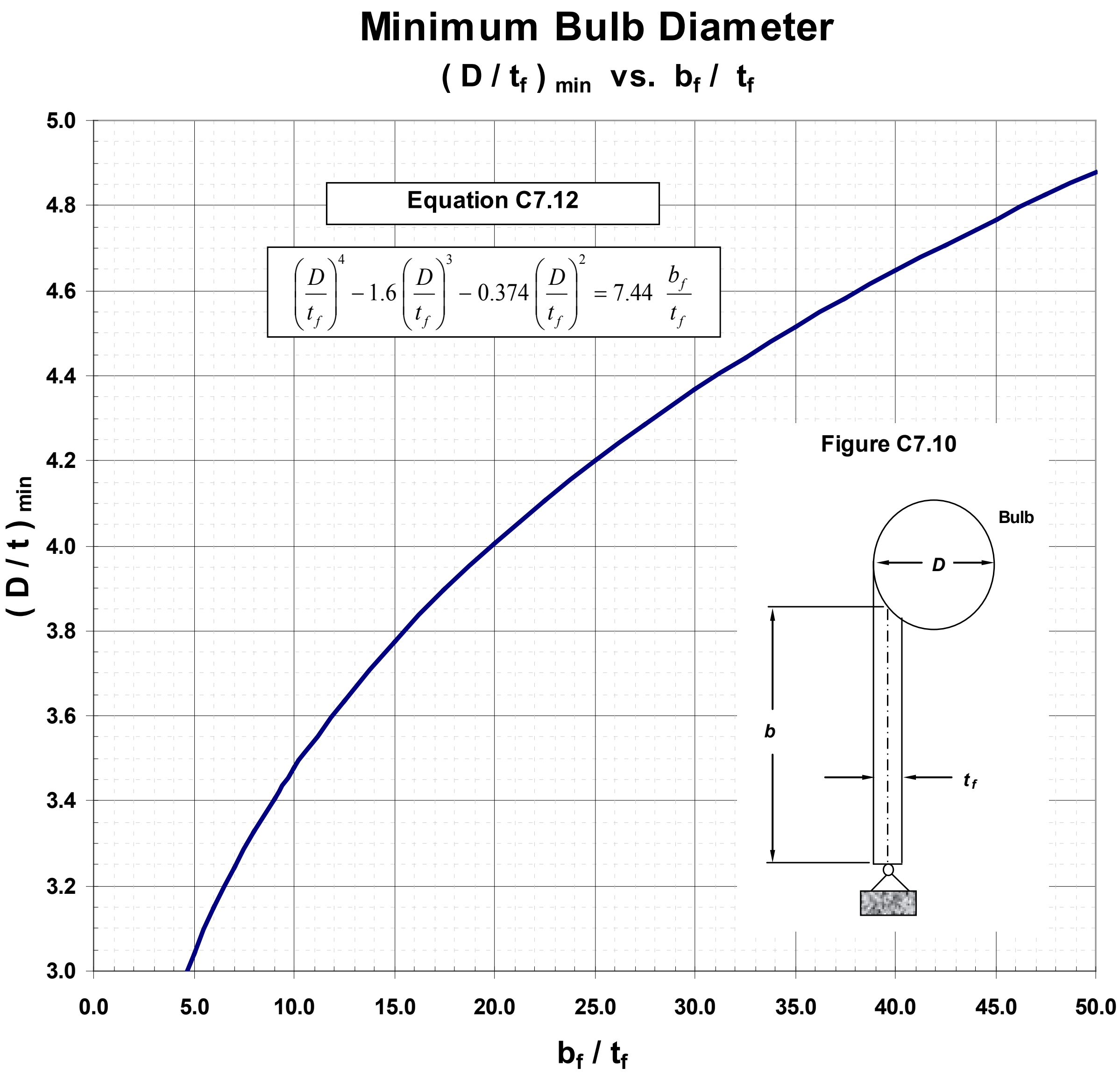
Minimum Bulb Diameter

D. F. Windenberg Proceedings. 5th International Congress for Applied Mechanics pages 54-61, 1939

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C7.12, page C7.6

Figure C7.12

Minimum Bulb Dimensions Required for Flange to Buckle as a Simply Supported Plate



Equation C7.12

$$\left(\frac{D}{t_f}\right)^4 - 1.6\left(\frac{D}{t_f}\right)^3 - 0.374\left(\frac{D}{t_f}\right)^2 = 7.44\ \frac{b_f}{t_f}$$

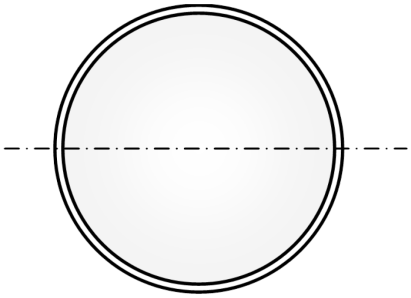
Maximum Shear Stress in Tubing

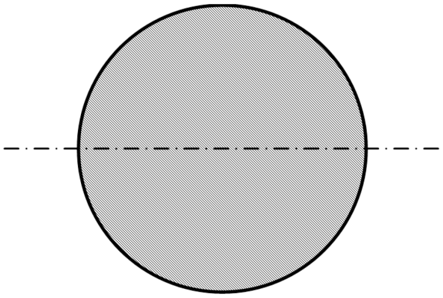
Tube

$$\tau_{\max} = \frac{4}{3}\frac{V}{A}\left(1 + \frac{D\ d}{D^2 + d^2}\right)$$

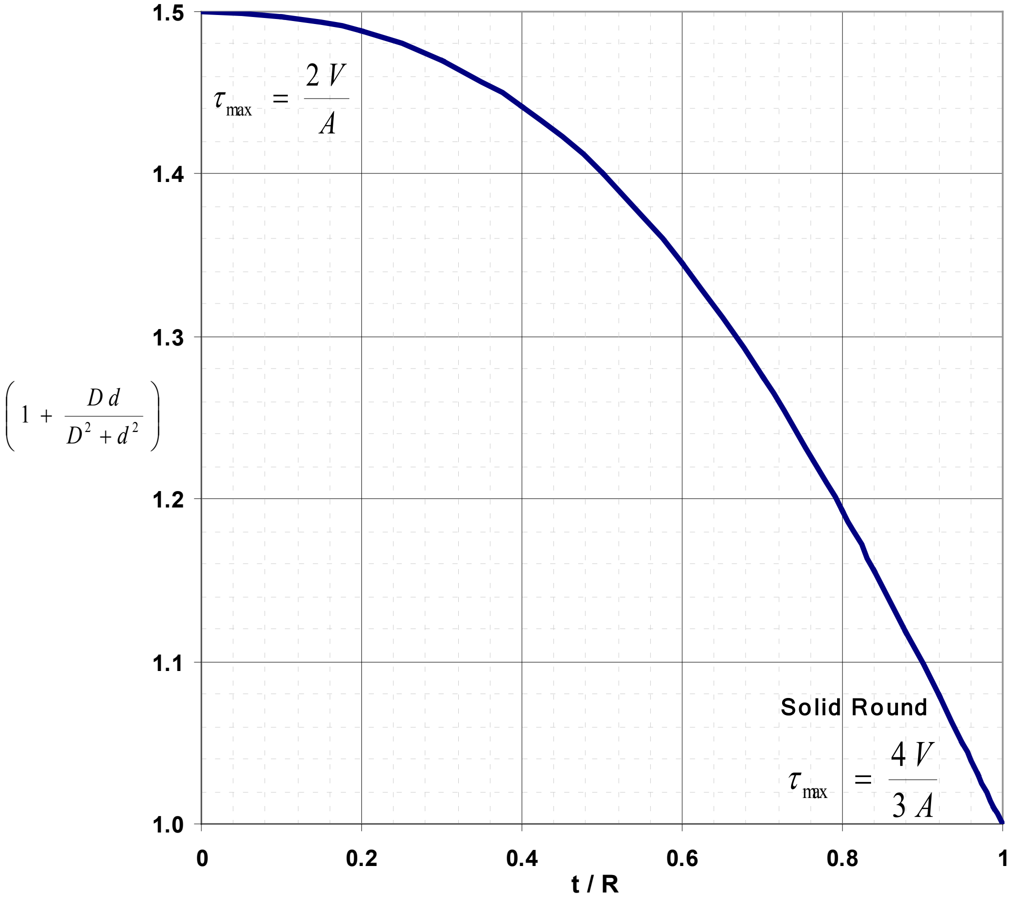
Solid Round

$$\tau_{\max} = \frac{4}{3}\frac{V}{A}$$





$$\left(1 + \frac{D\ d}{D^2 + d^2}\right) \text{ vs. } t / R$$



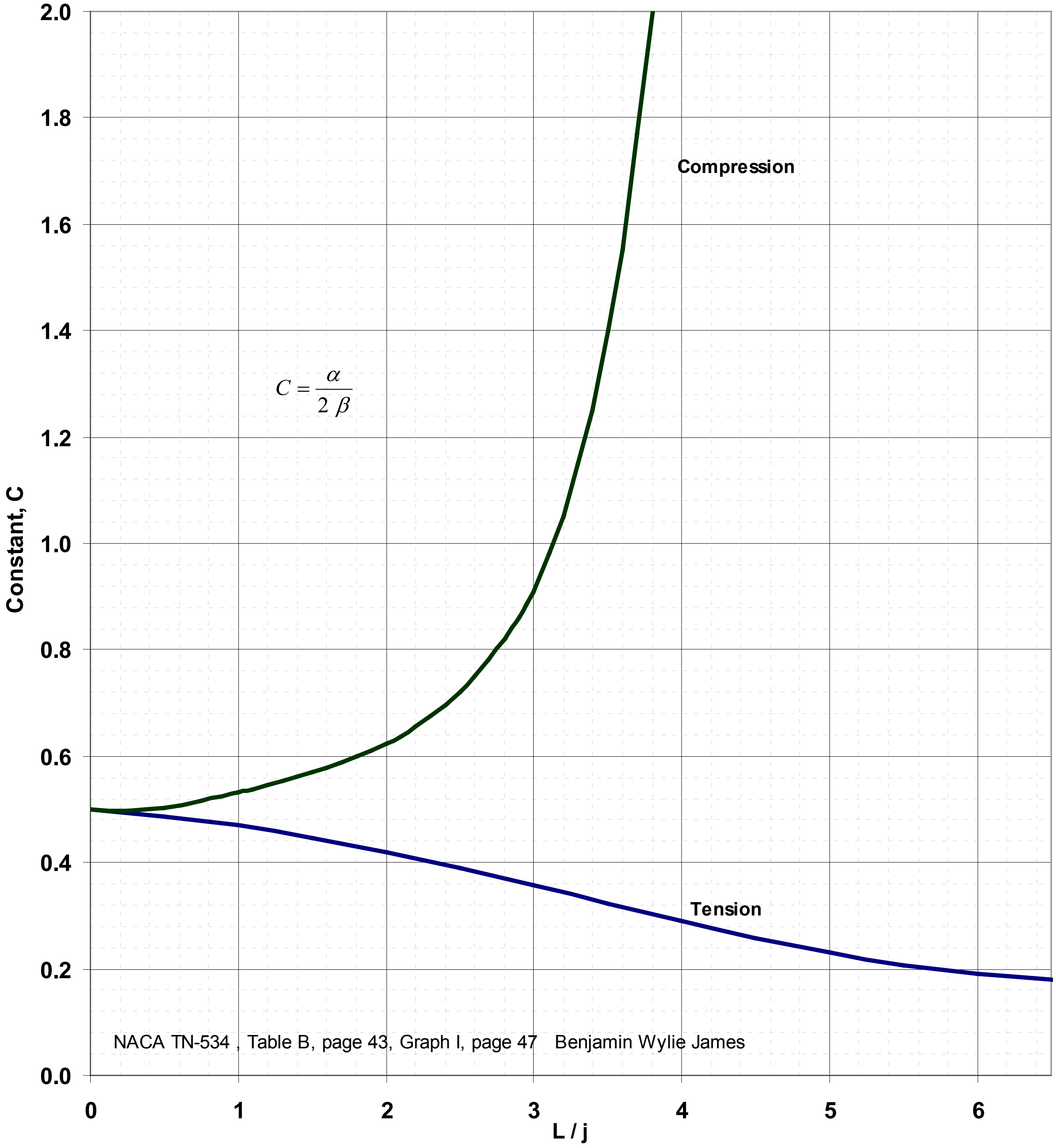
Carry-Over Factor

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure A11.46 page A11.23

NACA TN-534 Benjamin Wylie James Graph I, page 47

Principal Effects of Axial Load on Moment-Distribution Analysis of Rigid Structures

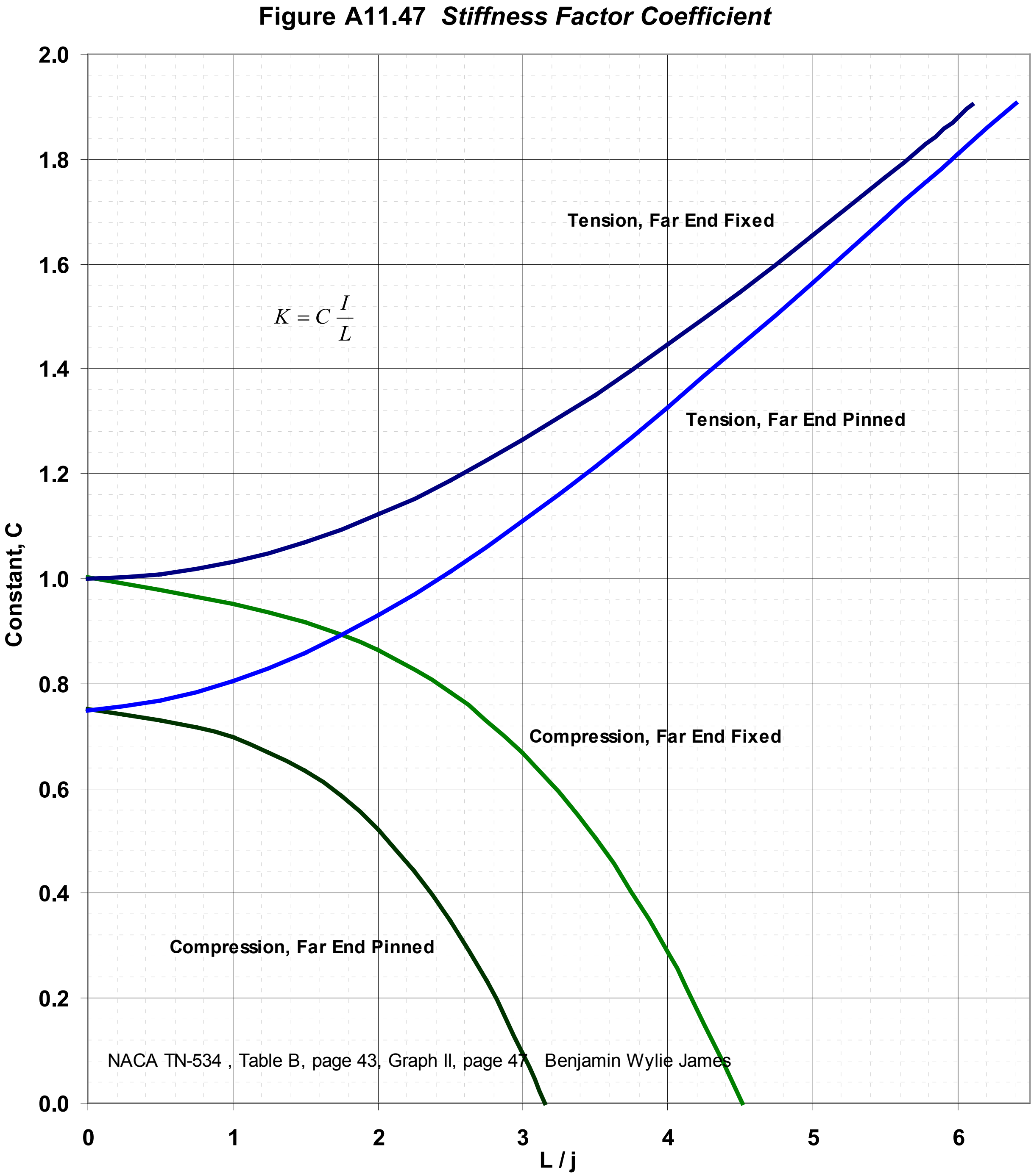
Figure A11.46 ***Carry-Over Factor***



Stiffness Factor Coefficient

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure A11.47 page A11.23

NACA TN-534 Graph II, page 47

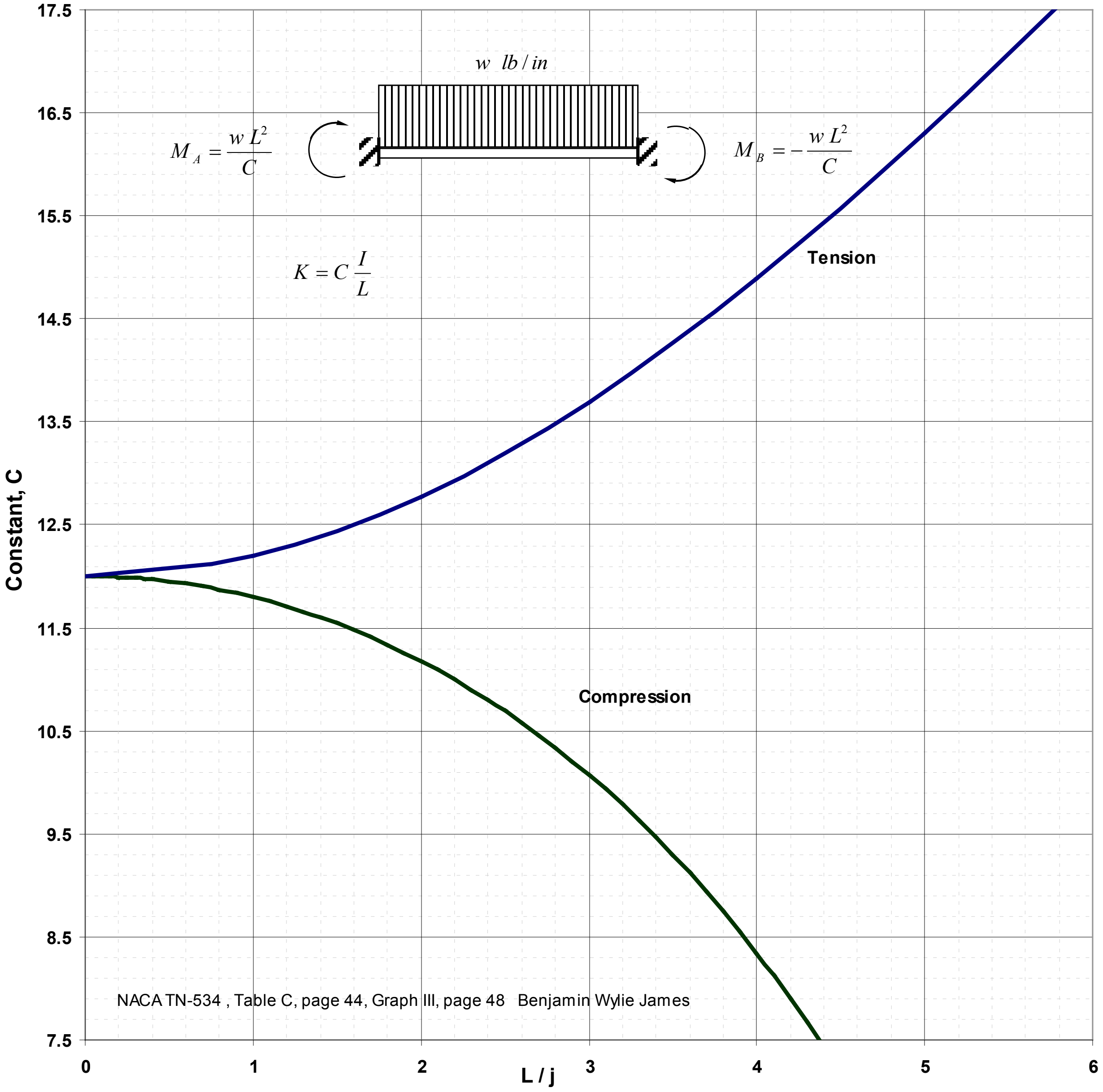


Fixed-End Moment Coefficient

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure A11.48 page A11.24

NACA TN-534 Graph **III**, page 48

Figure A11.48 ***Fixed-End Moment Coefficient, Uniform Load***



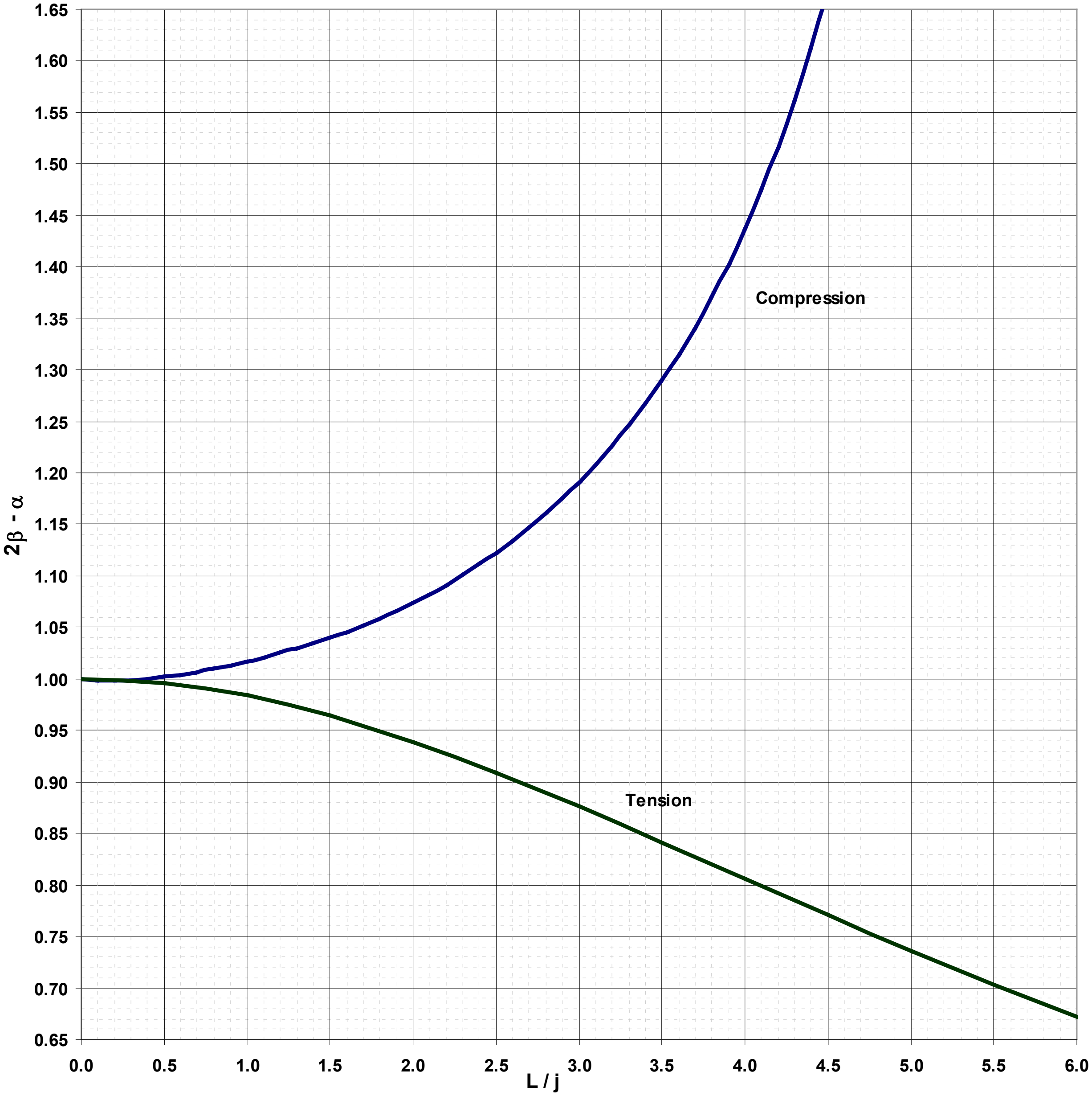
NACA TN-534 , Table C, page 44, Graph III, page 48 Benjamin Wylie James

Column Distribution Coefficient

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure A11.48 page A11.24

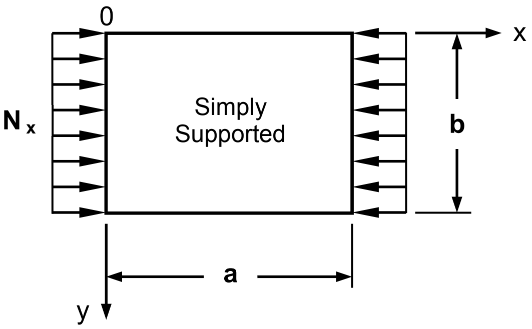
NACA TN-534 Graph XI, page 52

Figure A11.56 ***Column Distribution Coefficient***



Rectangular Plate

Stephen Timoshenko *Theory of Plates and Shells* Second Edition, pages 387-389



General Expression for the Deflection

$$w = \sum_{m=1,2,3...}^{\infty} \sum_{n=1,2,3...}^{\infty} a_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}$$

Fourier series where m and n are integers.

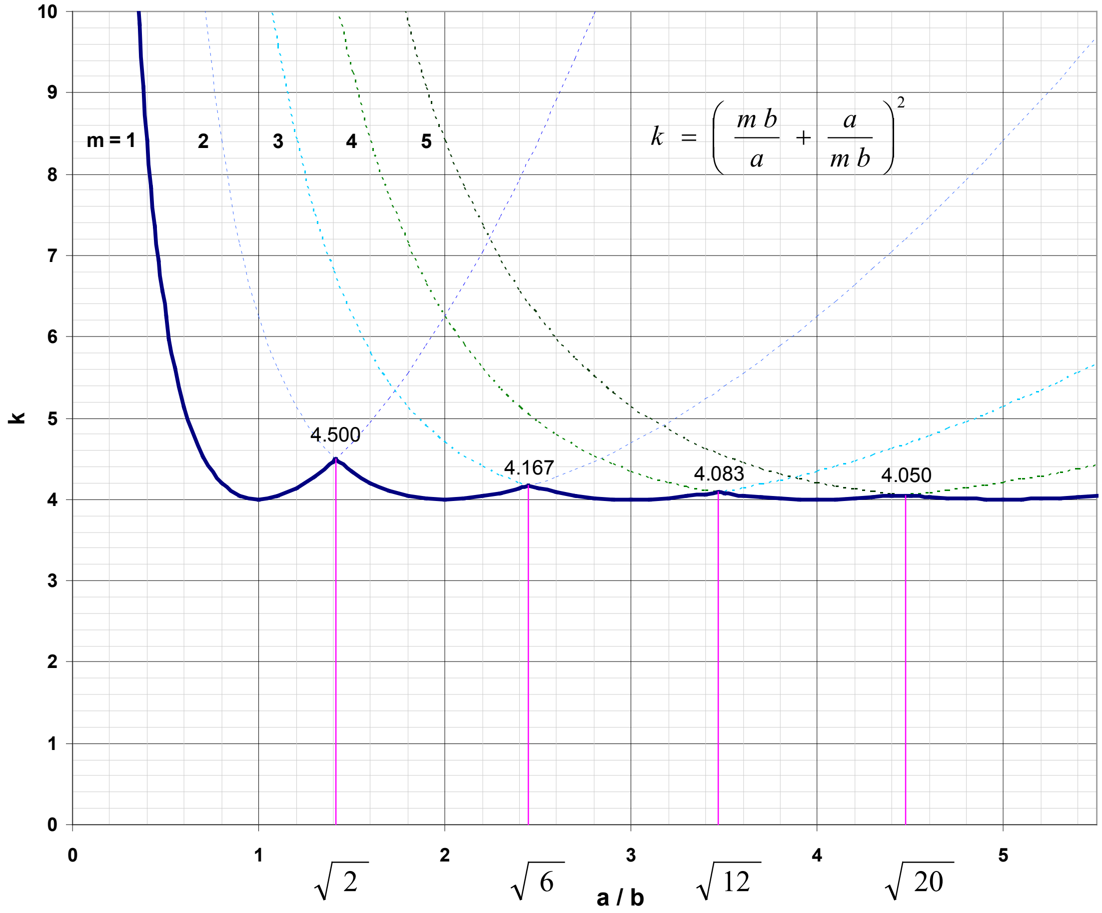
Critical Value of the Compressive Force

$$(N_x)_{cr} = \frac{\pi^2 a^2 D}{m^2} \left(\frac{m^2}{a^2} + \frac{1}{b^2} \right)^2 = \frac{\pi^2 D}{b^2} \left(\frac{m b}{a} + \frac{a}{m b} \right)^2 \qquad k = \left(\frac{m b}{a} + \frac{a}{m b} \right)^2$$

Flexural Rigidity $D = \dfrac{E t^3}{12 \left(1 - \nu^2\right)}$

Graph

Analysis and Design of Flight Vehicle Structures See Figure 12, page A18.22



Section Shape Factor

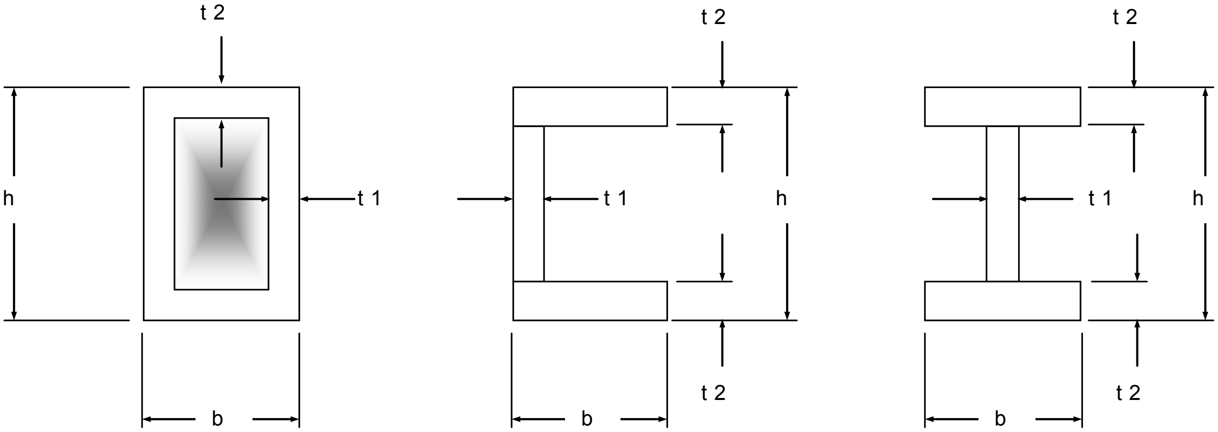
Analysis and Design of Flight Vehicle Structures Figure C3.8, page C3.3

Nomenclature - Strong Axis

t1 = Web Thickness

$$K = \frac{2\ Q}{Z} = \frac{2\ Q\ c}{I}$$

t2 = Flange Thickness

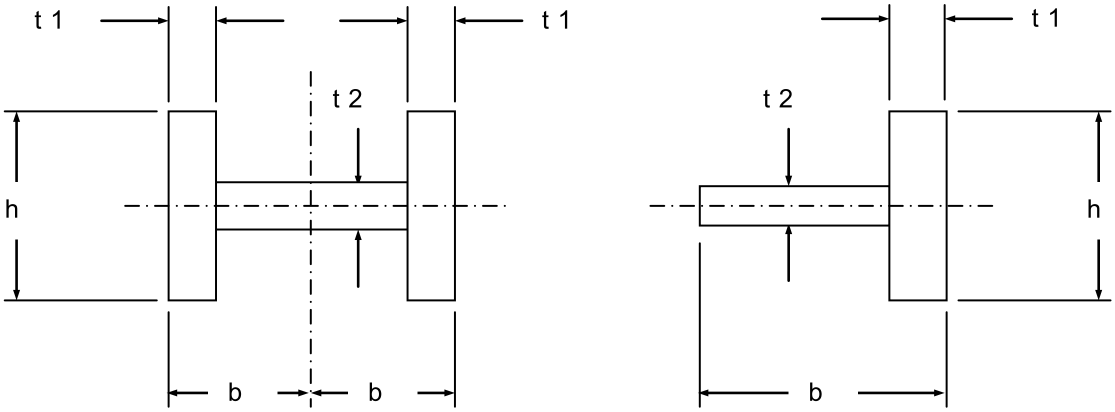


Nomenclature - Weak Axis

t1 = Flange Thickness

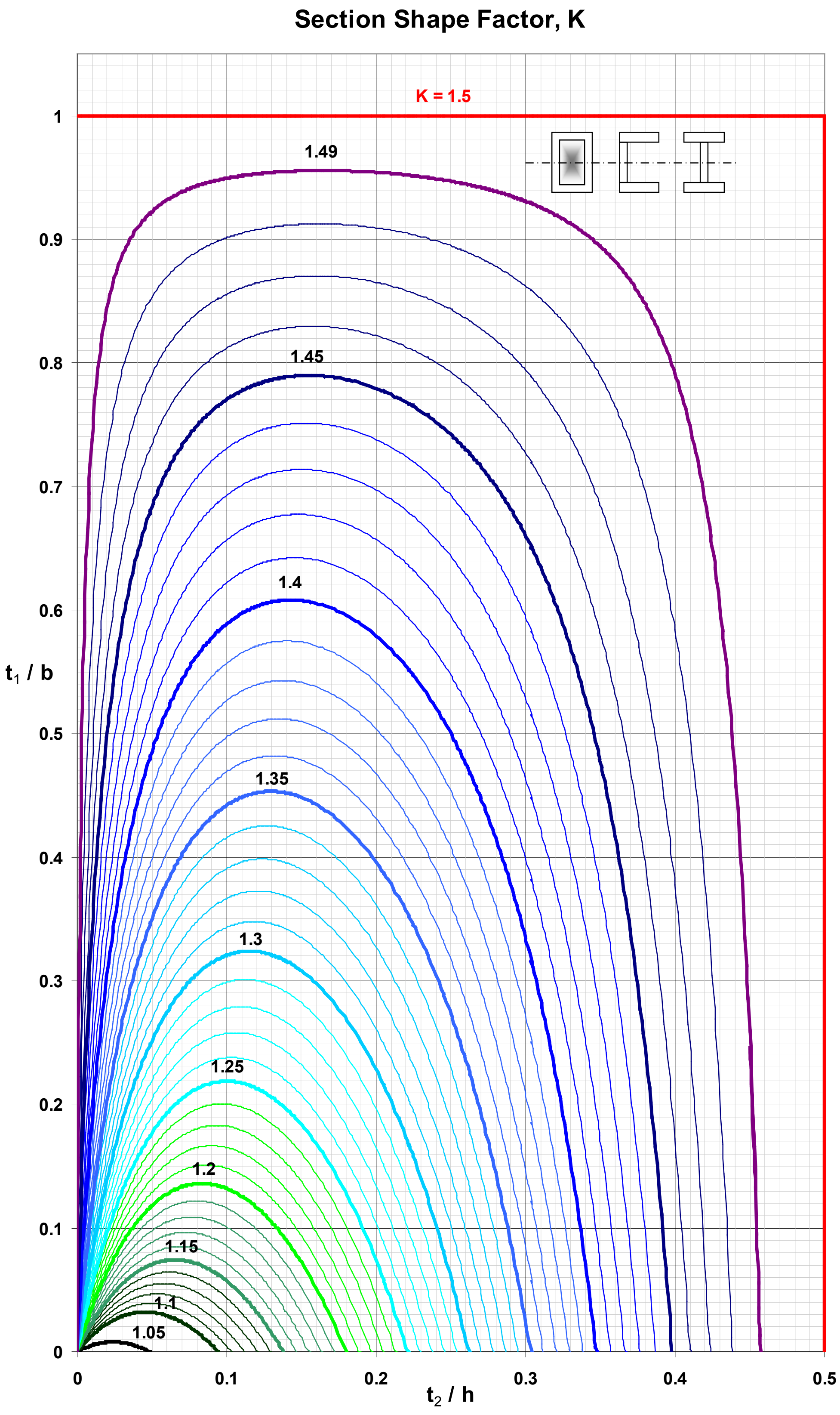
$$K = \frac{2\ Q}{Z} = \frac{2\ Q\ c}{I}$$

t2 = Web Thickness



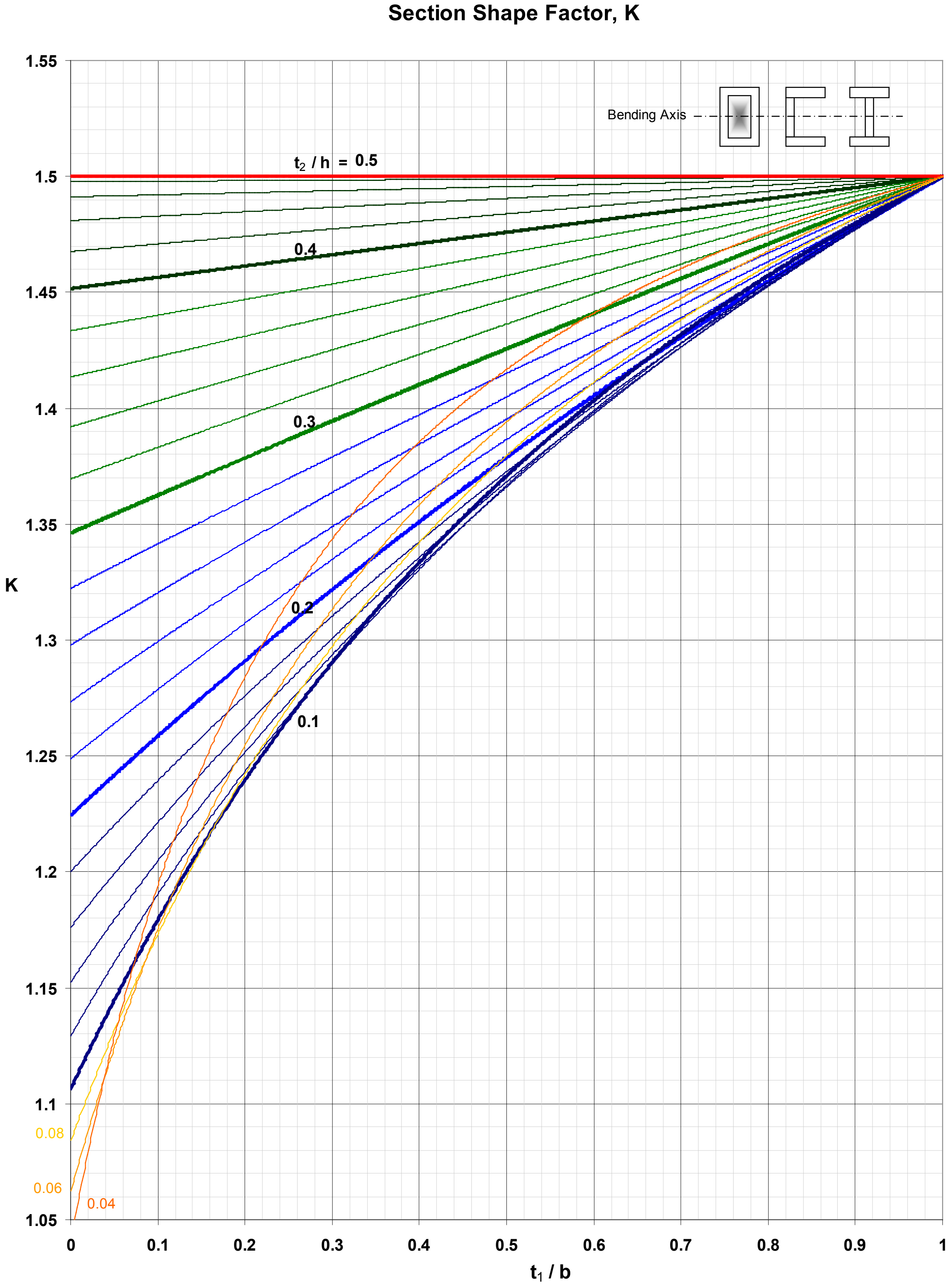
Section Shape Factor I – Strong Axis

Analysis and Design of Flight Vehicle Structures Figure C3.8, page C3.3 (Modified)



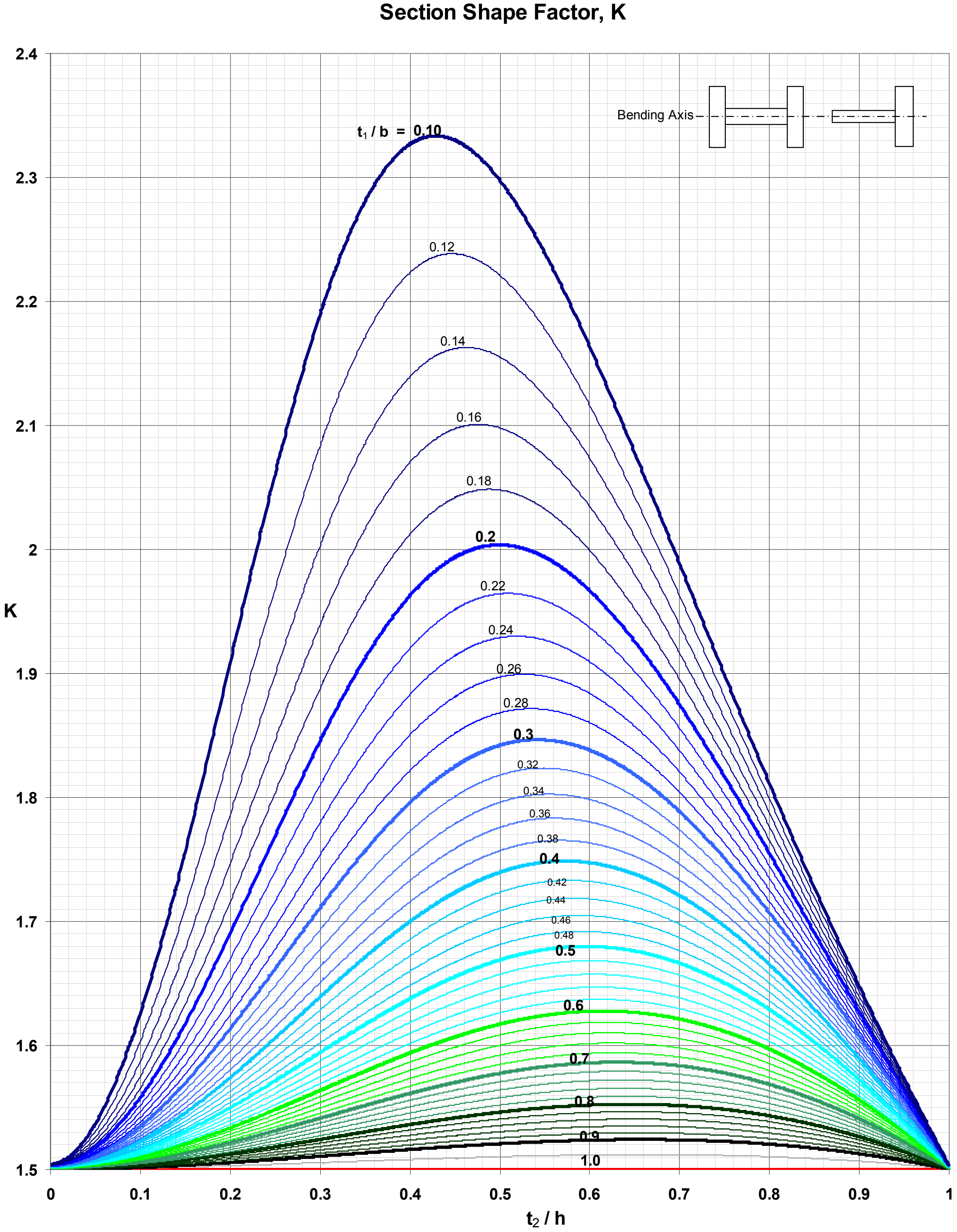
Section Shape Factor II – Strong Axis

Analysis and Design of Flight Vehicle Structures Figure C3.8, page C3.3 (Modified)



Section Shape Factor III – Weak Axis

Analysis and Design of Flight Vehicle Structures Figure C3.8, page C3.3 (Modified)



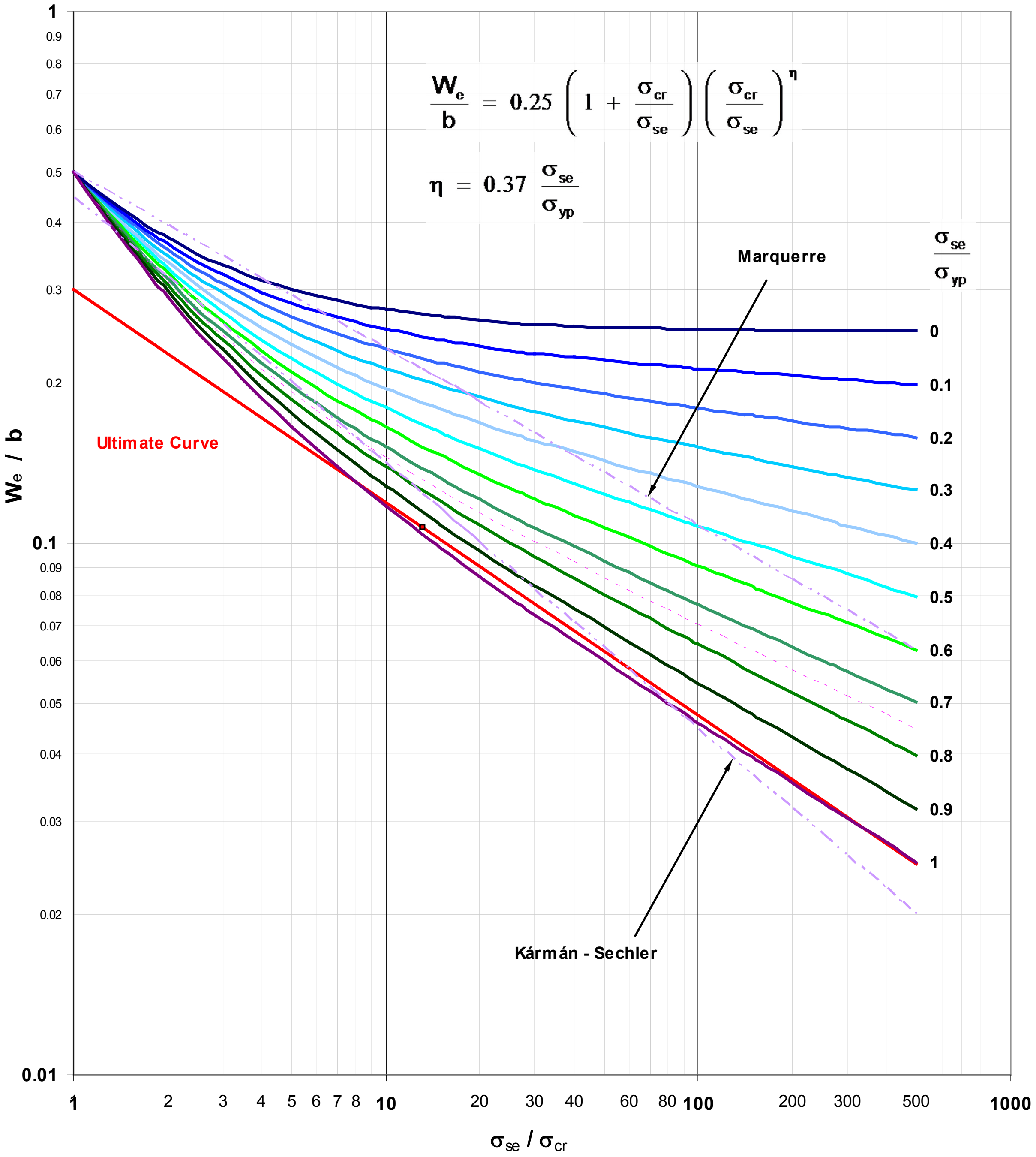
Effective Width

Ernest E. Sechler and Louis G. Dunn *Airplane Structural Analysis and Design* Figure 6-2, page 205

Equation 6-5

$$\frac{W_e}{b} = 0.25\left(1+\frac{\sigma_{cr}}{\sigma_{se}}\right)\left(\frac{\sigma_{cr}}{\sigma_{se}}\right)^{\eta} \qquad \eta = 0.37\,\frac{\sigma_{se}}{\sigma_{yp}}$$

Figure 6-2 Curve for Determining the Effective Width

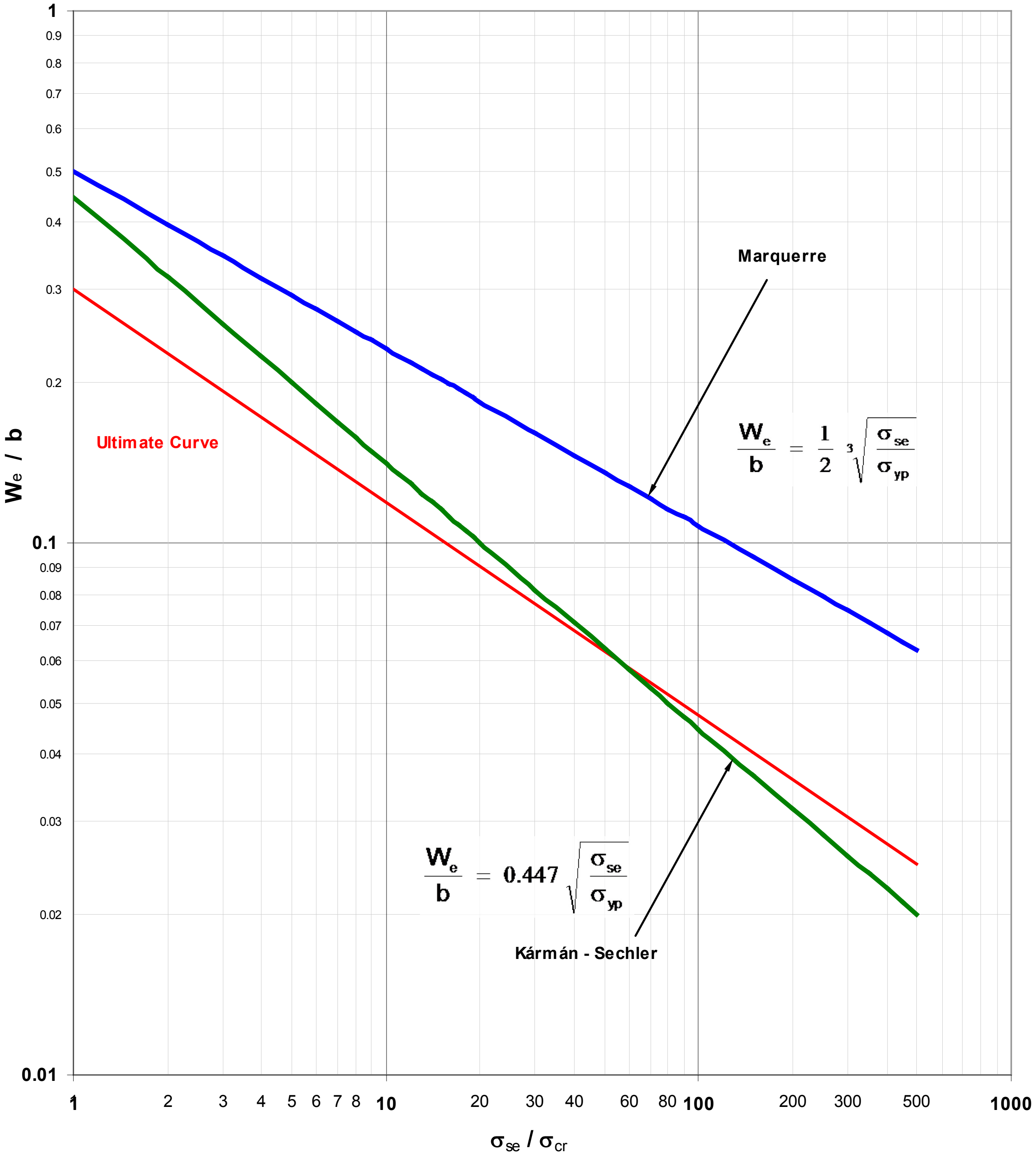


Marquerre and Kármán-Sechler

Marquerre $$\frac{W_e}{b} = \frac{1}{2}\sqrt[3]{\frac{\sigma_{se}}{\sigma_{yp}}}$$

Kármán-Sechler $$\frac{W_e}{b} = 0.447\sqrt{\frac{\sigma_{se}}{\sigma_{yp}}}$$

Figure 6-2 Curve for Determining the Effective Width



Flange Flexibility Factor

Ernest E. Sechler and Louis G. Dunn *Airplane Structural Analysis and Design* Figure 6-30, page 239

The family of curves shifts up with increasing values of the web buckling angle, α. It is interesting to see how cleverly the shift was accounted for in the days of static graphs. With a spreadsheet we can simply input a new value for α and the curves will shift as shown on the following three pages.

Equation 6-33, page 238

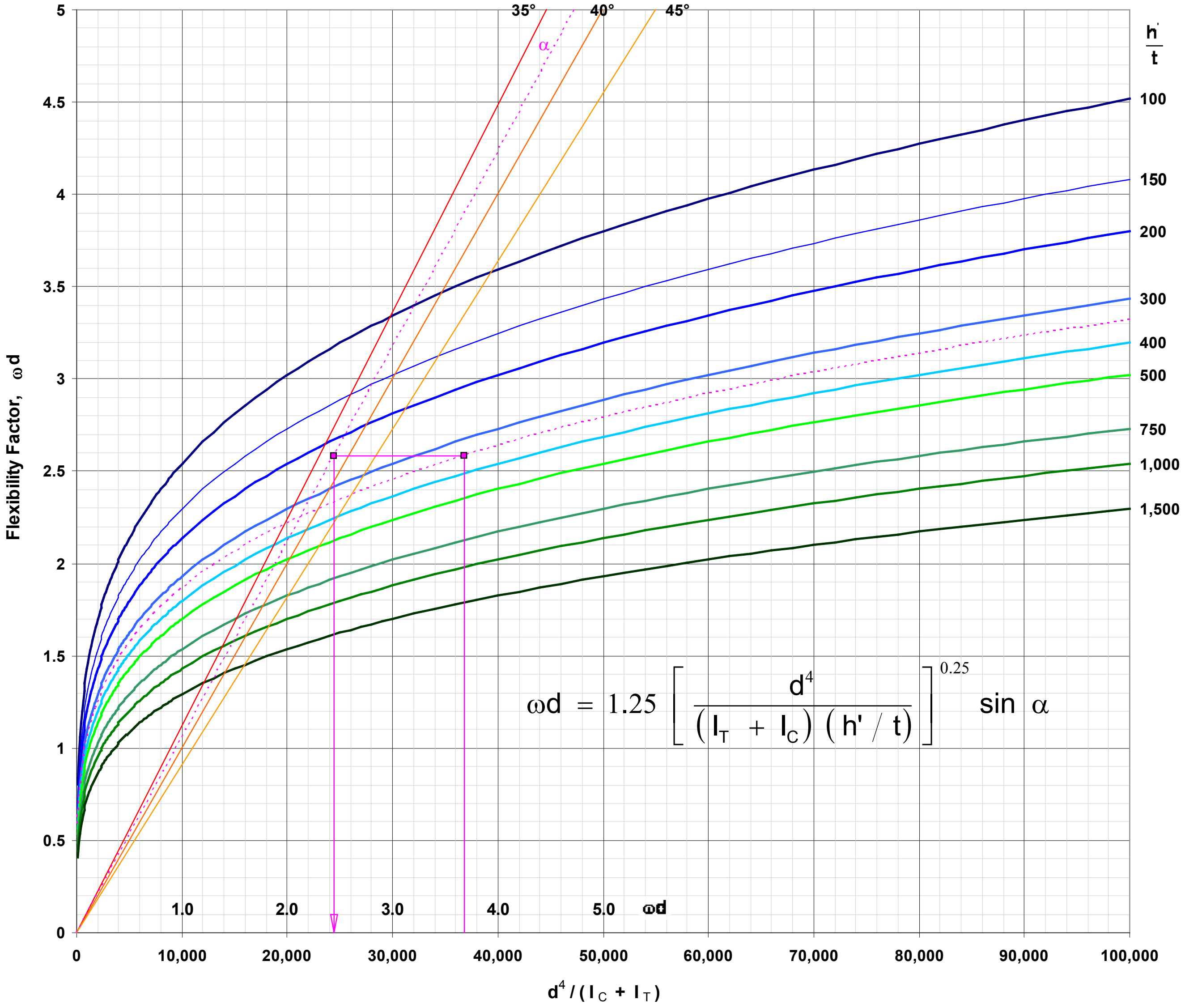
$$\omega d = 1.25 \ d \left[\frac{t}{(I_C + I_T)\ h} \right]^{0.25} \sin \alpha$$

Old School Nomograph

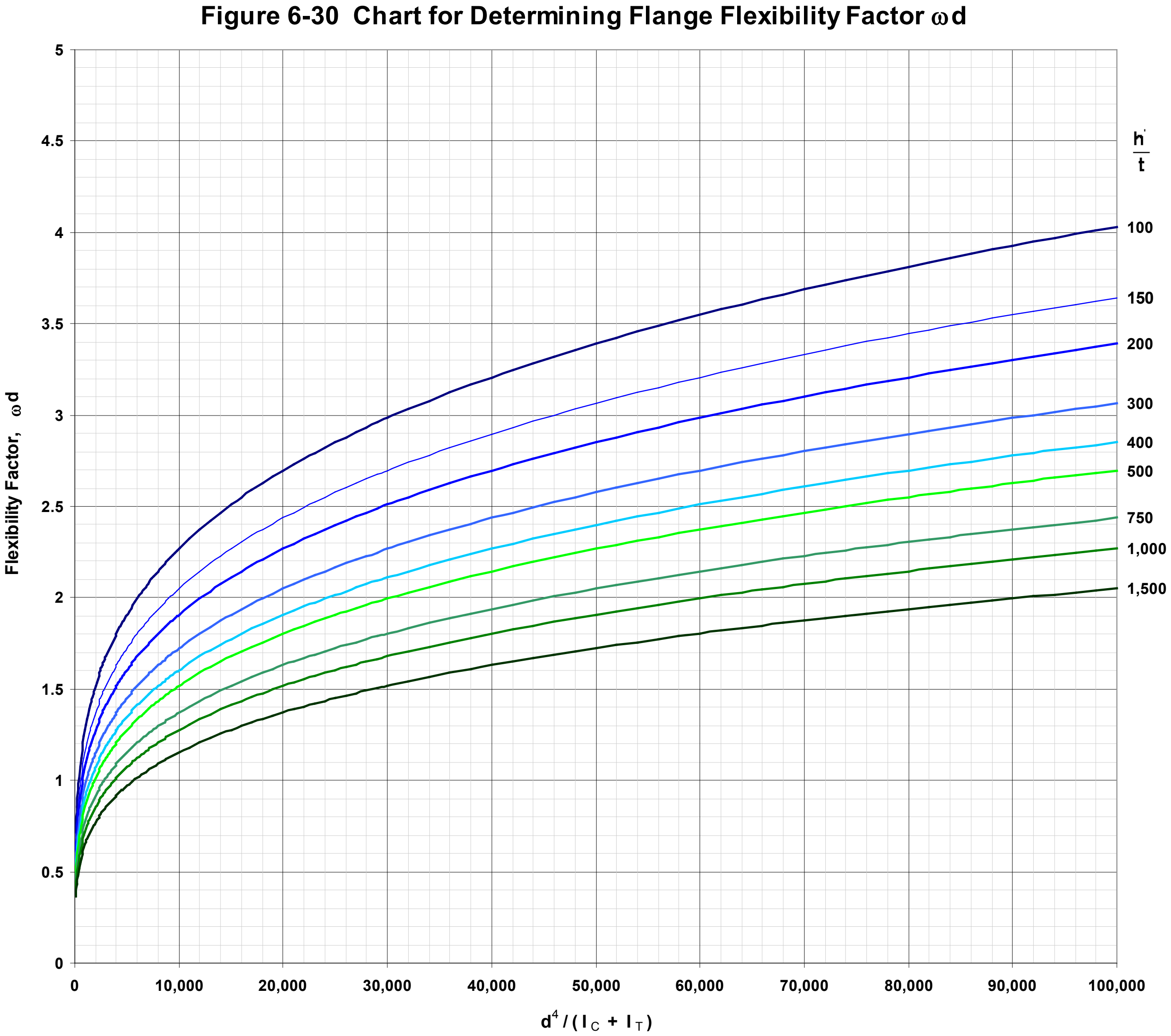
Web Buckling Angle, α = 40 degrees is plotted for the family of curves.

In this example, for α = 37.5 degrees ωd = 2.45

Figure 6-30 Chart for Determining Flange Flexibility Factor ωd

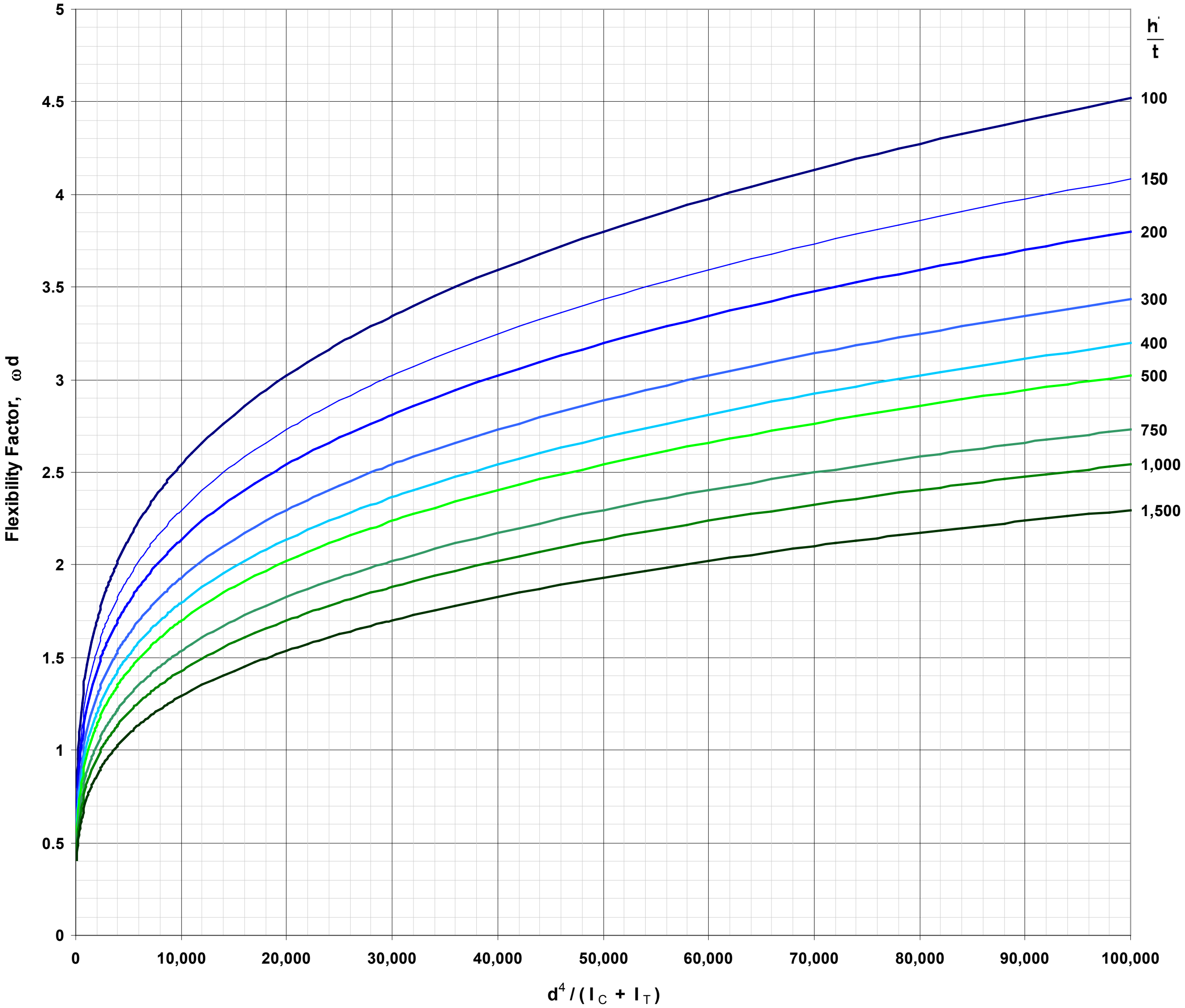


Web Buckling Angle, α = 35 degrees



Web Buckling Angle, α = 40 degrees

Figure 6-30 Chart for Determining Flange Flexibility Factor ωd



Web Buckling Angle, α = 45 degrees

Figure 6-30 Chart for Determining Flange Flexibility Factor ωd

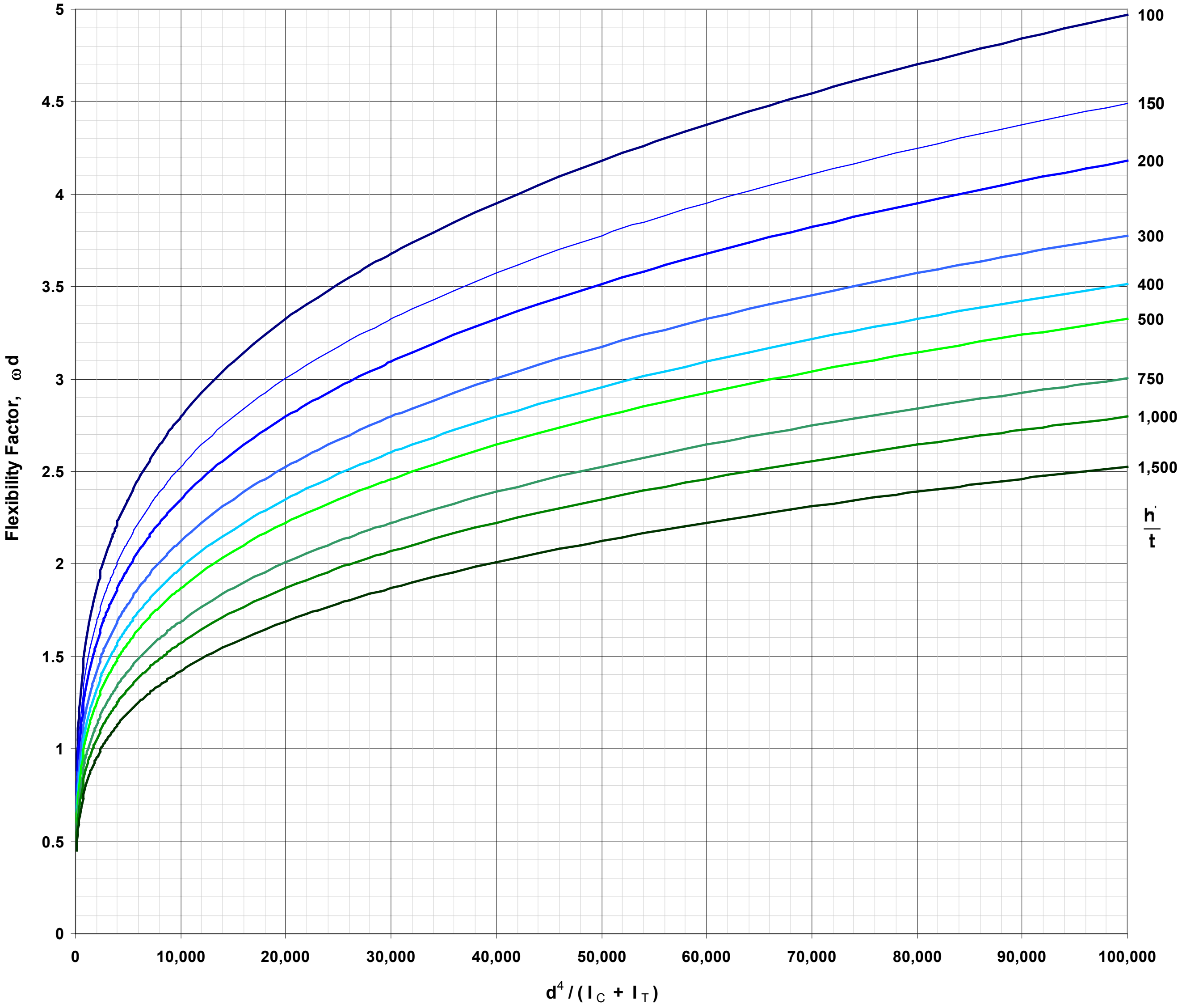


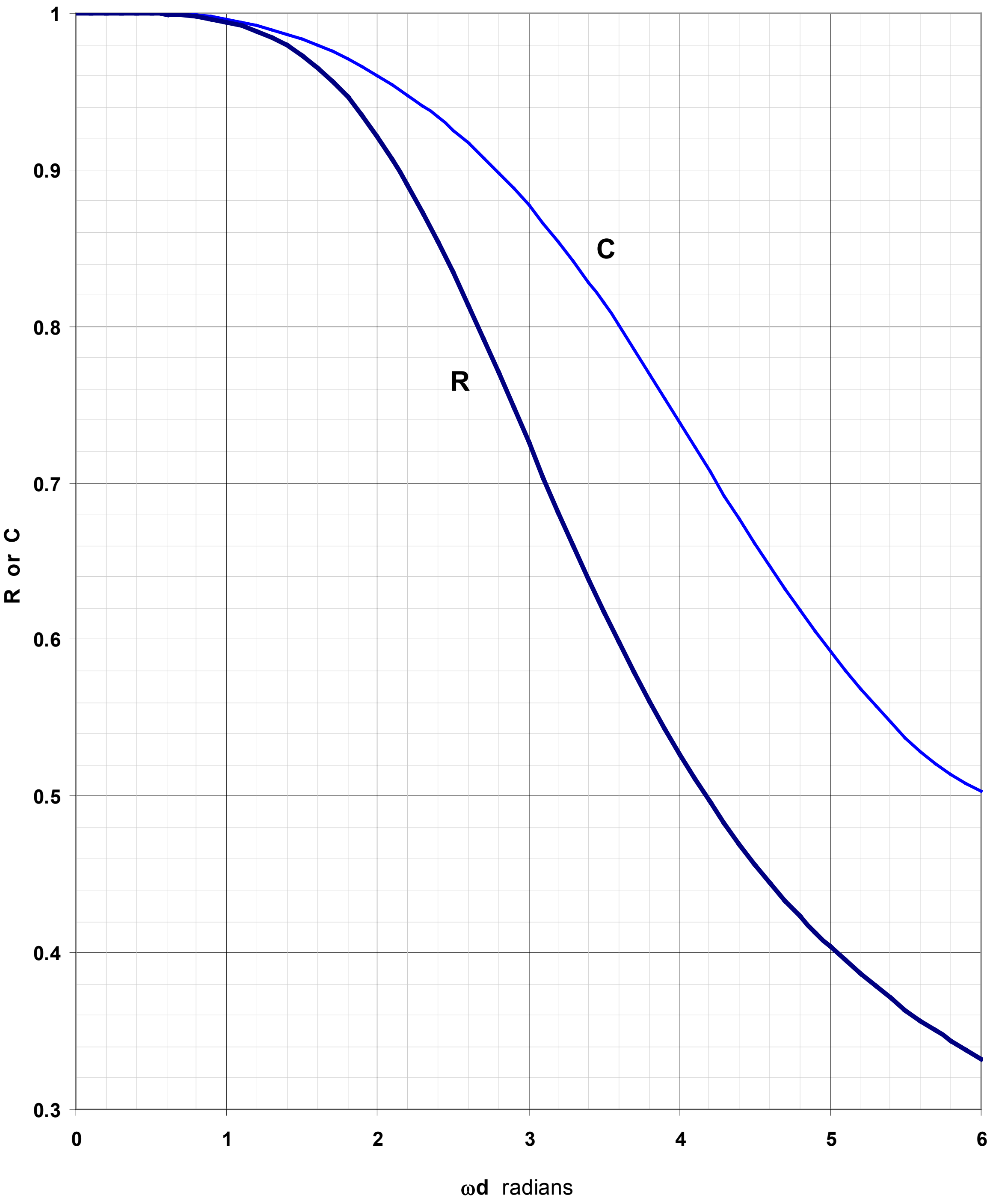
Chart for Determining Correction Factor R

Ernest E. Sechler and Louis G. Dunn *Airplane Structural Analysis and Design* Figure 6-31, page 20

Herbert Wagner NACA TM-606 *Flat Sheet Metal Girders With Very Thin Metal Web, Part III*

From *Zeitschrift für Flugtechnik und Motorluftschiffahrt* Volume 20, Nos. 11 and 12, June 14 & 28, 1929

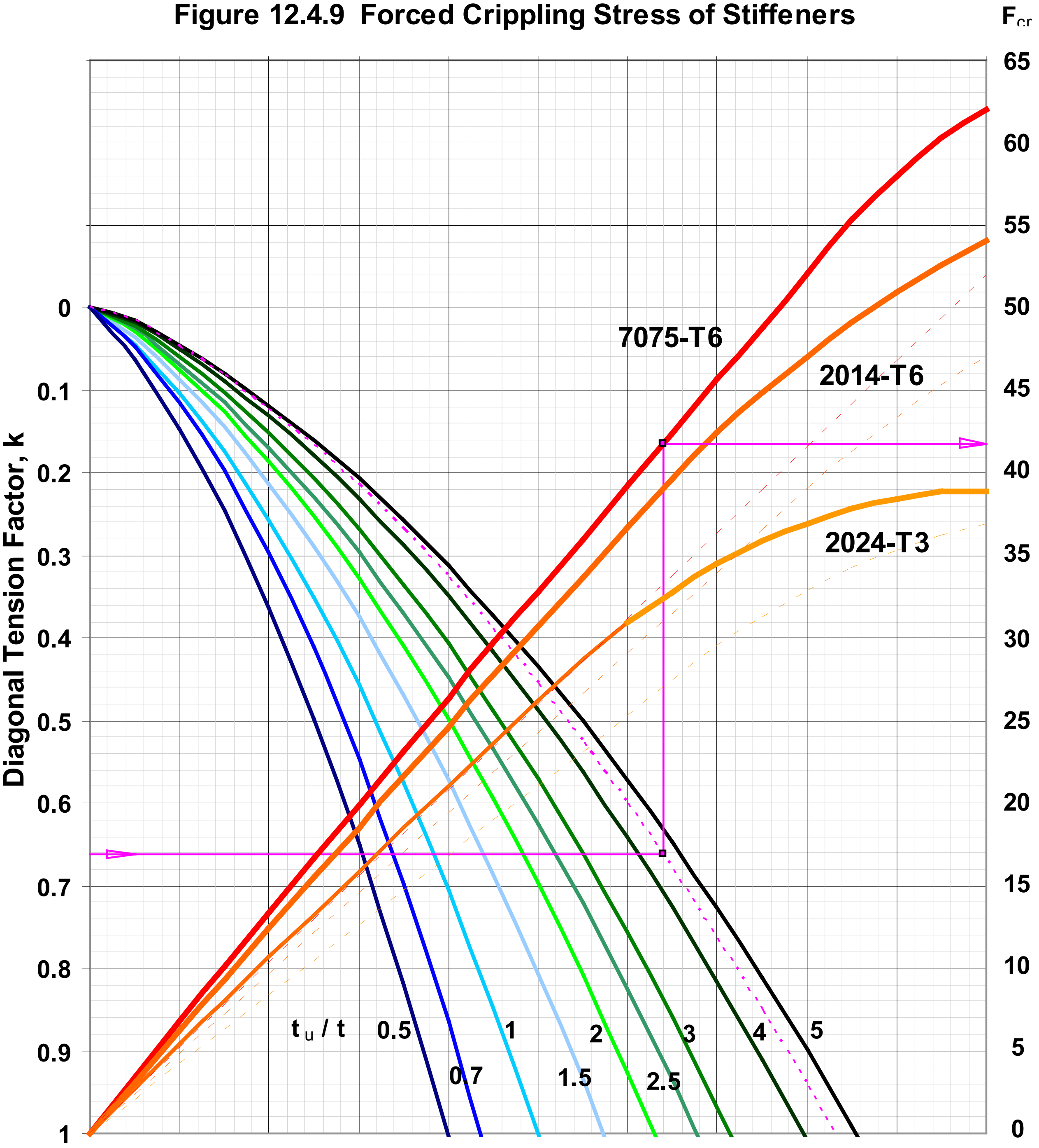
Figure 6-31 Chart for Determining R and C



Forced Crippling Stress of Stiffeners

Michael Chun-Yung Niu *Airframe Stress Analysis and Sizing* Figure 12.4.9, page 492

Figure 12.4.9 Forced Crippling Stress of Stiffeners



Calculation of Fastened-Joint Spring Constants

William F. McCombs *Engineering Column Analysis – The Analysis of Compression Members*
Appendix E, page E1

AFFDL-TR-67-184

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W. F. McCombs, J. C. McQueen, J. L. Perry

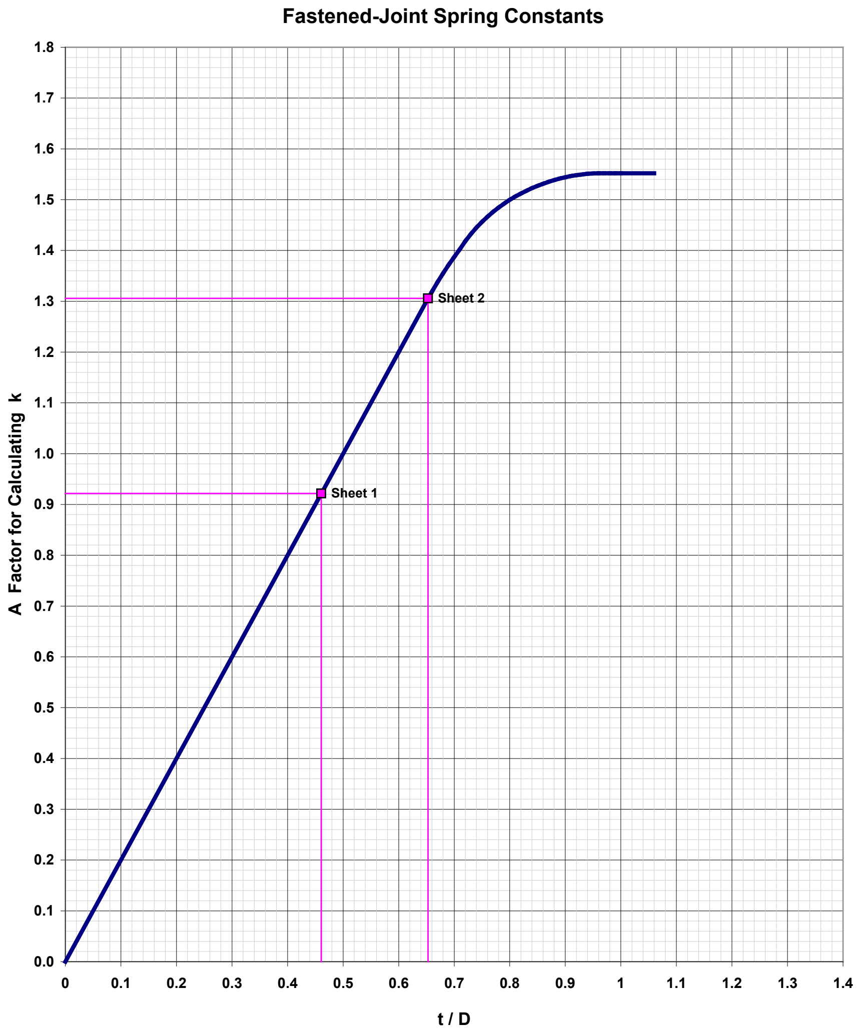


Table E-2

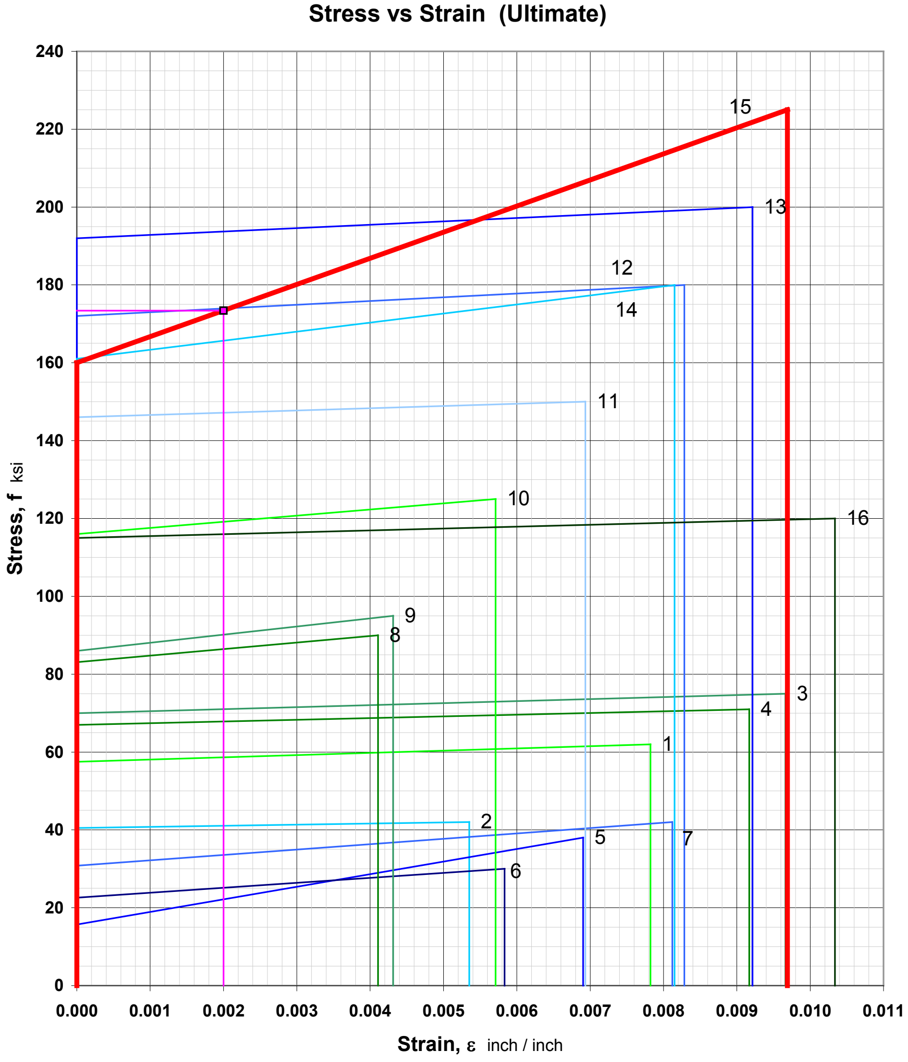
	k_n x 10^{-6}		
D	**Alum**	**Steel**	**Other**
0.125	0.212	4.71	0.715
0.156	**0.264**	5.88	0.892
0.188	0.317	7.07	1.073
0.250	0.423	9.43	1.431
0.313	0.528	11.80	1.790
0.375	0.633	14.20	2.154
0.438	0.732	16.40	2.488
0.540	0.845	18.90	2.867
0.563	0.952	21.20	3.216
0.625	1.060	23.50	3.565

	5/32" Hi-Lok		
Sheets	**Test**	**Calc**	**Ratio**
Qty 2 - 0.072	127,800	121,651	1.05
Qty 2 - 0.102	153,200	172,339	0.89
0.072 & 0.102	136,800	**142,626**	0.96
2 - 0.072 & 0.102	291,300	285,251	1.02

	1/4" NAS 464 Bolts		
Sheets	**Test**	**Calc**	**Ratio**
Qty 2 - 0.072	163,000	121,824	1.34
Qty 2 - 0.102	200,000	172,584	1.16
0.072 & 0.102	183,000	142,828	1.28
2 - 0.072 & 0.102	318,000	285,656	1.11

Ultimate Bending Strength

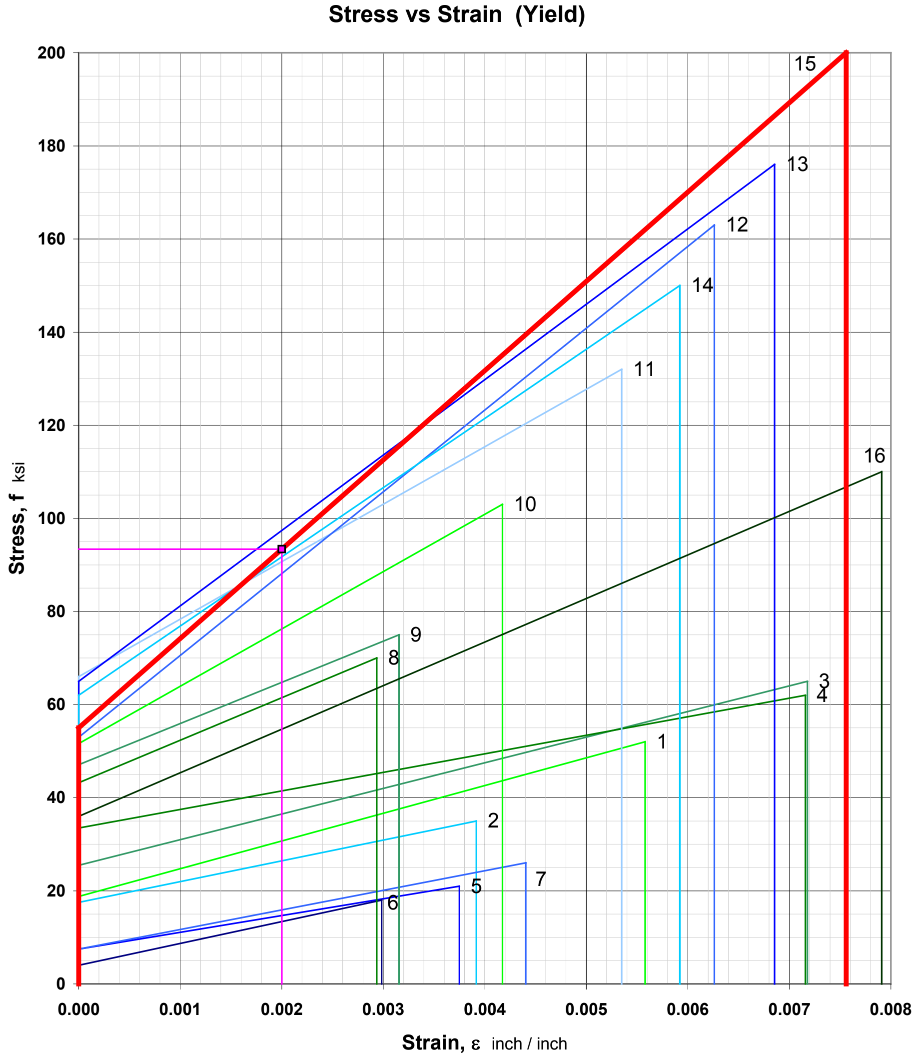
William F. McCombs *Engineering Column Analysis – The Analysis of Compression Members*
Appendix A, page A.1



No.	Material	Size	Yield		Ultimate	
			$f_m = F_{ty}$	f_o	$f_m = F_{tu}$	f_o
1	*2014-T6 Alum Die Forgings*	t ≤ 4 in	*52*	*18.8*	*62*	*57.5*
2	*6061-T6 Aluminum Sheet*	t > 0.020 in	*35*	*17.5*	*42*	*40.5*
3	*7075-T6 Alum Die Forgings*	t ≤ 2 in	*65*	*25.5*	*75*	*70*
4	*7075-T6 Alum Hand Forging (L)*	t ≤ 6 in	*62*	*33.5*	*71*	*67*
5	*AZ61A Magnesium Extr (L)*	t ≤ 0.25 in	*21*	*7.5*	*38*	*15.7*
6	*HK31A-O Magnesium Sheet*	0.016-0.25	*18*	*4*	*30*	*22.6*
7	*ZK60AMargnesium Forging*		*26*	*7.5*	*42*	*30.8*
8	*AISI Alloy Steel (Normalized)*	> 0.188 in	*70*	*43.2*	*90*	*83.1*
9	*AISI Alloy Steel (Normalized)*	t ≤ 0.188 in	*75*	*47.1*	*95*	*86*
10	*AISI Alloy Steel (Heat Treated)*		*103*	*51.6*	*125*	*116*
11	*AISI Alloy Steel (Heat Treated)*		*132*	*66*	*150*	*146*
12	*AISI Alloy Steel (Heat Treated)*		*163*	*53*	*180*	*172*
13	*AISI Alloy Steel (Heat Treated)*		*176*	*65*	*200*	*192*
14	*17-7 PH Stainless Steel*		*150*	*62*	*180*	*161*
15	**PH15-7 MO (RH950) Stainless**		**200**	**55**	**225**	**160**
16	*TI - 8MN Titanium Alloy*		*110*	*36*	*120*	*115*

Yield Bending Strength

William F. McCombs *Engineering Column Analysis – The Analysis of Compression Members*
Appendix A, page A.1



No.	Material	Size	Yield		Ultimate	
			$f_m = F_{ty}$	f_o	$f_m = F_{tu}$	f_o
1	*2014-T6 Alum Die Forgings*	t ≤ 4 in	*52*	*18.8*	*62*	*57.5*
2	*6061-T6 Aluminum Sheet*	t > 0.020 in	*35*	*17.5*	*42*	*40.5*
3	*7075-T6 Alum Die Forgings*	t ≤ 2 in	*65*	*25.5*	*75*	*70*
4	*7075-T6 Alum Hand Forging (L)*	t ≤ 6 in	*62*	*33.5*	*71*	*67*
5	*AZ61A Magnesium Extr (L)*	t ≤ 0.25 in	*21*	*7.5*	*38*	*15.7*
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7	*ZK60AMargnesium Forging*		*26*	*7.5*	*42*	*30.8*
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10	*AISI Alloy Steel (Heat Treated)*		*103*	*51.6*	*125*	*116*
11	*AISI Alloy Steel (Heat Treated)*		*132*	*66*	*150*	*146*
12	*AISI Alloy Steel (Heat Treated)*		*163*	*53*	*180*	*172*
13	*AISI Alloy Steel (Heat Treated)*		*176*	*65*	*200*	*192*
14	*17-7 PH Stainless Steel*		*150*	*62*	*180*	*161*
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16	*TI - 8MN Titanium Alloy*		*110*	*36*	*120*	*115*

Ratio of Maximum Stress to Average Stress in Stiffener

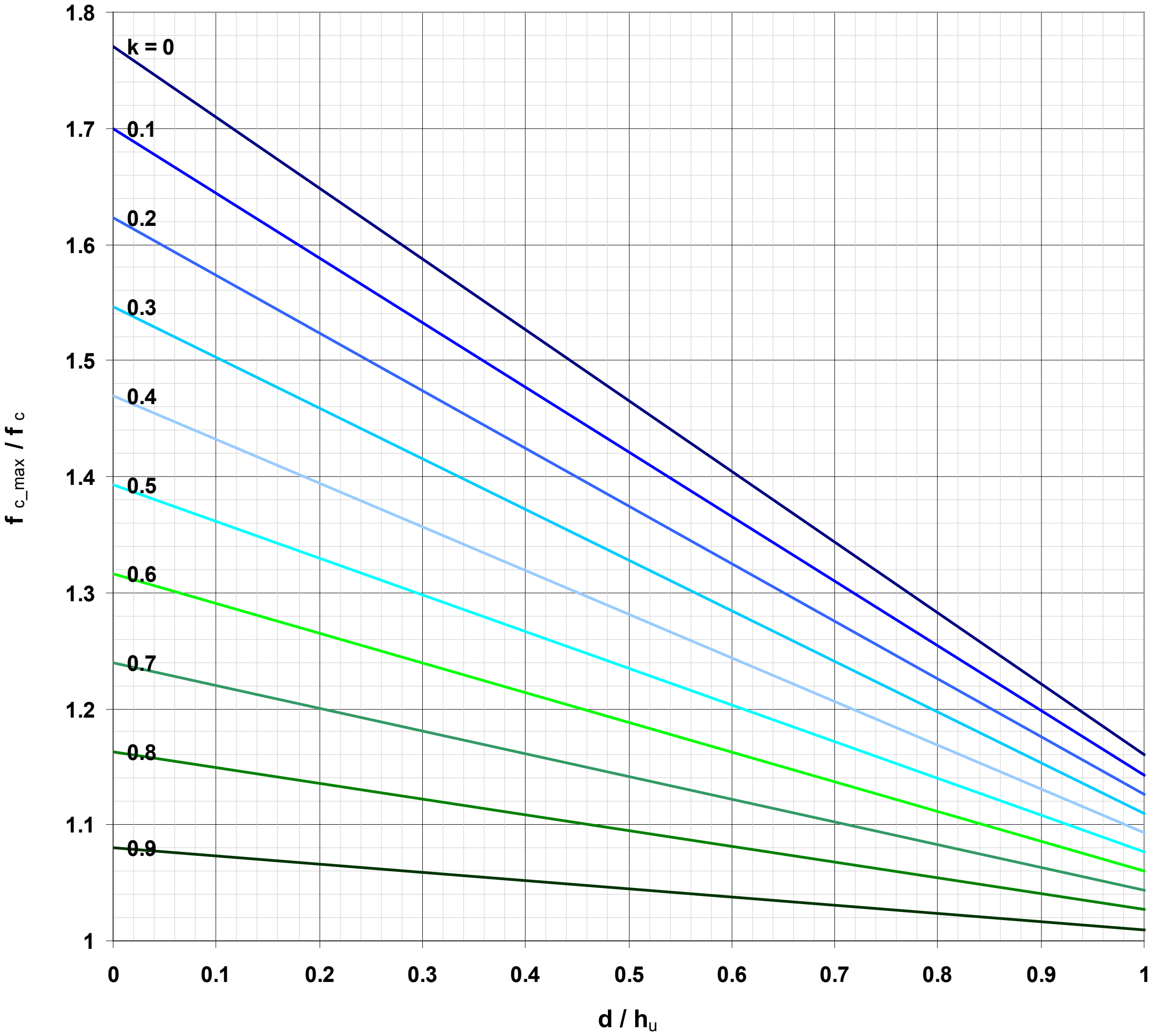
Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Figure C11.21, page C11.27

NACA TN-2661 *A Summary of Diagonal Tension Part I - Methods of Analysis* Figure 15, page 111

http://naca.central.cranfield.ac.uk/report.php?NID=5043

....Paul Kuhn, James P. Peterson, L. Ross Levin

Figure C11.21 Ratio of Maximum Stress to Average Stress in Stiffener



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http://naca.central.cranfield.ac.uk/reports/1947/naca-tn-1346.pdf

.... S. B. Batdorf, Murry Schildcrout, Manuel Stein

NACA TN-1347 *Shear And Longitudinal Direct Stress For Long Plates With Transverse Curvature*

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.... S. B. Batdorf, Manuel Stein, Murry Schildcrout

NACA TN-1348 *Critical Shear Stress of Curved Rectangular Panels*

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NACA TN-1928 *Critical Combinations of Shear and Direct Axial Stress… Curved Rectangular Panels*

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....Murry Schildcrout and Manuel Stein

NACA WR-L-57 *Effect of Normal Pressure …Critical Compressive and Shear Stress of Curved Sheet*

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....Norman Rafel and Charles W. Sandlin, Jr

Originally issued as NACA ARR-L5B10

ARR = Advanced Restricted Report

WR = Wartime Report

Analysis of Stiffened Curved Panels Under Shear and Compression

Journal of the Aeronautical Sciences, February, 1953 ....M. A. Melcon and A. F. Ensrud

Simplified Analysis of General Instability of Stiffened Shells in Pure Bending

Journal of the Aeronautical Sciences, Volume 16, October, 1949............................Francis R. Shanley

Plates

NACA ARR-L6A05

Charts for Critical Combinations of Longitudinal and Transverse Stress for Flat Rectangular Plates

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....Charles Libove and Manuel Stein

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Tables Of Stiffness And Carry-Over Factor For Flat Rectangular Plates Under Compression

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930093351_1993093351.pdf

....Kroll, W D

NACA TN-1817 *Plastic Buckling of Simply Supported Compressed Plates*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930082491_1993082491.pdf

.... Richard A. Pride and George J. Heimerl

ARR = Advanced Restricted Report

Shells

NACA TN-691

Some Elementary Principles of Shell Stress Analysis with Notes on the Use of the Shear Center

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930081541_1993081541.pdf

.... Paul Kuhn

Cylinders

NACA TN-951 *Tests on Thin-Walled Celluloid Cylinders to Determine the Interaction Curves*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930087870_1993087870.pdf

....Elmer F. Bruhn

NACA TN-1219 *Stress Analysis By Recurrence Formula… Circular Cylinders Under Lateral Loads*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930082107_1993082107.pdf

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NACA TN-1343 *Critical Stress Of Thin-Walled Cylinders In Axial Compression*

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.... George Gerard and Herbert Becker

NACA TR-887 *Critical Stress of Thin-Walled Cylinders in Axial Compression*

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Spanwise Lift Distribution

NACA TN-948 *A Simple Approximation Method for Obtaining the Spanwise Lift Distribution*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094469_1993094469.pdf .... O. Schrenk

Buckling

NACA TN-534 *Principal Effects of Axial Load on Moment-Distribution Analysis of Rigid Structures*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930081286_1993081286.pdf

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NACA TN-733 *Experimental Study of Torsional Column Failure*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930081491_1993081491.pdf

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NACA TN-902 *Description of Stress-Strain Curves of Three Parameters*

http://www.apesolutions.com/spd/public/NACA-TN902.pdf

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NACA TN-2994 *Column Strength of H–Sections and Square Tubes in Postbuckling Range of Component Plates*

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NACA TN-3781 *Handbook of Structural Stability Part I - Buckling of Flat Plates*

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.... George Gerard and Herbert Becker

NACA TN-3782 *Handbook of Structural Stability Part II - Buckling of Composite Elements*

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....Herbert Becker

NACA TN-3783 *Handbook of Structural Stability Part III - Buckling of Curved Plates and Shells*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930084510_1993084510.pdf

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NACA TN-3784 *Handbook of Structural Stability Part IV - Failure Of Plates and Composite Elements*

http://ntrs.nasa.gov/search.jsp?R=209695&id=8&qs=N%3D4294800010

....George Gerard

NACA TN-3785 *Handbook of Structural Stability Part V - Comp. Strength of Flat Stiffened Panels*

http://ntrs.nasa.gov/search.jsp?R=573939&id=6&qs=N%3D4294800010 ....George Gerard

NACA TN-3786 *Handbook of Structural Stability Part VI -Strength of Stiffened Curved Plates and Shells*

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Non-Dimensional Buckling Curves

Journal of the Aeronautical Sciences, October 1946 ....Cozzone & Melcon, Jr.

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NACA TN-530 *Bending Stresses Due to Torsion in Cantilever Box Beams*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930081277_1993081277.pdf

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NACA TN-1066 *Stresses Around Large Cut-Outs in Torsion Boxes*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930081723_1993081723.pdf

....Paul Kuhn, Edwin M. Moggio

NACA TN-1361 *Deformation Analysis of Wing Structures*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930081987_1993081987.pdf .... Paul Kuhn

Doublers

Holes with Reinforced Doublers

Aero Digest, August 1955, pages 24 -29 .... Paul K. M. King

Lightening Holes

NACA-WR-L-323 *The Strength and Stiffness of Shear Webs With Round Lightening Holes Having 45 Degree Flanges*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930093634_1993093634.pdf

.... Paul Kuhn

NACA-WR-L-402 *The Strength and Stiffness of Shear Webs With and Without Lightening Holes*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930093371_1993093371.pdf

.... Paul Kuhn

Technical Memorandums

NACA TM-490 *Structures of Thin Sheet Metal, Their Design and Construction*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930090854_1993090854.pdf . Herbert Wagner

NACA TM-604 *Flat Sheet Metal Girders With Very Thin Metal Web. Part I : General Theories and Assumptions*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094812_1993094812.pdf

.... Herbert Wagner

NACA TM-605 *The Flat Sheet Metal Girders With Very Thin Metal Web. Part II : Sheet Metal Girders With Spars Resistant To Bending - Oblique Uprights - Stiffness*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094811_1993094811.pdf

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NACA TM-606 *Flat Sheet Metal Girders With Very Thin Metal Web. Part III : Sheet Metal Girders With Spars Resistant To Bending - The Stress In Uprights - Diagonal Tension Fields*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094810_1993094810.pdf

.... Herbert Wagner

NACA TM-774 *The Tension Fields In Originally Curved, Thin Sheets During Shearing Stresses*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094643_1993094643.pdf

.... H. Wagner and W. Ballerstedt

NACA TM-784 *Torsion and Buckling of Open Sections*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094633_1993094633.pdf

.... H. Wagner and W. Pretschner

NACA TM-809 *Tests for the Determination of the Stress Condition In Tension Fields*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094606_1993094606.pdf

.... R. Lahde and H. Wagner

NACA TM-838 *The Strength of Shell Bodies – Theory and Practice*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094578_1993094578.pdf

.... H. Ebner

NACA TM-870 *Behavior of a Plate Strip Under Shear and Compressive Stresses Beyond the Buckling Limit*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094546_1993094546.pdf

.... A. Kromm and K. Marguerre

NACA TM-965 *Rectangular Shell Plating Under Uniformly Distributed Hydrostatic Pressure*

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930094451_1993094451.pdf

.... M. Neubert and A. Sommer

Lug Analysis

Product Engineering

Analysis of Lugs and Shear Pins Made of Aluminum and Steel Alloys

F.P.Cozzone, M.A. Melcon and F.M.Hoblit

Product Engineering, Volume 21, Number 5, pages 113-117, May 1950

Developments in the Analysis of Lugs and Shear Pins

M.A. Melcon and F.M.Hoblit

Product Engineering, Volume 24, Number 6, pages 160-170, June 1953

Efficiency Factor, Shear-Bearing

Stress Analysis Manual AFFDL-TR-69-42

Air Force Flight Dynamics Laboratory

Air Force Systems Command, Wright-Patterson Air Force Base

Figure 9-3, page 9-5

Efficiency Factor, Tension

Product Engineering, Volume 21, May, 1950 Cozzone, Melcon and Hoblit

Analysis of Lugs and Shear Pins Made of Aluminum and Steel Alloys

Elmer F. Bruhn   *Analysis and Design of Flight Vehicle Structures*

Figure D1.12, page D1.7

Efficiency Factor, Transverse Loads

Elmer F. Bruhn   *Analysis and Design of Flight Vehicle Structures*

Figure D1.15, page D1.8

Average Area

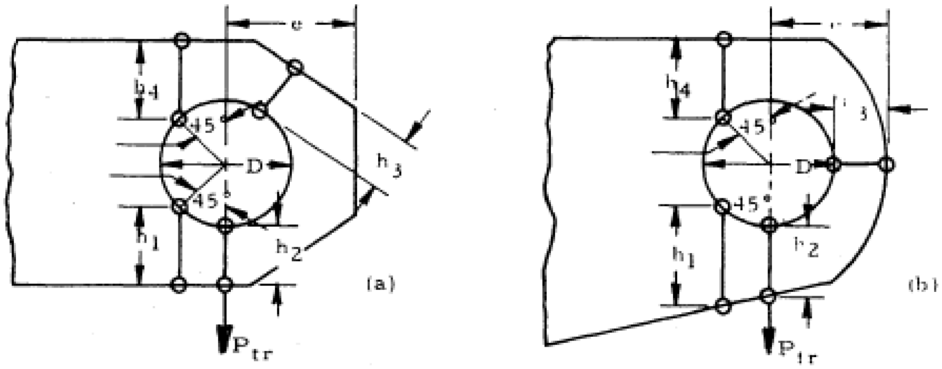

Figure 9-7. Schematic of Lugs Under Transverse Loads

$$A_{avg} = \frac{6}{\frac{3}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}}$$

Stress Analysis Manual AFFDL-TR-69-42

Air Force Flight Dynamics Laboratory

Air Force Systems Command, Wright-Patterson Air Force Base

Figure 9-7, page 9-20

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures*

Sketch on Figure D1.15, page D1.8 and equation on page D1.11

Interaction of Axial and Transverse Load

Stress Analysis Manual AFFDL-TR-69-42

Air Force Flight Dynamics Laboratory

Air Force Systems Command

Wright-Patterson Air Force Base

Figure 9-12, page 9-25

Military Handbooks

MIL-HDBK

MIL-HDBK-5J	*Metallic Materials and Elements for Aerospace Vehicle Structures*
MIL-HDBK-17-1F	*Composite Materials Handbook* *Volume 1. Polymer Matrix Composites Guidelines for Characterization*
MIL-HDBK-17-2E	*Composite Materials Handbook,* *Volume 2. Polymer Matrix Composites Materials Properties*
MIL-HDBK-17-3F	*Composite Materials Handbook* *Volume 3. Polymer Matrix Composites Materials Usage, Design and Analysis*
MIL-HDBK-17-4	*Composite Materials Handbook* *Volume 4. Metal Matrix Composites*
MIL-HDBK-17-5	*Composite Materials Handbook* *Volume 5. Ceramic Matrix Composites*
MIL-HDBK-23A	*Structural Sandwich Composites*
MIL-HDBK-700A	*Plastics*
MIL-HDBK-754	*Plastic Matrix Composite with Continuous Fiber Reinforcement*

MIL-HDBK-17 Online

http://euler9.tripod.com/materials/mil17v1.html

MMPDS-04

Metallic Materials Properties Development and Standardization (MMPDS)

http://store.ihs.com/specsstore/controller;jsessionid=+ip-IzSUbqB8JBFbIK-qgg**.app2?event=DOCUMENT_DETAILS&docId=HLAAHCAAAAAAAAAA

Army-Navy-Civil Committee on Aircraft Requirements (ANC)

ANC-5	*Strength of Aircraft Elements*
ANC-18	*Design of Wood Aircraft Structures*
ANC-19	*Wood Aircraft Inspection and Fabrication*

http://www.eflightmanuals.com/search/searchResults.asp

MMPDS and MIL-HDBK-5

Online Information Resource

http://www.hdbk-5.battelle.org/

Public Version

http://www.hdbk-5.battelle.org/Default2.htm

MMPDS

http://www.mmpds.org/

MIL-HDBK-5J

http://assist.daps.dla.mil/quicksearch/basic_profile.cfm?ident_number=53876

Background

MIL-HDBK-5 has been the primary source of statistically based, design allowables for metallic materials and fastened joints used in the design of aerospace vehicle structures in the United States for over 50 years. In 2001/2002, the US Air Force transitioned custodianship of the handbook to the FAA. The FAA, in conjunction with Battelle Memorial Labs and an Industrial Steering Group (ISG), established the Metallic Materials Properties Development and Standardization (MMPDS) handbook as the ultimate replacement for MIL-HDBK-5.

History

http://www.hdbk-5.battelle.org/history.pdf

NASGRO

Fracture Mechanics and Fatigue Crack Growth Analysis Software

http://www.nasgro.swri.org/

Southwest Research Institute® (SwRI®)

http://www.swri.org/

Address

SwRI Main Office
6220 Culebra Road
P.O. Drawer 28510
San Antonio, Texas 78228-0510

Fatigue Analysis

NAVWEPS 00-25-559 *Tips on Fatigue* Clarence R. Smith
Department of the Navy Prepared for the Bureau of Naval Weapons

ASTM International – Special Technical Publications

STP 9	*References on Fatigue*
STP 91	*Manual on Fatigue Testing*
STP 203	*Fatigue on Aircraft Structures*
STP 237	*Symposium on Basic Mechanism of Fatigue*
STP 274	*Symposium on Fatigue of Aircraft Structures*
STP 284	*Symposium on Acoustical Fatigue*
STP 338	*Symposium on Fatigue Tests of Aircraft Structures: Low-cycle, Full-scale, and Helicopters*

Note: ASTM International was originally called the American Society for Testing and Materials.

http://www.astm.org/COMMIT/filtrexx40.cgi?-P+COMMIT+E08+commitpubs_stp.frm

American Society of Mechanical Engineers – ASME

Conference (International) on Fatigue of Metals – Proceedings

ASM International (American Society for Metals)

Metals Handbook

Society for Experimental Mechanics – SEM

Handbook of Experimental Stress .... Edited by M. Hetenyi

Note: "The Society for Experimental Mechanics, originally called The Society for Experimental Stress Analysis, was founded in 1943 as a nonprofit scientific and educational organization …" (Wikipedia)

Miscellaneous

Fatigue of Metals	P.G. Forrest
Fatigue in Aircraft Structure	A.M. Freudenthal
The Fatigue of Metals and Structures	Gordon Grover & Jackson
Metallic Fatigue With Particular Reference to Significance of Certain Standard Aircraft Fabrication and Finishing Process	W.J. Harris
Designing by Photoelasticity	R.B. Heywood
Designing Against Fatigue of Metals	R.B. Heywood
Stress Concentration Design Factors	R.E. Peterson
Full-Scale Fatigue Testing of Aircraft Structures	R.J. Plantema & Schijve (Editors)
Metal Fatigue	George Sines and J.L. Waisman
Fatigue Testing and Analysis of Results	W. Weibull

AFGRO

Crack Growth Life Prediction Program

http://afgrow.wpafb.af.mil/

AFGRO History

http://afgrow.wpafb.af.mil/about/history.php

Download

http://afgrow.wpafb.af.mil/downloads/afgrow/pdownload.php

CASI – Center for Aerospace Information

http://www.sti.nasa.gov/RECONselect.html

AIAA Publications and Papers

http://www.aiaa.org/content.cfm?pageid=2

Public Domain Aeronautical Software

http://www.pdas.com/index.htm

Properties Of The U.S. Standard Atmosphere

http://www.pdas.com/atmos.htm

Atmosphere Table – Sample

http://www.pdas.com/e2.htm

COSPAR International Reference Atmosphere

http://badc.nerc.ac.uk/data/cira/

Mathcad Files

http://www.mathcad.com/library/mathcad_files.asp

MacNeal-Schwendler Documentation

http://www.mscsoftware.com/support/documentation/

Numerical Recipes

http://www.library.cornell.edu/nr/

MIT OpenCourseWare

http://ocw.mit.edu/index.html

Eng-Tips Forums

http://www.eng-tips.com/

Time

http://tycho.usno.navy.mil/cgi-bin/timer.pl

NASA Links

http://mscweb.gsfc.nasa.gov/543web/designref.html

Magazines

Aviation Week

http://www.aviationnow.com/avnow/

Scientific American

http://www.sciam.com/

NASA Tech Briefs

http://www.nasatech.com/

Internet

Elmer F. Bruhn

https://engineering.purdue.edu/AAE/History

http://aae.www.ecn.purdue.edu/AAE/History/HistPics/HistL.html

The Atanasoff-Berry Computer

The final product was the size of a desk, weighed 700 pounds, had over 300 vacuum tubes, and contained a mile of wire. It could calculate about one operation every 15 seconds…

http://inventors.about.com/library/weekly/aa050898.htm?once=true&

http://www.scl.ameslab.gov/ABC/Progress.html

John Vincent Atanasoff

(1903 - 1995)

The obsession of finding a solution to the computer problem had built to a frenzy in the winter months of 1937. One night, frustrated after many discouraging events, he got into his car and started driving without destination. Two hundred miles later, he pulled onto a roadhouse in the state of Illinois. Here, he had a drink of bourbon and continued thinking about the creation of the machine. No longer nervous and tense, he realized that this thoughts were coming together clearly. He began generating ideas on how to build this computer! After receiving a grant of $650 from Iowa State College in March 1939, Atanasoff was ready to embark in this exciting adventure. To help him accomplish his goal, he hired a particularly bright electrical engineering student, Clifford E. Berry. From 1939 until 1941 they worked at developing and improving the ABC, Atanasoff-Berry Computer, as it was later named. When World War II started on 7 December 1941, the work on the computer came to a halt. Although Iowa State College had hired a Chicago patent lawyer, Richard R. Trexler, the patenting of the ABC was never completed.

http://www.scl.ameslab.gov/ABC/Biographies.html

The idea of building an electronic digital computer came to him while he was sitting in a tavern. Dr. Atanasoff came up with four principles for his electronic digital computer.

> *He would use electricity and electronics as the medium for the computer.*
>
> *In spite of custom, he would use base-two numbers (the binary system) for his computer.*
>
> *He would use condensers for memory and would use a regenerative or "jogging" process to avoid lapses that might be caused by leakage of power.*
>
> *He would compute by direct logical action and not by enumeration as used in analog calculating devices.*

http://ei.cs.vt.edu/~history/do_Atanasoff.html

Clifford E. Berry

http://www.scl.ameslab.gov/ABC/Biographies.html

Stephen Prokofyevich Timoshenko

http://en.wikipedia.org/wiki/Stephen_Timoshenko

Hardy Cross

http://en.wikipedia.org/wiki/Hardy_Cross

http://www.nexusjournal.com/Eaton.html#anchor316605

Francis R. Shanley

http://content.cdlib.org/xtf/view?docId=hb229003hz&doc.view=frames&chunk.id=div00008&toc.depth=1&toc.idd=

Joseph Raphson

http://numericalmethods.eng.usf.edu/anecdotes/raphson.html

Pafnuty Lvovich Chebyshev

http://www-history.mcs.st-andrews.ac.uk/Biographies/Chebyshev.html

http://www-history.mcs.st-and.ac.uk/Extras/Chebyshev_nets.html

http://en.wikipedia.org/wiki/Chebyshev

André-Louis Cholesky

http://en.wikipedia.org/wiki/Andr%C3%A9-Louis_Cholesky

Jean Baptiste Joseph Fourier

http://en.wikipedia.org/wiki/Joseph_Fourier

Richard von Mises

In aerodynamics, Richard von Mises made notable advances in boundary-layer-flow theory and airfoil design. He developed the distortion energy theory of stress, which is one of the most important concepts used by engineers in material strength calculations. On probability, he posed the well-known birthday problem.

http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Mises.html

http://en.wikipedia.org/wiki/Richard_von_mises

Birthday Problem

http://en.wikipedia.org/wiki/Birthday_problem

http://www.damninteresting.com/?p=402

von Mises Yield Criterion

http://en.wikipedia.org/wiki/Von_Mises_stress

Ludwig von Mises

http://en.wikipedia.org/wiki/Ludwig_von_Mises

The Cathedral and the Bazaar

http://www.catb.org/~esr/writings/cathedral-bazaar/

Famous Curves Index

http://www-gap.dcs.st-and.ac.uk/~history/Curves/Curves.html

Chebyshev Linkage

The Chebyshev linkage is a mechanical linkage that converts rotational motion to approximate straight-line motion.

http://en.wikipedia.org/wiki/Chebyshev_linkage

Peaucellier-Lipkin Linkage

The Peaucellier-Lipkin linkage (or Peaucellier-Lipkin cell), invented in 1864, was the first linkage capable of transforming rotary motion into perfect straight-line motion, and vice versa.

http://en.wikipedia.org/wiki/Peaucellier-Lipkin_linkage

Regulations.gov

http://www.regulations.gov/

5.0 Symbols and Abbreviations

General

A	Area
A_{ij}	Extensional Stiffness Matrix (Composites)
$[\ A\ ]$	Extensional Stiffness Matrix (Composites)
a	Dimension
α	Angle
α	α = a ratio obtained from Bleich’s solution (Curved Beams)
B_{ij}	Coupling Stiffness Matrix (Composites)
$[\ B\ ]$	Coupling Stiffness Matrix (Composites)
BL	Butt Line
b	Dimension (e.g. width)
β	Angle
β	β = a ratio obtained from Bleich’s solution (Curved Beams)
C_{ij}	Constants for Stress-Strain Relationship (Composites) $\sigma_i = C_{ij}\ \varepsilon_j$
c	Fixity Coefficient for Effective Length (Buckling)
c	Distance from Neutral Axis to Extreme Fiber
D	Diameter
D	Bending/Flexural Rigidity (Plates)
D_{ij}	Bending Stiffness Matrix (Composites)
$[\ D\ ]$	Bending Stiffness Matrix (Composites)
d	Dimension (e.g. depth)
δ	Deflection
∇^2	Differential Operator
Δ	Allowable Shear Correction, Curved Webs
Δ	Laplacian Differential Operator $\Delta = \nabla^2$
$\Delta\ \Delta$	Biharmonic Differential Operator $\Delta\ \Delta = \nabla^2\ \nabla^2 = \nabla^4$
E	Modulus of Elasticity (Young’s Modulus)

E_c	Modulus of Elasticity, Compression
E_L	Modulus of Elasticity, Longitudinal (Composites)
E_T	Modulus of Elasticity, Transverse (Composites)
e	Dimension (e.g. Distance from center of hole to an edge as in e/D)
ε	Strain or Tensor Shear Strain $\gamma = \varepsilon_{xy} + \varepsilon_{yx} = 2\,\varepsilon_{xy}$ (Half of the engineering shear strain, γ)
F_{bru}	Ultimate Bearing Allowable
F_{bry}	Yield Bearing Allowable
F_{cc}	Crippling Allowable
F_{cy}	Compressive Yield Strength
F_{ir}	Inter-Rivet Buckling Stress
F_{su}	Ultimate Shear Allowable
F_{tu}	Ultimate Tensile Strength
F_{ty}	Tensile Yield Strength
F_{s_cr}	Critical Shear Buckling Stress
$F_{0.7}$	Secant Yield Stress (Intersection of 0.70 E)
$F_{0.85}$	Secant Yield Stress (Intersection of 0.85 E)
f_b	Bending Stress
f_s	Shear Stress
G	Shear Modulus
G_s	Secant Shear Modulus
γ	Engineering Shear Strain $\gamma = \varepsilon_{xy} + \varepsilon_{yx} = 2\,\varepsilon_{xy}$ (Twice the tensor shear strain, ε)
h	Dimension (e.g. height)
I	Moment of Inertia
I_x	Moment of Inertia about x-axis
I_y	Moment of Inertia about y-axis
I_{xy}	Product of Inertia
K_t	Tension Efficiency Factor of Lug
k_s	Shear Buckling Coefficient
κ	Curvature of a Two Dimensional Curve
L	Length

L	Longitudinal Grain Direction
LT	Long Transverse Grain Direction
L’	Effective Length
λ	$\lambda = 1 - \nu^2$
M	Moment
M_{max}	Maximum Bending Moment
m	Slope
μ	Poisson’s Ratio (υ is more common)
M.S.	Margin of Safety
N	Running Load (lb/in)
NACA	National Advisory Committee for Aeronautics
NASA	National Aeronautics and Space Administration
n	Ramberg-Osgood Shape Parameter
n_i	Initial Ramberg-Osgood Shape Factor
η	Inelastic Reduction Factor
P	Load
P_s	Shear Load
P_t	Tensile Load
P_{allow}	Allowable Load
P_{ax}	Axial Load
P_{tr}	Transverse Load
p_z	Pressure Loading
Q	First Area Moment
Q_{ij}	Reduced Stiffness Matrix for a Plane Stress State (Composites)
$\overline{Q}_{ij}$	Transformed Reduced Stiffness Matrix for a Plane Stress State (Composites)
R	Reaction
R_{ax}	Axial Stress Ratio
R_{tr}	Transverse Stress Ratio
R_s	Stress Ratio, Shear

R_t	Stress Ratio, Tension
ρ	Radius of Curvature
ρ	Radius of Gyration
S_{ij}	Constants for Strain-Stress Relationship (Composites) $\varepsilon_i = S_{ij}\ \sigma_j$
ST	Short Transverse Grain Direction
STA	Fuselage Station
σ_{cr}	Critical Stress
τ_{cr}	Critical Shear Stress
$[T]$	Transformation Matrix (Composites)
$[T]^{-1}$	Inverse of the Transformation Matrix (Composites)
t	Thickness
V	Shear
υ	Poisson’s Ratio (sometimes μ is used)
WL	Water Line
w	Deflection of a Plate
w	Width
Z	Section Property for Curved Beams $Z = -1 + \frac{a}{A}\left[b \ln\left(\frac{a_o}{a_i}\right) - \left(b - t_w\right) \ln\left(\frac{a_o - t}{a_i + t}\right)\right]$

Composites

Extensional Stiffness Matrix

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k - h_{k-1}\right)$$

Coupling Stiffness Matrix

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h^2{}_k - h^2{}_{k-1}\right)$$

Bending Stiffness Matrix

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h^3{}_k - h^3{}_{k-1}\right)$$

Stiffness Matrix

C_{ij} Constants for Stress-Strain Relationship $\sigma_i = C_{ij}\ \varepsilon_j$

$C_{ij} = C_{ji}$ 36 Independent Constants become 21 Independent Constants

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

Compliance Matrix

S_{ij} Constants for Strain-Stress Relationship $\varepsilon_i = C_{ij}\ \sigma_j$

$S_{ij} = S_{ji}$ 36 Independent Constants become 21 Independent Constants

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

Transformation Matrix

$$[\,T\,] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Inverse of the Transformation Matrix

$$[\,T\,]^{-1} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Reduced Stiffness Matrix

Q_{ij} Reduced Stiffness Matrix for a Plane Stress State

$$Q_{ij} = C_{ij} - \frac{C_{i3}\ C_{j3}}{C_{33}}$$

Where $\sigma_3 = 0 \qquad \tau_{23} = 0 \qquad \tau_{31} = 0$

$$Q_{11} = \frac{S_{22}}{S_{11}\ S_{22} - S_{12}^{\ 2}} = \frac{E_L}{1 - \upsilon_{LT}\ \upsilon_{TL}}$$

$$Q_{22} = \frac{S_{11}}{S_{11}\ S_{22} - S_{12}^{\ 2}} = \frac{E_T}{1 - \upsilon_{LT}\ \upsilon_{TL}}$$

$$Q_{12} = \frac{S_{12}}{S_{11}\ S_{22} - S_{12}^{\ 2}} = \frac{\upsilon_{LT}\ E_T}{1 - \upsilon_{LT}\ \upsilon_{TL}}$$

$$Q_{66} = \frac{1}{S_{66}} = \frac{E}{2\ (1 + \upsilon)} = G_{LT}$$

Transformed Reduced Stiffness Matrix

$\overline{Q}_{ij}$ Transformed Reduced Stiffness Matrix

$$\overline{Q}_{11} = Q_{11} \cos^4\theta + Q_{22} \sin^4\theta + 2\left(Q_{12} + 2\,Q_{66} \right)\sin^2\theta \cos^2\theta$$

$$\overline{Q}_{22} = Q_{11} \sin^4\theta + Q_{22} \cos^4\theta + 2\left(Q_{12} + 2\,Q_{66} \right)\sin^2\theta \cos^2\theta$$

$$\overline{Q}_{12} = \left(Q_{11} + Q_{22} - 4\,Q_{66} \right)\sin^2\theta \cos^2\theta + Q_{12}\left(\sin^4\theta + \cos^4\theta \right)$$

$$\overline{Q}_{66} = \left(Q_{11} + Q_{22} - 2\,Q_{12} - 2\,Q_{66} \right)\sin^2\theta \cos^2\theta + Q_{66}\left(\sin^4\theta + \cos^4\theta \right)$$

$$\overline{Q}_{16} = \left(Q_{11} - Q_{12} - 2\,Q_{66} \right)\cos^3\theta \sin\theta - \left(Q_{22} - Q_{12} - 2\,Q_{66} \right)\cos\theta \sin^3\theta$$

$$\overline{Q}_{26} = \left(Q_{11} - Q_{12} - 2\,Q_{66} \right)\cos\theta \sin^3\theta - \left(Q_{22} - Q_{12} - 2\,Q_{66} \right)\cos^3\theta \sin\theta$$

NACA Symbols – Diagonal Tension

A	Cross Sectional Area (square inches)
E	Young's Modulus (ksi)
G	Shear Modulus (ksi)
G_e	Effective Shear Modulus (ksi) Note: Includes effects of diagonal tension and plasticity.
H	Force in Flange Beam due to Horizontal Component of Diagonal Tension
I	Moment of Inertia (in^4)
J	Torsion Constant (in^4)
L	Length of Beam (inches)
L_e	Effective Column Length of Upright (inches)
P	Force (kips)
P_U	Internal Force in Upright (kips)
Q	First Area Moment about Neutral Axis of Parts of Cross-Section as Specified by Subscripts (in^3)
R	Total Shear Strength (kips in single shear) of all upright–to-web rivets in one upright
R	Coefficient of Edge Restraint
S	Transverse Shear Force (kips)
T	Torque (in-kips)
d	Spacing of Uprights (inches)
e	Distance From Median Plane of Web to Centroid of Single Upright (inches)
f_n	Normal Stress
f_s	Shear Stress
h	Depth of Beam (inches)
k	Diagonal Tension Factor
q	Shear Flow (kips per inch)
t	Thickness (inches) Note: Used without subscript signifies thickness of web.
α	Angle between Neutral Axis of Beam and Direction of Diagonal Tension (degrees)
δ	Deflection of Beam (inches)
ε	Normal Strain
μ	Poisson's Ratio

ρ	Centroidal Radius of Gyration of Cross-Section of Upright about Axis Parallel to Web (inches) (No sheet should be included.)
σ	Normal Stress (ksi)
σ_o	“Basic Allowable” Stress for Forced Crippling of Uprights (ksi) defined by Formula (37) of NACA TN-2661
τ	Shear Stress (ksi)
τ^*_{all}	“Basic Allowable” Value of Web Shear Stress (ksi) given by Figure (19) in NACA TN-2661
ωd	Flange Flexibility Factor defined by expression (19a) in NACA TN-2661

Subscripts

DT	Diagonal Tension
IDT	Incomplete Diagonal Tension
PDT	Pure Diagonal Tension
F	Flange
S	Shear
U	Upright
W	Web
all	Allowable
avg	Average
cr	Critical
cy	Compressive Yield
e	Effective
max	Maximum
ult	Ultimate

Special Combinations

P_u	Internal Force in Upright (kips)	
R''	Shear Force on Rivets per inch of run (kips per inch)	
R_R	Value of R Required by Formula (40) in NACA TN-2661	
R_d	Restraint Coefficients for Shear Buckling of Web	Note: See Equation (32) of NACA TN-2661

R_h	Restraint Coefficients for Shear Buckling of Web Note: See Equation (32) of NACA TN-2661
R_{tot}	Total Shear Strength in Single Shear of All Rivets in One Upright (kips)
d_c	Clear Upright Spacing (inches) Note: Measured as shown in Figure 12(a) of NACA TN-2661.
h_c	Clear Depth of Web (inches) Note: Measured as shown in Figure 12(a) of NACA TN-2661.
h_e	Effective Depth of Beam Measured between Centroid of Flanges (inches)
h_R	Depth of Beam Measured between Centroids of Web-to-Flange Rivet Patterns (inches)
h_U	Length of Upright Measured between Centroids of Upright-to-Flange Rivet Patterns (inches)
k_{ss}	Theoretical Buckling Coefficient for Plates with Simply Supported Edges
F_u	"Basic" Allowable Stress for Forced Crippling of Uprights
ωd	Flange Flexibility Factor defined by expression (19a) in NACA TN-2661

Curved Web Systems Only

R	Radius of Curvature (inches)
Z	Curvature Parameter defined in NACA TN-2661, Figure 30
d	Spacing of Rings (inches)
h	Length of Arc between Stringers (inches)

Subscripts for Curved Web Systems

RG	Ring
ST	Stringer

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Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures*, page C11.14 to C11.15

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.... Paul Kuhn, James P. Peterson, L. Ross Levin

Aerodynamics

A	Wing Aspect Ratio = b^2 / S
α	Angle of Attack
α_{aR}	From Reference Line
α_{w0}	Reference Plane to Plane of Zero Lift
b	Wing Span (inches)
C_D	Wing Drag Coefficients
C_L	Wing Lift Coefficients
C_M	Wing Moment Coefficients
C_t	Dimensionless Tail Force Coefficients
C_{za}	Normal Force Coefficients for the Aircraft
c	Chord
c_d	Drag Coefficient
c_{di}	Section Induced Drag Coefficient
c_{di1}	Induced Drag Coefficient for $C_L = 1.0$
c_l	Section Lift Coefficient
c_{lb}	Basic Lift
c_{la1}	Additional Lift for $C_L = 1.0$
c_n	Normal Force Coefficients
K	Gust Effectiveness Factor
L_t	Distance from Aircraft Center of Gravity to Air Load on Tail
M	Wing Bending Moment
m_0	Slope of Section Lift Coefficients
n	Limit Maneuver Load Factor
q	Dynamic Pressure (lb / ft^2)
ρ	Mass Density of Air ($slugs / ft^3$)
S	Wing Area ($inch^2$)
U	Gust Vertical Velocity
V	Velocity

V	Wing Shear
W	Gross Weight of Aircraft
y	Point Along Span

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