Ellipse Perimeter

The Quest for a Simple, Exact Expression

brought to you by

The Midwest Norwegian-American
Bachelor Farmer Preservation Guild

Ellipse Perimeter

The Quest for a Simple, Exact Expression

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www.AircraftStressAnalysis.com

www.SuperDuperGenius.com

www.Super-Genius.com

www.Sexiest-Man-Alive.us

Mestrius Plutarchus (Plutarch)

"The mind is not a vessel to be filled but a fire to be kindled."

Albert Einstein

“Anyone who has never made a mistake has never tried anything new.”

“Example isn't another way to teach, it is the only way to teach.”

“It is a miracle that curiosity survives formal education.”

References

Numericana

Gérard P. Michon, Ph.D.

http://www.numericana.com/answer/ellipse.htm#elliptic

Paul Bourke

University of Western Australia

http://www.paulbourke.net/geometry/ellipsecirc/

The Math Forum at Drexel

Ask Dr. Math

http://mathforum.org/dr.math/faq/formulas/faq.ellipse.circumference.html

Math is Fun

http://www.mathsisfun.com/geometry/ellipse-perimeter.html

Construction Methods

Auxillary Circles Method

Pin and String Method

Trammel Method

Parallelogram Method

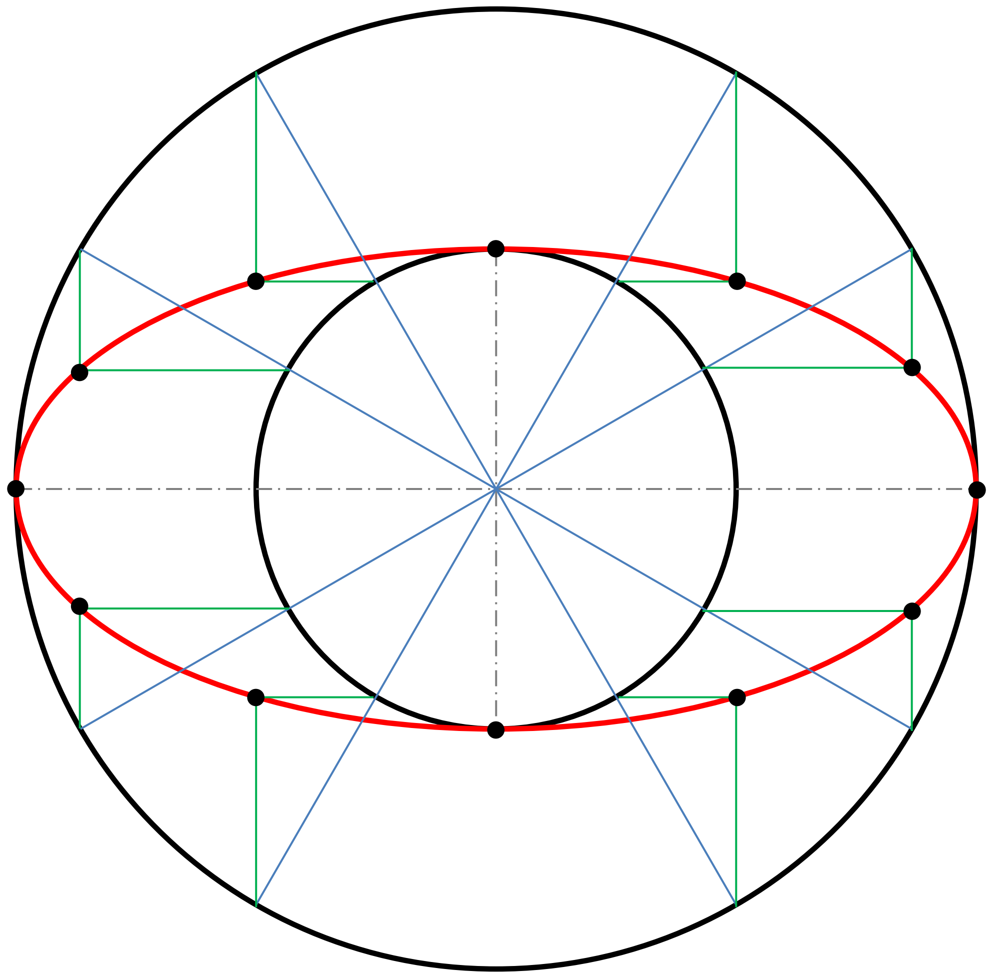
Ellipse in a Rectangle Method

Plane Cutting a Cone Method

Thanks to Anthony Rynne University of Limerick http://www3.ul.ie/~rynnet/swconics/SE.htm

Construction Methods

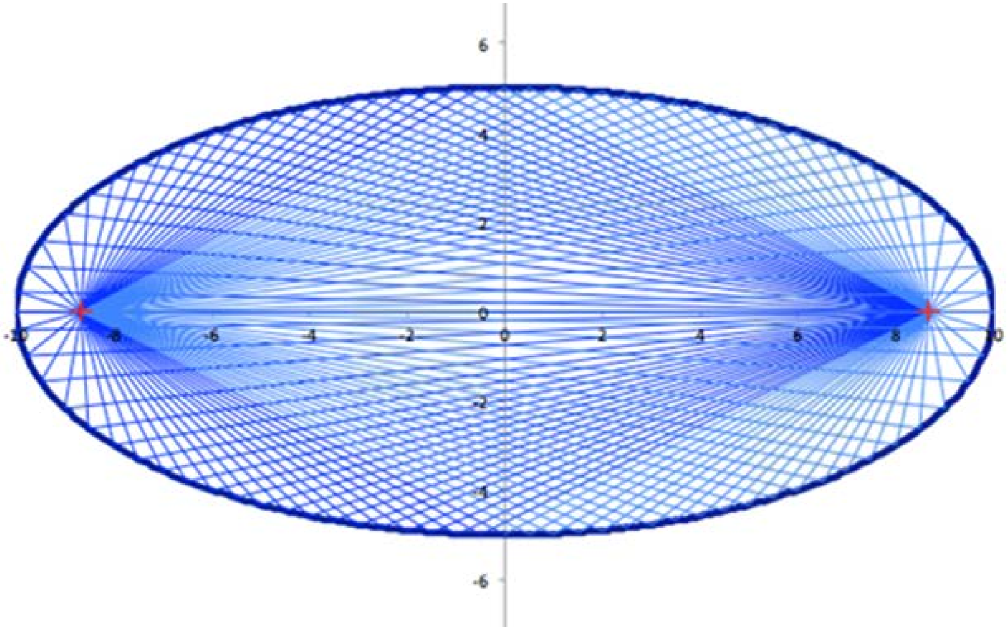
Auxillary Circles Method



Thanks to Anthony Rynne University of Limerick http://www3.ul.ie/~rynnet/swconics/SE.htm

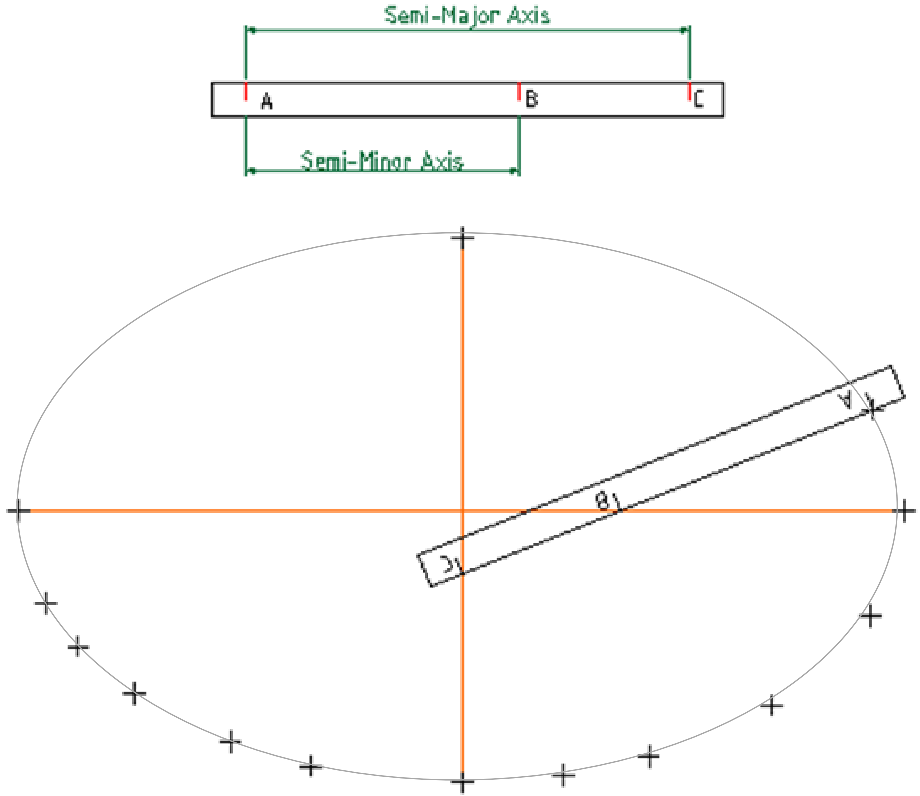
Construction Methods

Pin and String Method



Construction Methods

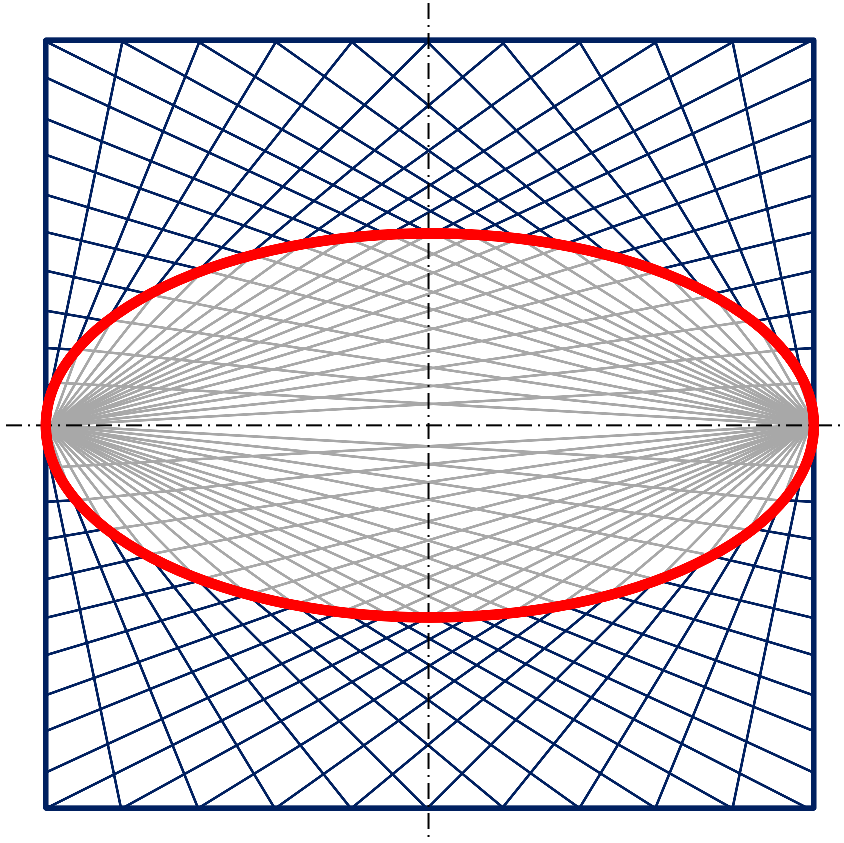
Trammel Method

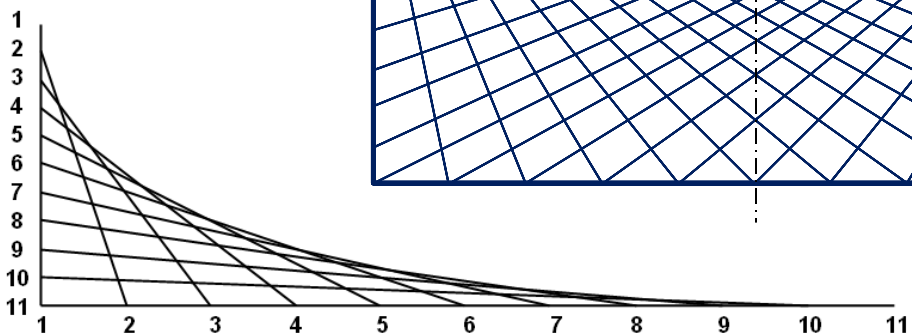


Thanks to Anthony Rynne University of Limerick http://www3.ul.ie/~rynnet/swconics/SE.htm

Construction Methods

Parallelogram Method



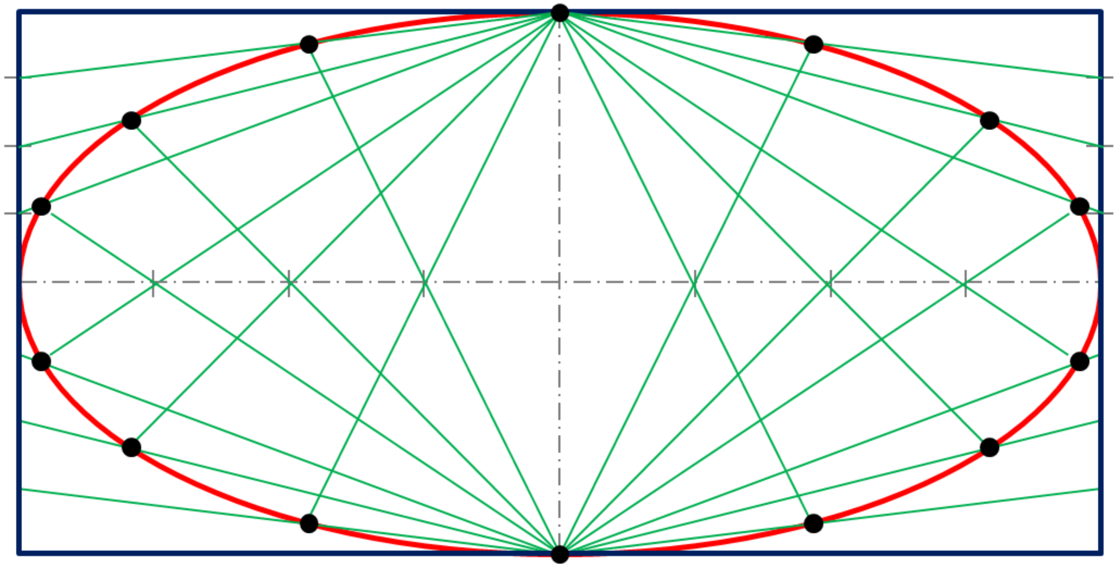


See also P. Grodzinski *Investigations on Shaft Fillets* 1941 Engineering (London), Volume 152, 1941, pp. 321-331

Streamlined Fillets by Grodzinski … *Peterson's Stress Concentration Factors* by Walter D. Pilkey Figure 3.8, page 142

Construction Methods

Ellipse in a Rectangle Method

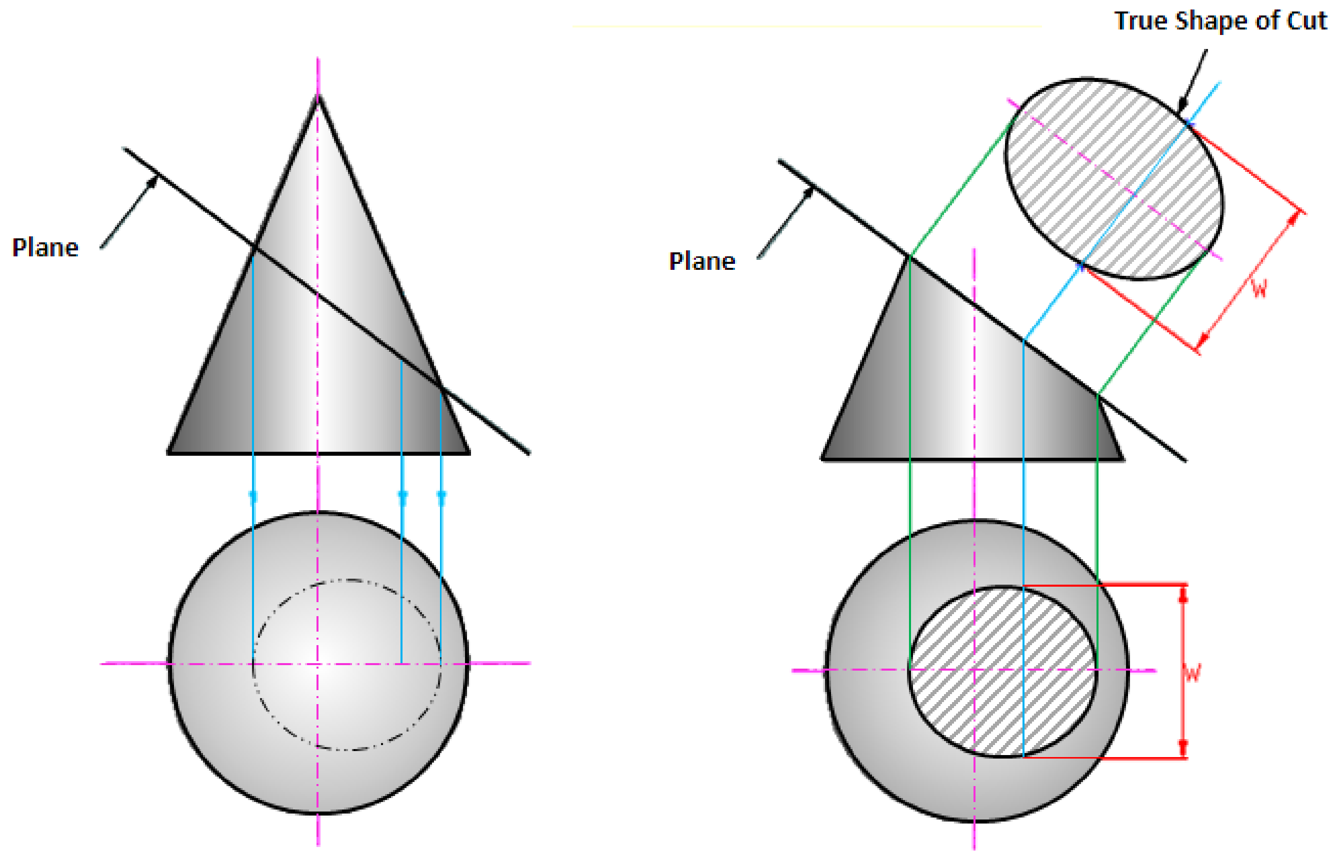


Is this another name for the Parallelogram Method ?

Thanks to Anthony Rynne University of Limerick http://www3.ul.ie/~rynnet/swconics/SE.htm

Construction Methods

Plane Cutting a Cone Method



Thanks to Anthony Rynne University of Limerick http://www3.ul.ie/~rynnet/swconics/SE.htm

Eccentricity

$$\varepsilon = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

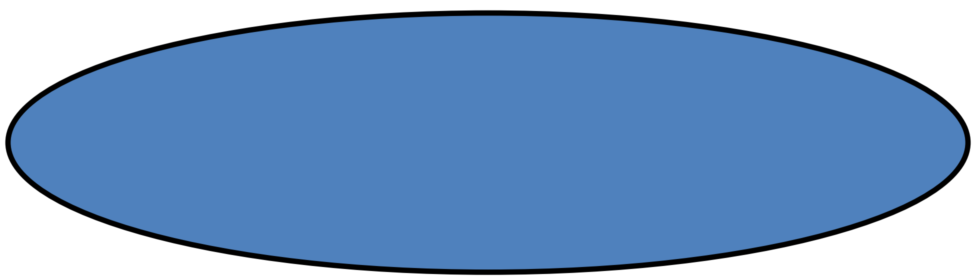
Haley's Comet

a = 17.8 AU   b = 4.8 AU

1 AU = 149,597,870,700 meters

Aspect Ratio, b / a ≈ 0.27

Eccentricity ≈ 0.963



Eccentricity

Colosseum

Approx 189 m x 156 m

Aspect Ratio, b / a = 0.825

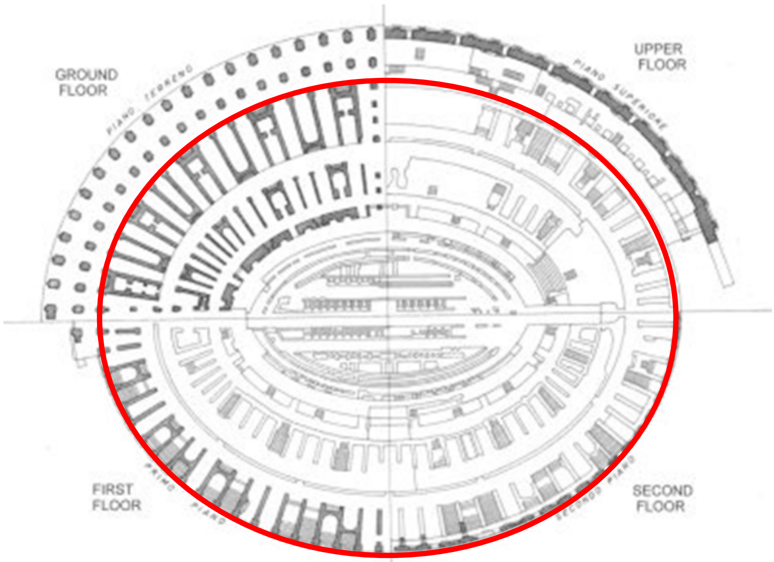
Many believe the Colosseum is actually an ovoid.

Eccentricity ≈ 0.565

The Amphitheatre Construction Problem

http://users.cs.cf.ac.uk/Paul.Rosin/resources/papers/amphitheatre2.pdf

"… the *parallels of an ellipse … are actually eighth order polynomials containing 104 coefficients!* "



Thanks to http://www.the-colosseum.net/architecture/ellipsis.htm

http://www.tribunesandtriumphs.org/colosseum/dimensions-of-the-colosseum.htm

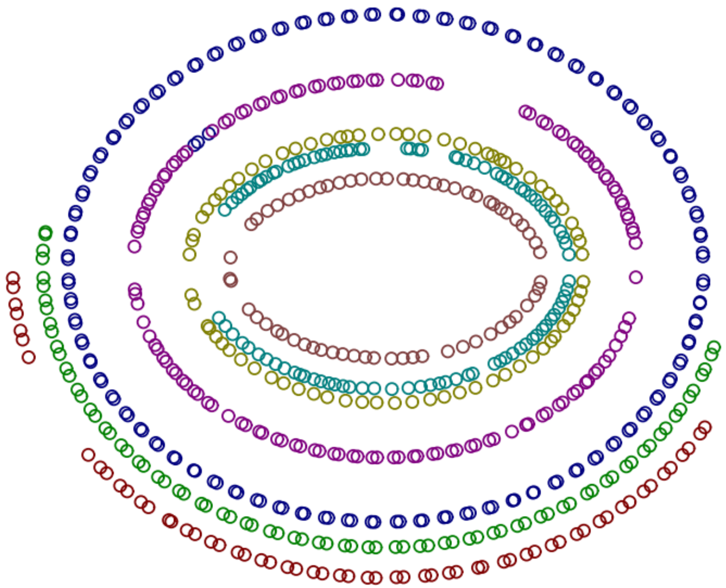
Four – Centered Oval

The Amphitheatre Construction Problem

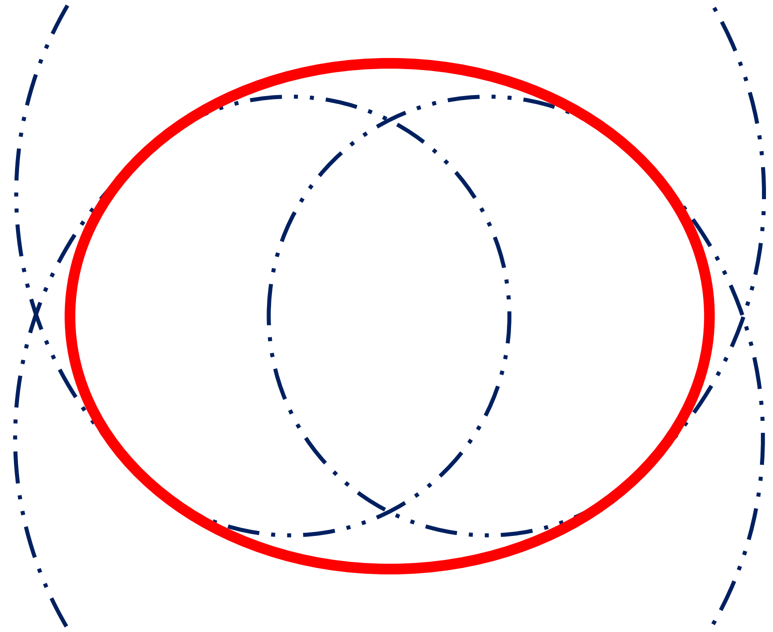
http://users.cs.cf.ac.uk/Paul.Rosin/resources/papers/amphitheatre2.pdf

Paul Rosin "… the *parallels of an ellipse … are actually eighth order polynomials containing 104 coefficients [13] !* "

[13] N.T. Gridgeman *Elliptic Parallels*, The Mathematics Teacher, 63:481-485, 1970.



Measured Data from the Colosseum



Four – Centered Oval

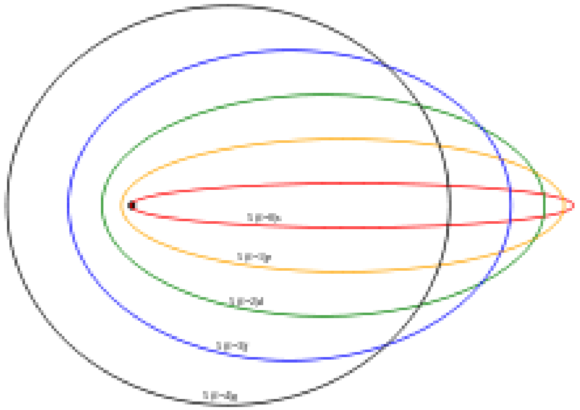
Thanks to http://www.the-colosseum.net/architecture/ellipsis.htm

http://www.tribunesandtriumphs.org/colosseum/dimensions-of-the-colosseum.htm

Applications

Elliptical Orbits

Quantum Theory



Elliptic Orbits with the Same Energy and Quantized Angular Momentum

Several enhancements to the Bohr model were proposed; most notably the Sommerfeld model or Bohr–Sommerfeld model, which suggested that electrons travel in elliptical orbits around a nucleus instead of the Bohr model's circular orbits.

http://en.wikipedia.org/wiki/Bohr_model

Elliptic Curve Cryptography

Elliptic curve cryptography (ECC) is an approach to public-key cryptography based on the algebraic structure of elliptic curves over finite fields. The use of elliptic curves in cryptography was suggested independently by Neal Koblitz[1] and Victor S. Miller[2] in 1985.

1 Koblitz, N. (1987). "Elliptic curve cryptosystems". *Mathematics of Computation* 48 (177): 203–209. JSTOR 2007884.
2 Miller, V. (1985). "Use of elliptic curves in cryptography". *CRYPTO* 85: 417–426. doi:10.1007/3-540-39799-X_31.

Thanks to http://en.wikipedia.org/wiki/Elliptic_curve_cryptography

Applications

Ring Doublers

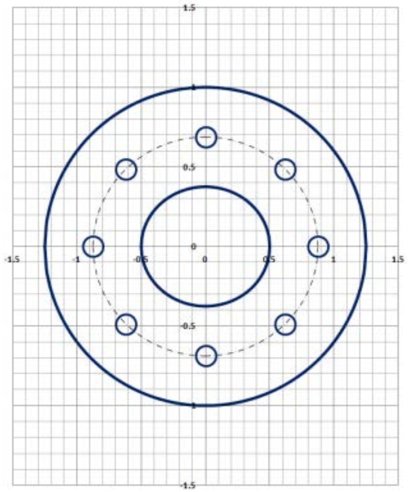
Paul Rosin "… the *parallels of an ellipse … are actually eighth order polynomials containing 104 coefficients !* "

Computer Aided Design allows us to create parallel offsets using splines and calculates the equal spacing for the fasteners.

Rules of Thumb for Structural Design

M. L. Hand's Rule of Thumb

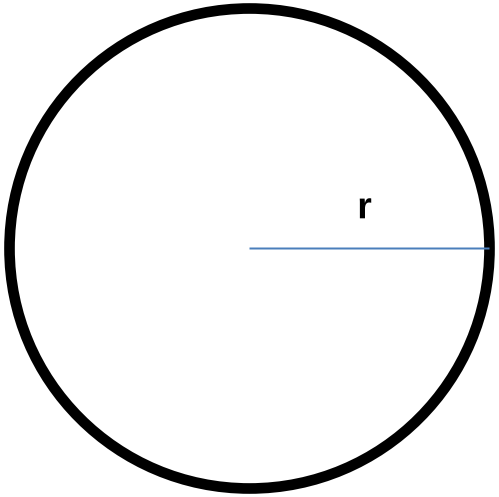
When designing doublers, (reinforcements around openings in a shell) an approximate sizing guide is to replace the removed material. In highly loaded structures or where fatigue is a concern, more reinforcement may be required. I've been told that ***replacing three times the removed material is the norm for commercial aircraft.***



Ring Doubler with Equal Fastener Spacing

Circle

$$(x - a)^2 + (y - b)^2 = r^2$$

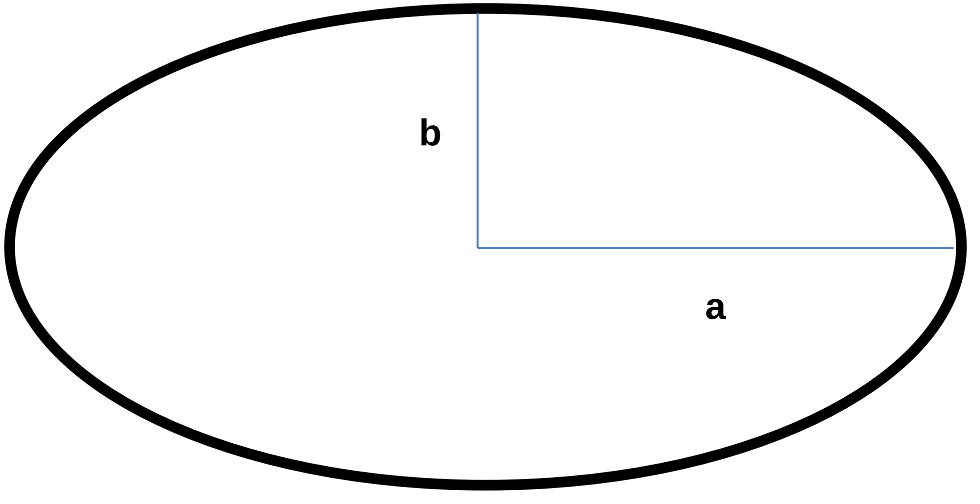


Area $= \pi r^2$

Circumference $= 2 \pi r$

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Area = π a b

Perimeter = ?

Ellipse Perimeter

Perimeter

$$P = 4a \int_0^{\pi/2} \sqrt{1 - \left(\frac{a^2 - b^2}{a^2}\right) \sin^2\theta} \; d\theta$$

or

$$P = 4a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta} \; d\theta$$

Eccentricity

$$\varepsilon = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

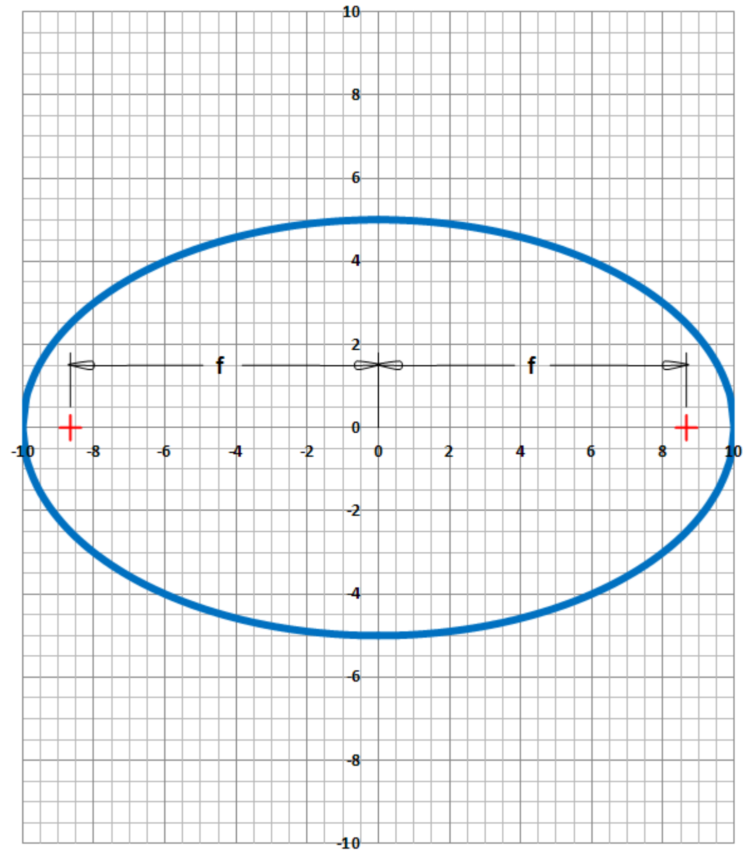
Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

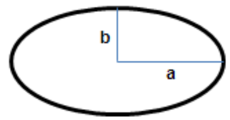
$$f = \sqrt{a^2 - b^2}$$

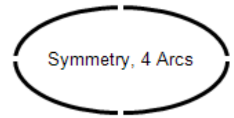
$$h = \frac{(a - b)^2}{(a + b)^2}$$

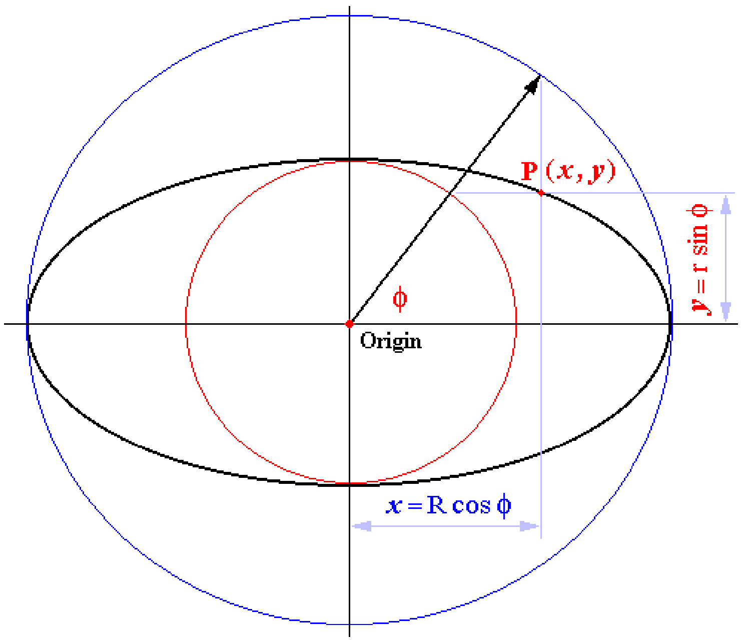
$$\varepsilon = \sqrt{1 - \left(\frac{b}{a}\right)^2} = \frac{f}{a}$$



Derivation of the Arc Length of an Ellipse







Eccentricity, ε

$$\varepsilon = \sqrt{\frac{a^2 - b^2}{a}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

Equation of an Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Parametric Equations

$$x = a \sin\theta \qquad y = b\cos\theta$$

$$\frac{dx}{d\theta} = a\cos\theta \qquad \frac{dy}{d\theta} = -b\sin\theta$$

Circumference

$$C = \int_0^{2\pi} \sqrt{a^2\cos^2\theta + b^2\sin^2\theta}\, d\theta$$

Symmetry

$$C = 4\int_0^{\pi/2} \sqrt{a^2\cos^2\theta + b^2\sin^2\theta}\, d\theta$$

Substitution

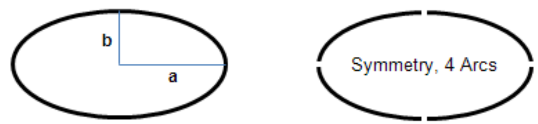
$\sin^2\theta + \cos^2\theta = 1$ Substitute $\cos^2\theta = (1 - \sin^2\theta)$ and eccentricity, ε

$$C = 4a\int_0^{\pi/2} \sqrt{1 - \varepsilon^2\sin^2\theta}\, d\theta = 4\,a\,E(\pi/2, \varepsilon)$$

Thanks to http://www.codeproject.com/Articles/566614/Elliptic-integrals

Solve for One Arc Length and Multiply by Four

Step 1 There are four arcs due to symmetry. Solve for one arc length.



Step 2 Calculate the Aspect Ratio, b / a

Step 3 Calculate the Eccentricity, ε

$$\varepsilon = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

Step 4 Determine the Complete Elliptic Integral of the Second Kind for ε.

Step 5 Arc Length

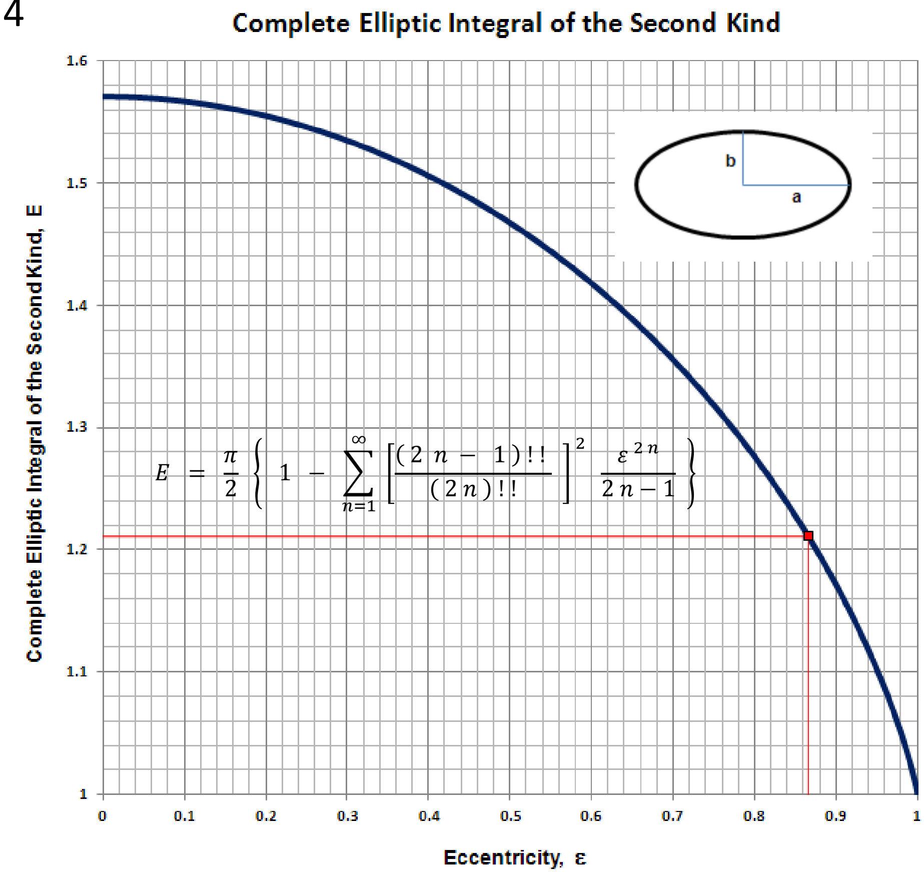
$$Arc\ Length = a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2 \theta}\ d\theta$$

Step 6 Multiply by 4.

$$P = 4\ a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2 \theta}\ d\theta$$

Complete Elliptic Integral of the Second Kind

Step 4



b / a	***ε***	**E** [π/2,ε]
0	1	1
0.05	0.99874922	1.00553186
0.10	0.99498744	1.01611306
0.15	0.98868600	1.03153564
0.20	0.97979590	1.05050268
0.25	0.96824584	1.07230273
0.30	0.95393920	1.09647752
0.35	0.93674970	1.12268489
0.40	0.91651514	1.15065563
0.45	0.89302855	1.18017231
0.50	0.86602540	1.21105603
0.55	0.83516465	1.24315735
0.60	0.8	1.27634994
0.65	0.75993421	1.31052590
0.70	0.71414284	1.34559225
0.75	0.66143783	1.38146826
0.80	0.6	1.41808339
0.85	0.52678269	1.45537562
0.90	0.43588989	1.49329011
0.95	0.31224990	1.53177816
1	0	1.57079633

For Aspect Ratio	Eccentricity	Complete Elliptic Integral
$\frac{b}{a} = 0.50$	$\varepsilon = 0.8660254$	$E\,[\pi/2\,,\varepsilon\,] = 1.21105603$

Complete Elliptic Integrals

First Kind

$$K[\pi/2, k] = \int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2\theta}}\, d\theta$$

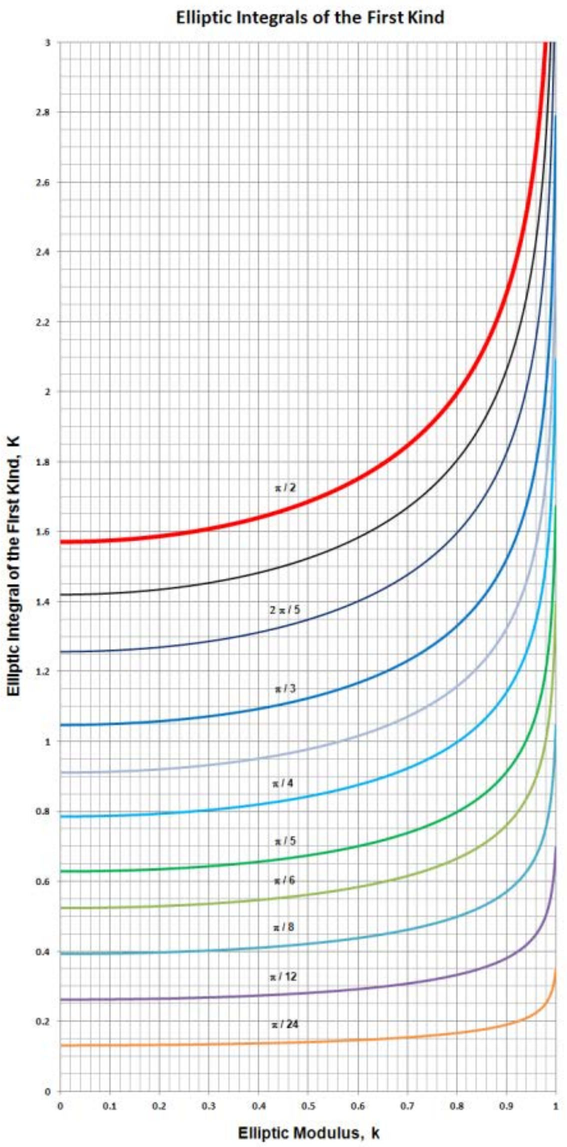
Second Kind

$$E[\pi/2, \varepsilon] = \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\, d\theta$$

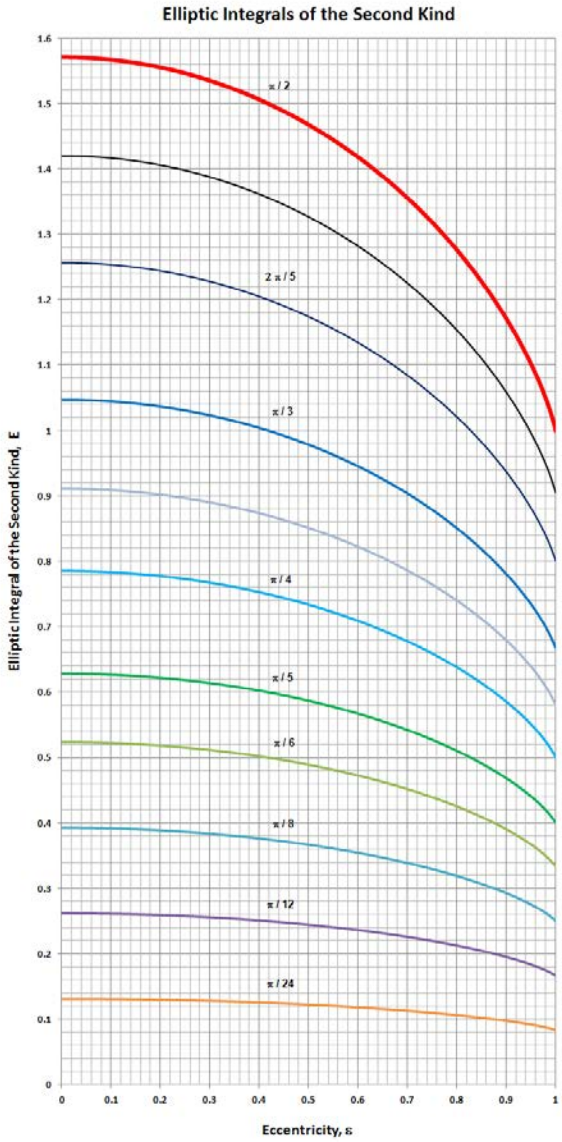
Third Kind

$$\prod[n; \pi/2 \mid k] = \int_0^{\pi/2} \frac{1}{(1 - n \sin^2\theta)\sqrt{1 - k^2 \sin^2\theta}}\, d\theta$$

Incomplete Elliptic Integrals of the First Kind



Incomplete Elliptic Integrals of the Second Kind



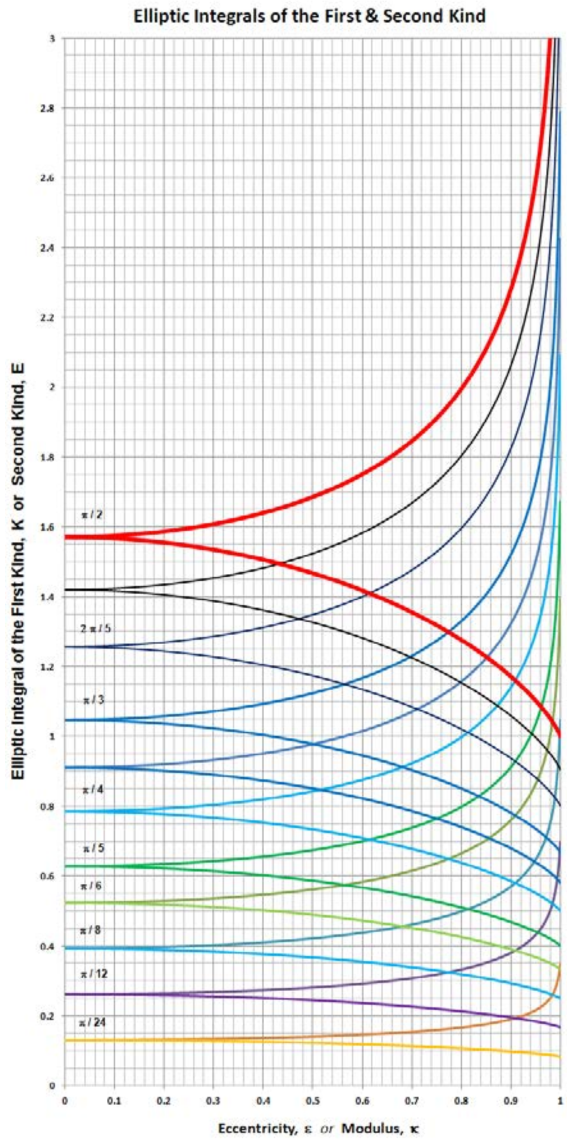
Quote

Bertrand Russell

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry."

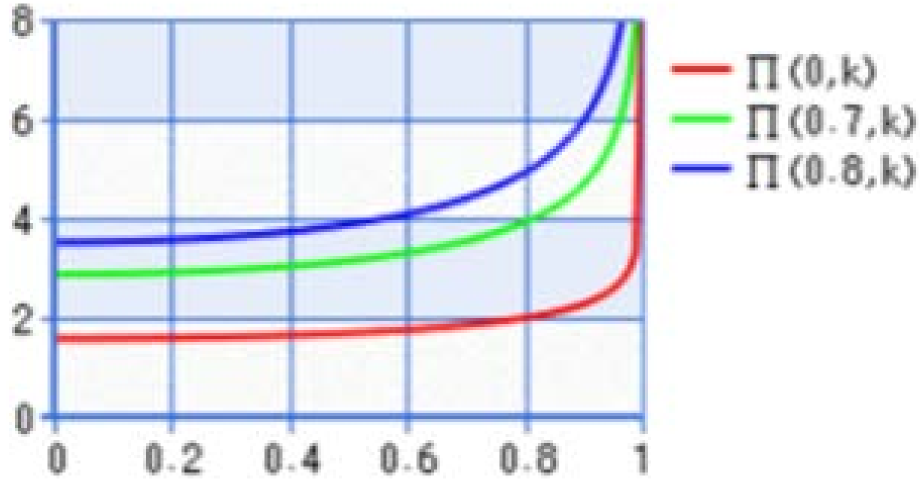
Elliptic Integrals of the First & Second Kind

Fine Art



Incomplete Elliptic Integrals of the Third Kind

$$\prod [\, n\, ;\, \pi/2 \,|\, k\,] = \int_{0}^{\pi/2} \frac{1}{(1 - n \sin^2\theta)\sqrt{1 - k^2 \sin^2\theta}}\, d\theta$$



http://keisan.casio.com/exec/system/1244988714

Complete Elliptic Integral of the First Kind – Pendulum Period

INPUT

L	37.780	inches
θ_0	30	degrees

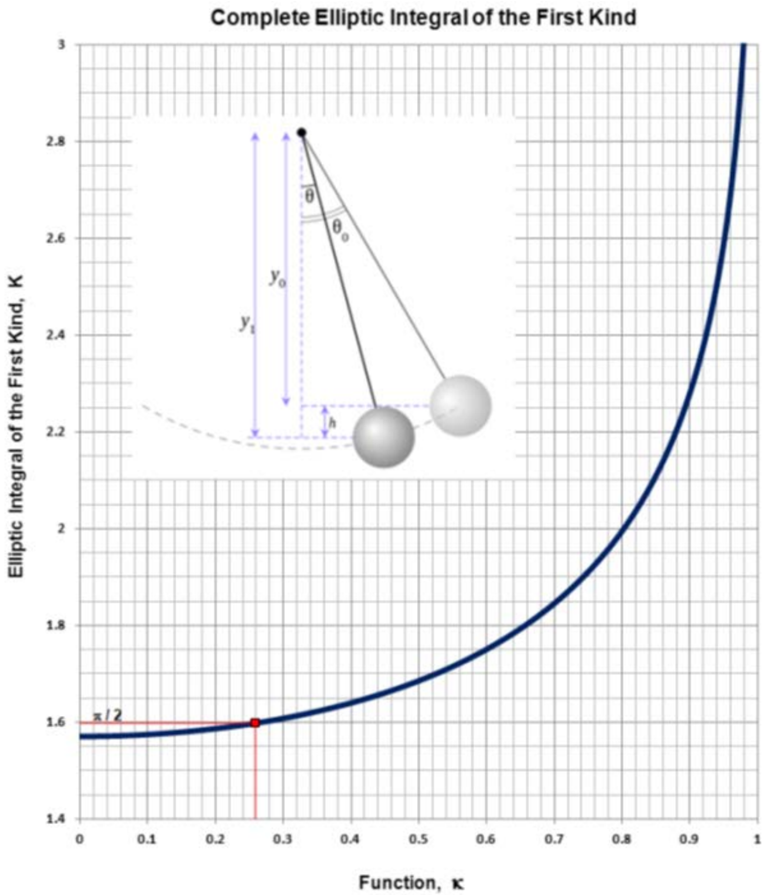
DATA

Σ	1.0174088	
$\theta_0 / 2$	15	degrees
$\sin(\theta_0/2)$	0.2588	
κ	0.2588	

OUTPUT

K	1.598	
T =	2.000	seconds
T ≈	1.965	seconds

Length	L	37.780	inch
Angle	θ	30	degrees
Gravity	g	32.174	ft / sec ^{2}
Gravity	g	386.088	inch / sec ^{2}
Period	T	2.000	seconds
Approximate	T	1.965	seconds
Frequency	f	0.500	cycles / second
Approximate	f	0.509	cycles / second



$$T \approx 2\pi\sqrt{\frac{L}{g}} \qquad T = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} = 4\sqrt{\frac{L}{g}}\,K(\pi/2, k)$$

Figure … thanks to http://www.codeproject.com/Articles/566614/Elliptic-integrals

Pendulum Period Error

θ	T / T_0	T / T_0
0	1	0%
5	1.00048	0.048%
10	1.00191	0.191%
15	1.0043	0.430%
20	1.0077	0.767%
25	1.0120	1.203%
30	1.0174	1.741%
35	1.0238	2.383%
40	1.0313	3.134%
45	1.0400	3.997%
50	1.0498	4.978%
55	1.0608	6.083%
60	1.0732	7.318%
65	1.0869	8.692%
70	1.1021	10.214%
75	1.1190	11.896%
80	1.1375	13.749%
85	1.1579	15.789%
90	1.1803	18.034%

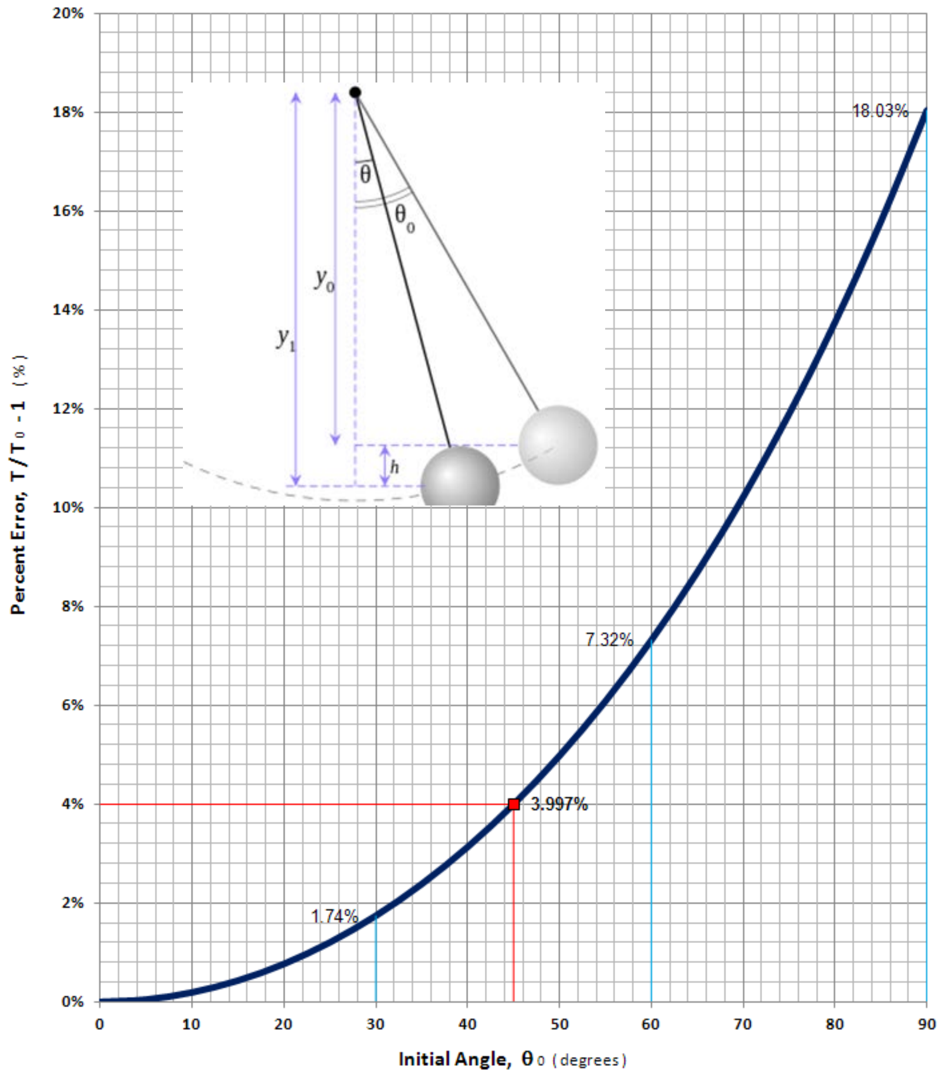


Figure … thanks to http://www.codeproject.com/Articles/566614/Elliptic-integrals

Complete Elliptic Integrals

The value of the **Complete Elliptic Integral of the First Kind** at k = 0 is … **π / 2**

The value of the **Complete Elliptic Integral of the Second Kind** at ε = 0 is …**π / 2**

For Eccentricity, ε = 0, the Aspect Ratio, b / a = 1 and the Ellipse is a Circle with Circumference, C = 2 π r

When a = b, the ratio b / a = 1 and the Complete Elliptic Integral of the Second Kind, **E** [π / 2, 0] = **π / 2**

$$P = 4\ a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\ d\theta = 4\ a\ (\pi/2) = 2\ \pi\ a$$

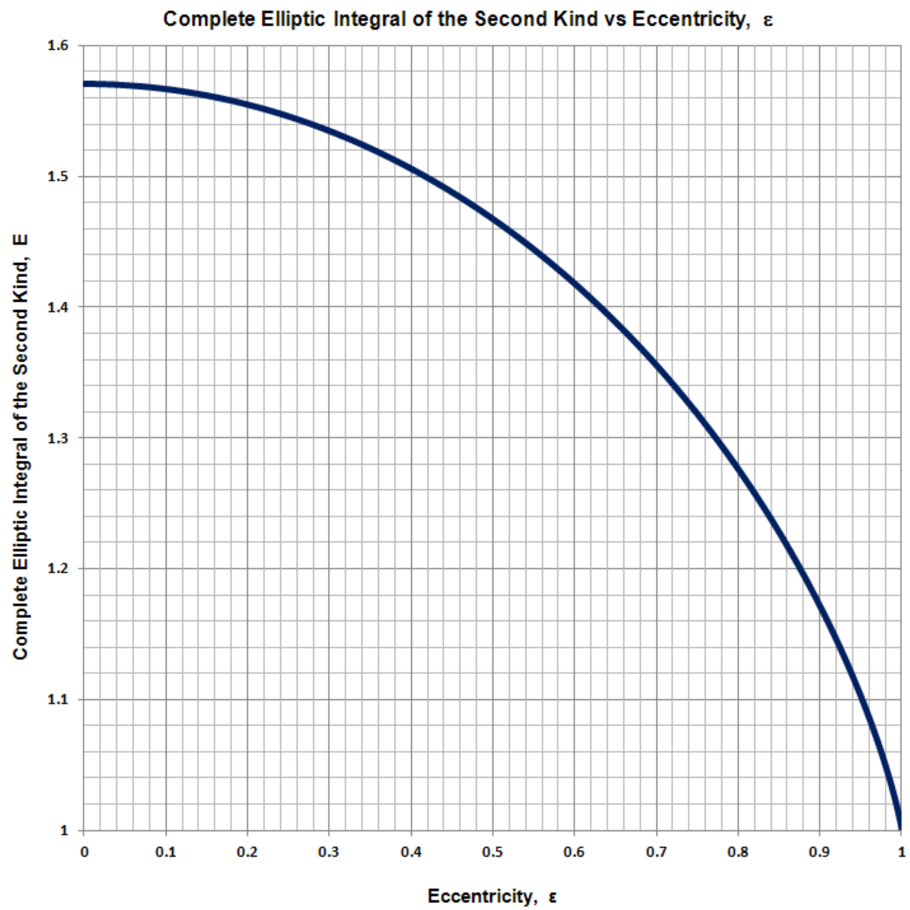
When b / a = 0, ε = 1 and the Complete Elliptic Integral of the Second Kind, **E** [π / 2, 1] = **1**

$$P = 4\ a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\ d\theta = 4\ a\ (1) = 4\ a$$

The value of the **Complete Elliptic Integral of the Third Kind** at the k = 0 is … **π / 2**

Complete Elliptic Integral of the Second Kind

$$E\left[\pi/2,\varepsilon\right] = \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\; d\theta$$



Complete Elliptic Integral of the Second Kind

$$P = 4\,a\,E[\pi/2, \varepsilon] = 4\,a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\; d\theta \qquad E \approx \left[A + B\left(\frac{b}{a}\right)^C\right]^{-1/D} + Offset$$

INPUT

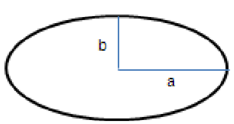
a	10	units
b	5	units

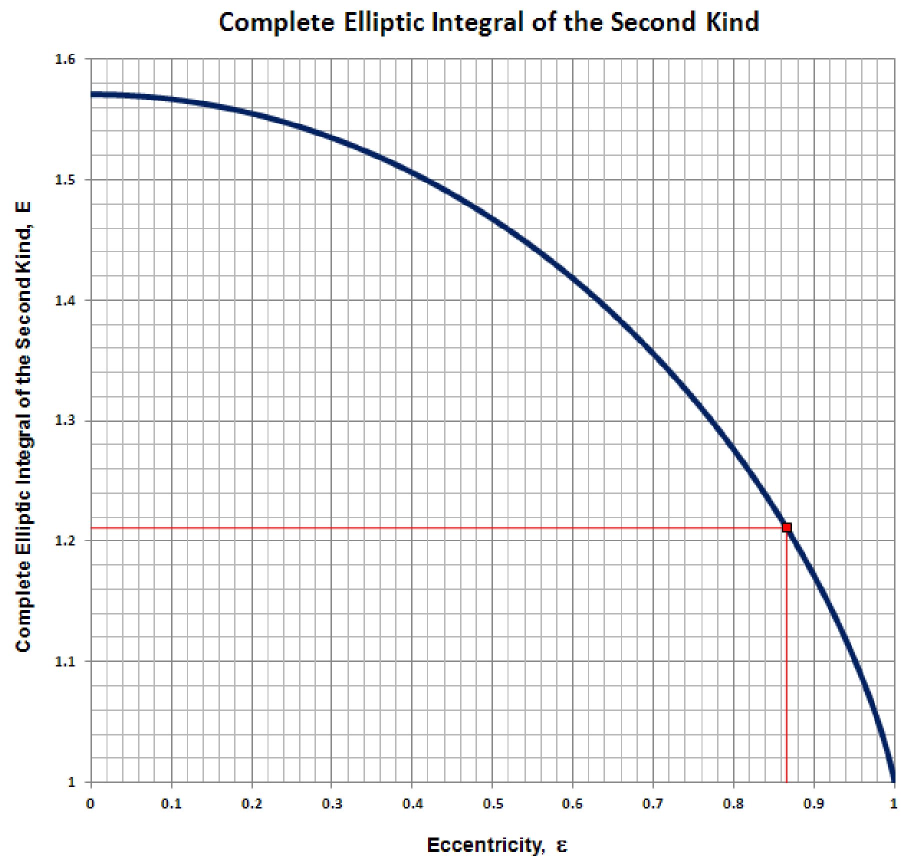
DATA

b / a	0.5
ε	0.8660254
1 - Σ	0.77098221

OUTPUT

E	1.21105603	
P	48.4422411	units





$$P = 4\ a\ E\,[\pi/2, \varepsilon\,] = 4\ (\,10.00\ inch\,)\ 1.21105603 = 48.4422\ inch$$

Exact Expressions – Ellipse Perimeter

Colin Maclaurin 1742

$$P = 2\pi a \sum_{n=0}^{\infty} \left\{ \left(\frac{-1}{2n-1} \right) \left[\frac{(2n)!}{(2^n n!)^2} \right]^2 e^{2n} \right\}$$

Leonhard Euler 1773

$$P = \pi \sqrt{2(a^2+b^2)} \left\{ 1 - \sum_{n=1}^{\infty} \left[\left(\frac{\delta}{16} \right)^n (4n-3)!! \,/\, (n!)^2 \right] \right\}$$

$$\delta = \left[\frac{(a^2-b^2)}{(a^2+b^2)} \right]^2$$

James Ivory 1796

$$P = 4aE[\pi/2, \varepsilon]$$

$$E = \frac{\pi}{2} \left\{ 1 - \sum_{n=1}^{\infty} \left[\frac{(2n-1)!!}{(2n)!!} \right]^2 \frac{\varepsilon^{2n}}{2n-1} \right\}$$

Arthur Cayley 1876

$$Perimeter = 4a \left\{ 1 + \left(\frac{x}{4} \right) \left[\ln\left(\frac{16}{x} \right) - 1 \right] + \left(\frac{3x^2}{32} \right) \left[\ln\left(\frac{16}{x} \right) - \frac{13}{6} \right] + \left(\frac{15x^3}{256} \right) \left[\ln\left(\frac{16}{x} \right) - \frac{12}{5} \right] + \left(\frac{175x^4}{4{,}096} \right) \left[\ln\left(\frac{16}{x} \right) - \frac{1{,}051}{420} \right] + \ldots \right\}$$

Colin Maclaurin

$$Perimeter = 2\,\pi\,a \sum_{n=0}^{\infty} \left\{ \left(\frac{-1}{2n-1} \right) \left[\frac{(2n)!}{(2^n\,n!)^2} \right]^2 e^{2n} \right\}$$

Example: For a = 10 inch b = 5 inch b / a = 0.50 Eccentricity, e = 0.866

n	2n	2n - 1	-1/2n-1	$\left[\frac{(2n)!}{(2^n\,n!)^2} \right]^2$	$\frac{-1}{2n-1} \left[\frac{(2n)!}{(2^n\,n!)^2} \right]^2 e^{2n}$
0	0	-1	1	1	1
1	2	1	-1	0.25	-0.1875
2	4	3	-0.333333333	0.140625	-0.026367188
3	6	5	-0.2	0.09765625	-0.008239746
4	8	7	-0.142857143	0.074768066	-0.003379583
5	10	9	-0.111111111	0.060562134	-0.001596853
6	12	11	-0.090909091	0.050889015	-0.000823377
7	14	13	-0.076923077	0.043878794	-0.000450547
8	16	15	-0.066666667	0.038565346	-0.000257393
9	18	17	-0.058823529	0.034399336	-0.000151933
10	20	19	-0.052631579	0.031045401	-9.20145E-05
...	...	...	...	...	...
75	150	149	-0.006711409	0.004230008	-1.2099E-14
76	152	151	-0.006622517	0.004174533	-8.83664E-15
77	154	153	-0.006535948	0.004120495	-6.45617E-15
78	156	155	-0.006451613	0.004067837	-4.71857E-15
79	158	157	-0.006369427	0.004016509	-3.44976E-15
80	160	159	-0.006289308	0.003966459	-2.52294E-15
81	162	161	-0.00621118	0.003917642	-1.8457E-15
82	164	163	-0.006134969	0.003870011	-1.35067E-15
83	166	165	-0.006060606	0.003823525	-9.88701E-16
84	168	167	-0.005988024	0.003778142	-7.23949E-16
85	170	169	-0.00591716	0.003733824	-5.30243E-16
				Σ	**0.7709822126**

$$Perimeter = 2\,\pi\,(10\,inch) = 2\,\pi\,(10\,inch)\,0.7709822126 = 48.4422\text{ inch}$$

Leonhard Euler

$$Perimeter = \pi \sqrt{2(a^2 + b^2)} \left\{ 1 - \sum_{n=1}^{\infty} \left[\left(\frac{\delta}{16} \right)^n (4n - 3)!! / (n!)^2 \right] \right\}$$

where

$$\delta = \left[\frac{(a^2 - b^2)}{(a^2 + b^2)} \right]^2$$

Example: a = 10 inch b = 5 inch

n	$(\delta / 16)^n$	4 n - 3	(4 n - 3) ! !	$n!^2$	$\sum_{n=1}^{\infty} \left[\left(\frac{\delta}{16} \right)^n (4n - 3)!! / (n!)^2 \right]$
1	0.0225	1	1	1	0.0225
2	0.000506	5	15	4	0.001898438
3	1.14E-05	9	945	36	0.000299004
4	2.56E-07	13	135,135	576	6.01278E-05
5	5.77E-09	17	34,459,425	14,400	1.37993E-05
6	1.3E-10	21	13,749,310,575	518,400	3.44121E-06
7	2.92E-12	25	7,905,853,580,625	25,401,600	9.08584E-07
8	6.57E-14	29	6,190,283,353,629,370	1,625,702,400	2.50109E-07
9	1.48E-15	33	6.33265987076285E+18	131,681,894,400	7.10727E-08
10	3.33E-17	37	8.20079453263789E+21	13,168,189,440,000	2.07088E-08
				Σ	**0.02477606**

$$Perimeter = \pi \sqrt{2[(10.00\ inch)^2 + (5.00\ inch)^2]}\ (1 - 0.02477606) = 48.4422\ inch$$

James Ivory

Gauss – Kummer Series

$$P = 4\ a\ E[\pi/2, \varepsilon]$$

$$E[\pi/2, \varepsilon] = \frac{\pi}{2}\left\{1 - \sum_{n=1}^{\infty}\left[\frac{(2n-1)!!}{(2n)!!}\right]^2 \frac{\varepsilon^{2n}}{2n-1}\right\}$$

Example: For a = 10 inch b = 5 inch b / a = 0.50 Eccentricity, ε = 0.866

n	2n	2 n - 1	$\sum_{n=1}^{\infty}\left[\frac{(2n-1)!!}{(2n)!!}\right]^2 \frac{\varepsilon^{2n}}{2n-1}$
1	2	1	0.1875
2	4	3	0.026367
3	6	5	0.00824
4	8	7	0.00338
5	10	9	0.001597
6	12	11	0.000823
7	14	13	0.000451
8	16	15	0.000257
9	18	17	0.000152
10	20	19	9.2E-05
...	...	...	...
140	280	279	2.62E-23
141	282	281	1.94E-23
142	284	283	1.43E-23
143	286	285	1.06E-23
144	288	287	7.85E-24
145	290	289	5.8E-24
146	292	291	4.29E-24
147	294	293	3.18E-24
148	296	295	2.35E-24
149	298	297	1.74E-24
150	300	299	1.29E-24
		Σ	**0.229018**
		1 - Σ	**0.770982**
		π / 2 (1 - Σ)	**1.211056**

$$P = 4\ (10.00\ inch)\ 1.211056 = 48.4422\ inch$$

Arthur Cayley

$$2E = 2 + \left[\ln\left(\frac{4}{b/a}\right) - \frac{1}{1\cdot 2} \right]\left(\frac{b}{a}\right)^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{1}{3\cdot 4} \right]\left(\frac{b}{a}\right)^4$$

$$+ \left(\frac{1\cdot 3^2\cdot 5}{2\cdot 4^2\cdot 6}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{1}{5\cdot 6} \right]\left(\frac{b}{a}\right)^6$$

$$+ \left(\frac{1\cdot 3^2\cdot 5^2\cdot 7}{2\cdot 4^2\cdot 6^2\cdot 8}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{2}{5\cdot 6} - \frac{1}{7\cdot 8} \right]\left(\frac{b}{a}\right)^8$$

$$+ \left(\frac{1\cdot 3^2\cdot 5^2\cdot 7^2\cdot 9}{2\cdot 4^2\cdot 6^2\cdot 8^2\cdot 10}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{2}{5\cdot 6} - \frac{2}{7\cdot 8} - \frac{1}{9\cdot 10} \right]\left(\frac{b}{a}\right)^{10}$$

$$+ \left(\frac{1\cdot 3^2\cdot 5^2\cdot 7^2\cdot 9^2\cdot 11}{2\cdot 4^2\cdot 6^2\cdot 8^2\cdot 10^2\cdot 12}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{2}{5\cdot 6} - \frac{2}{7\cdot 8} - \frac{2}{9\cdot 10} - \frac{1}{11\cdot 12} \right]\left(\frac{b}{a}\right)^{12}$$

$$+ \left(\frac{1\cdot 3^2\cdot 5^2\cdot 7^2\cdot 9^2\cdot 11^2\cdot 13}{2\cdot 4^2\cdot 6^2\cdot 8^2\cdot 10^2\cdot 12^2\cdot 14}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{2}{5\cdot 6} - \frac{2}{7\cdot 8} - \frac{2}{9\cdot 10} - \frac{2}{11\cdot 12} - \frac{1}{13\cdot 14} \right]\left(\frac{b}{a}\right)^{14}$$

$$+ \left(\frac{1\cdot 3^2\cdot 5^2\cdot 7^2\cdot 9^2\cdot 11^2\cdot 13^2\cdot 15}{2\cdot 4^2\cdot 6^2\cdot 8^2\cdot 10^2\cdot 12^2\cdot 14^2\cdot 16}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{2}{5\cdot 6} - \frac{2}{7\cdot 8} - \frac{2}{9\cdot 10} - \frac{2}{11\cdot 12} - \frac{2}{13\cdot 14} - \frac{1}{15\cdot 16} \right]\left(\frac{b}{a}\right)^{16}$$

$$+ \left(\frac{1\cdot 3^2\cdot 5^2\cdot 7^2\cdot 9^2\cdot 11^2\cdot 13^2\cdot 15^2\cdot 17}{2\cdot 4^2\cdot 6^2\cdot 8^2\cdot 10^2\cdot 12^2\cdot 14^2\cdot 16^2\cdot 18}\right)\left[\ln\left(\frac{4}{b/a}\right) - \frac{2}{1\cdot 2} - \frac{2}{3\cdot 4} - \frac{2}{5\cdot 6} - \frac{2}{7\cdot 8} - \frac{2}{9\cdot 10} - \frac{2}{11\cdot 12} - \frac{2}{13\cdot 14} - \frac{2}{15\cdot 16} - \frac{1}{17\cdot 18} \right]\left(\frac{b}{a}\right)^{18}$$

$$\ldots$$

Example: For a = 10 inch b = 5 inch b / a = 0.50 2 E = 2.4221120

n	Numerator	Denominator	Fraction	Fractions - Right Side of Natural Logorithm														
1	1	1	1	1 2														
2	3	8	0.375	2 2	1 12													
3	45	192	0.2343750	2 2	2 12	1 30												
4	1,575	9,216	0.1708984	2 2	2 12	2 30	1 56											
5	99,225	737,280	0.1345825	2 2	2 12	2 30	2 56	1 90										
6	9,823,275	88,473,600	0.1110306	2 2	2 12	2 30	2 56	2 90	1 132									
7	1,404,728,325	14,863,564,800	0.0945082	2 2	2 12	2 30	2 56	2 90	2 132	1 182								
8	273,922,023,375	3,329,438,515,200	0.0822727	2 2	2 12	2 30	2 56	2 90	2 132	2 182	1 240							
9	69,850,115,960,625	958,878,292,377,600	0.0728457	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	1 306						
10	22,561,587,455,281,900	345,196,185,255,936,000	0.0653587	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	2 306	1 380					
11	9,002,073,394,657,470,000	151,886,321,512,612,000,000	0.0592685	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	2 306	2 380	1 462				
12	4,348,001,449,619,560,000,000	80,195,977,758,659,100,000,000	0.0542172	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	2 306	2 380	2 462	1 552			
13	2,500,100,833,531,250,000,000,000	50,042,290,121,403,200,000,000,000	0.0499598	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	2 306	2 380	2 462	2 552	1 650		
14	1,687,568,062,633,590,000,000,000,000	36,430,787,208,381,600,000,000,000,000	0.0463226	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	2 306	2 380	2 462	2 552	2 650	1 756	
15	1,321,365,793,042,100,000,000,000,000,000	30,601,861,255,040,500,000,000,000,000,000	0.0431793	2 2	2 12	2 30	2 56	2 90	2 132	2 182	2 240	2 306	2 380	2 462	2 552	2 650	2 756	1 870

$$P = 4\,a\,E = 2\,a\,(2E) = 2\,(10.00\ inch)\ 2.4221120 = 48.4422\ inch$$

Friedrich Bessel

$$P \approx \pi (a + b) \left\{ 1 + \left(\frac{1}{2}\right)^2 h + \sum_{n=2}^{\infty} \left[\frac{(2n - 3)!!}{(2n)!!}\right]^2 h^n \right\}$$

where

$$h = \frac{(a - b)^2}{(a + b)^2}$$

n	2 n	j 2 n ! !	(2 n - 3)	k (2 n - 3) ! !	(k / j) ^{2}	(k / j) ^{2} h ^{n}
2	4	8	1	1	0.015625	0.00019290123456790
3	6	48	3	3	0.0039063	0.00000535836762689
4	8	384	5	15	0.0015259	0.00000023256803936
5	10	3840	7	105	0.0007477	0.00000001266203770
6	12	46080	9	945	0.0004206	0.00000000079137736
7	14	645120	11	10395	0.0002596	0.00000000005428382
8	16	10321920	13	135135	0.0001714	0.00000000000398176
9	18	185794560	15	2027025	0.000119	0.00000000000030723
10	20	3.716E+09	17	34459425	8.6E-05	0.00000000000002466
11	22	8.175E+10	19	654729075	6.414E-05	0.00000000000000204
12	24	1.962E+12	21	1.3749E+10	4.911E-05	0.00000000000000017
13	26	5.101E+13	23	3.1623E+11	3.843E-05	0.00000000000000002
14	28	1.428E+15	25	7.9059E+12	3.064E-05	0.00000000000000000
15	30	4.285E+16	27	2.1346E+14	2.482E-05	0.00000000000000000
...	...	...	...	...	...	...
50	100	3.424E+79	97	2.7529E+76	6.463E-07	0.00000000000000000
51	102	3.493E+81	99	2.7254E+78	6.088E-07	0.00000000000000000
52	104	3.633E+83	101	2.7526E+80	5.742E-07	0.00000000000000000
53	106	3.85E+85	103	2.8352E+82	5.422E-07	0.00000000000000000
54	108	4.159E+87	105	2.977E+84	5.125E-07	0.00000000000000000
55	110	4.574E+89	107	3.1854E+86	4.849E-07	0.00000000000000000
56	112	5.123E+91	109	3.4721E+88	4.593E-07	0.00000000000000000
57	114	5.841E+93	111	3.854E+90	4.354E-07	0.00000000000000000
58	116	6.775E+95	113	4.355E+92	4.132E-07	0.00000000000000000
59	118	7.995E+97	115	5.0083E+94	3.925E-07	0.00000000000000000
60	120	9.59E+99	117	5.8597E+96	3.731E-07	0.00000000000000000
					Σ	**0.00019850568225**

$$P \approx \pi (a + b) [1 + 0.25 (0.111111111) + (0.00019850568)] = 48.4422$$

Approximate Expressions – Ellipse Perimeter

Srinivasa Ramanujan I

$$P \approx \pi\left[3(a+b) - \sqrt{3(a+b)(a+3b)}\right] = \pi(a+b)\left[3 - \sqrt{4-h}\right]$$

$$h = \frac{(a-b)^2}{(a+b)^2}$$

Srinivasa Ramanujan II

$$P \approx \pi(a+b)\left[1 + \frac{3h}{\left(10 + \sqrt{4-3h}\right)}\right]$$

David W. Cantrell

$$P \approx \pi(a+b)\left[1 + \frac{3h}{\left(10 + \sqrt{4-3h}\right)} + 4h^6\left(\frac{1}{\pi} - \frac{7}{22}\right)h^6\right]$$

Ramanujan Improvement

Lindner

$$P \approx \pi(a+b)\left(\frac{4}{\pi}\right)^h$$

$$P \approx 4(a+b)\left(\frac{\pi}{4}\right)^{[(4ab)/(a+b)^2]}$$

Approximate Expressions – Ellipse Perimeter

Shahram Zafary

$$P \approx \pi (a + b) \left(\frac{4}{\pi}\right)^{h} \approx \pi (a + b) \left(\frac{4}{\pi}\right)^{\frac{(a-b)^2}{(a+b)^2}}$$

Shahram Zafary

$$P \approx 4 (a + b) \left(\frac{\pi}{4}\right)^{[(4ab) / (a+b)^2]}$$

David F. Rivera

$$P \approx 4 \left[\frac{\pi a b + (a - b)^2}{a + b}\right] - \frac{89}{146} \left(\frac{a\sqrt{b} - b\sqrt{a}}{a + b}\right)^2$$

Approximate Expressions – Ellipse Perimeter

Padé Approximates

A Manual of Mathematics by Ralph G. Hudson and Joseph Lipka 1955

$$P \approx \pi (a + b) \frac{64 - 3h^2}{64 - 16h}$$

Truncation at order m = 3 of Padé Approximates

$$P \approx \pi (a + b) \frac{64 + 16h}{64 - h^2}$$

Jacobsen and Waadeland 1985

$$P \approx \pi (a + b) \frac{256 - 48h - 21h^2}{256 - 112h + 3h^2}$$

Next Level

$$P \approx \pi (a + b) \frac{3{,}072 - 1{,}280h - 252h^2 + 33h^3}{3{,}072 - 2{,}048h + 212h^2}$$

Next Level

$$P \approx \pi (a + b) \frac{135{,}168 - 85{,}760h - 5{,}568h^2 + 3{,}867h^3}{135{,}168 - 119{,}552h + 22{,}208h^2 - 345h^3}$$

Approximate Expressions – Ellipse Perimeter

Giuseppe Peano

$$P \approx \pi \left[\frac{3\,(a + b)}{2} - \sqrt{a\,b} \right] = \pi\,(a + b)\,\frac{\left[3 - \sqrt{1 - h}\,\right]}{2}$$

YNOT Formula by Roger Maertens

$$P \approx 4\,(a^{y} + b^{y})^{1/y}$$

$$y = \frac{\ln(2)}{\ln(\pi/2)}$$

Approximate Expressions – Ellipse Perimeter

Naïve Formula

$$P \approx \pi\ (a + b)$$

Johannes Kepler

$$P \approx 2\pi \sqrt{a\,b}$$

Leonhard Euler

$$P \approx \pi \sqrt{2\,(a^2 + b^2)}$$

Thomas Muir

where p = 3 / 2

$$P \approx 2\pi \left[\frac{(a^p + b^p)}{2} \right]^{1/p}$$

Approximate Expressions – Ellipse Perimeter

Circumference Formulas

$$P \approx \pi \sqrt{2\left(a^2 + b^2\right) - \frac{(a-b)^2}{D}} = \pi (a+b) \sqrt{1 + h\left(1 - 1/D\right)}$$

Takakazu Seki

$$P \approx 2\sqrt{\pi^2 a b + 4 (a-b)^2}$$

Standard Mathematical Tables and Formulae

Errata to the 30th Edition of the Standard Mathematical Tables and Formulae (CRC Press). Thanks to David F. Rivera and www.numericana.com
http://www.mathtable.com/errata/smtf30_errata_p2/index.html

$$P \approx 2a\left[2 + (\pi - 2)\left(\frac{b}{a}\right)^{1.456}\right]$$

Anonymous

$$P \approx \pi \sqrt{2\left[\left(a^2 + b^2\right) - 0.50\,(a-b)^2\right]}$$

Approximate Expressions – Ellipse Perimeter

Sipos

$$P \approx \pi (a + b) \frac{2}{1 + \mu^2}$$

where

$$\mu = \frac{(a - b)}{(a + b)}$$

Bronshtein

$$P \approx \pi (a + b) \frac{64 - 3\mu^4}{64 - 16\mu^2}$$

Ernst S. Selmer

$$P \approx \pi (a + b) \left[1 + \frac{\mu^2}{4} \left(\frac{16}{16 - \mu^2} \right) \right]$$

Ernst S. Selmer

$$P \approx \pi (a + b) \left[\frac{3}{2} + \frac{\mu^2}{8} - \frac{1}{2} \sqrt{\mu^2} \right]$$

Ernst S. Selmer

$$P \approx \pi (a + b) \left[\frac{4 (a - b)^2}{(5a + 3b)(3a + 5b)} \right] = \pi (a + b) \left[\frac{16 + 3h}{16 - h} \right]$$

where

$$h = \frac{(a - b)^2}{(a + b)^2}$$

Approximate Expressions – Ellipse Perimeter

Almkvist

$$P \approx 2\pi(a+b)\left\{\frac{2(a+b)^2-\left(\sqrt{a}-\sqrt{b}\right)^4}{(a+b)\left[\left(\sqrt{a}-\sqrt{b}\right)^2+2\sqrt{2}\sqrt{(a+b)\sqrt[4]{ab}}\right]}\right\}$$

Lu Chee Ket

$$P \approx \pi\sqrt{2(a^2+b^2)}\sum_{n=1}^{\infty}\left(\frac{\delta}{16}\right)^n\frac{(4n-3)!!}{(n!)^2}$$

$$\delta = \left[\frac{(a^2-b^2)}{(a^2+b^2)}\right]^2 = \frac{4h}{(1+h)^2}$$

Approximate Expressions – Ellipse Perimeter

Gauss – Kummer Series

See James Ivory

$$P = \pi\,(a + b)\sum_{n=0}^{\infty}\binom{0.5}{n}^2 h^n$$

$$P \approx \pi\,(a + b)\left[1 + \frac{h}{4} + \frac{h^2}{64} + \frac{h^3}{256} + \frac{25\,h^4}{16{,}384} + \frac{49\,h^5}{65{,}536} + \frac{441\,h^6}{1{,}048{,}576} + \frac{1{,}089\,h^7}{4{,}194{,}304} + \ldots\right]$$

a = 10 units b = 5 units

n		Num		Denom	$\sum_{n=0}^{\infty}\binom{0.5}{n}^2 h^n$
0			0	1	1
1	1	1	2	4	0.0277777777777778
2	1	1	6	64	0.0001929012345679
3	1	1	8	256	0.0000053583676269
4	5	25	14	16,384	0.0000002325680394
5	7	49	16	65,536	0.0000000126620377
6	21	441	20	1,048,576	0.0000000007913774
7	33	1,089	22	4,194,304	0.0000000000542838
8	429	184,041	30	1,073,741,824	0.0000000000039818
				Σ	1.027976283460
				$\pi\,(a + b)$	47.1238898
				Perimeter	48.4422411

Approximate Expressions – Ellipse Perimeter

David W. Cantrell 2001

$$P \approx 4(a+b) - \frac{2(4-\pi)ab}{H_p} = 4(a+b) - 2(4-\pi)ab\,H_p$$

Holder Mean

$$H_p = \left[\frac{(a^p + b^p)}{2}\right]^{1/p}$$

p	0.82990	$p = \dfrac{3\pi - 8}{8 - 2\pi}$
p	0.82506	
p	0.81949	$p = \dfrac{\ln(2)}{\ln[2/(4-\pi)]}$

David W. Cantrell 2004

$$P \approx 4(a+b) - \frac{2(4-\pi)ab}{f}$$

where

$$f = p(a+b)[(1-2p)/(k+1)]\sqrt{(a+kb)(ka+b)}$$

k = 74

Approximate Expressions – Ellipse Perimeter

Hassan Abed

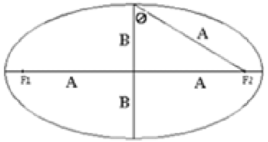
$\lambda^4/(\sin\theta+\cos\theta)^{16}$ **This is what i added to Ramanujan's formula, where** $\lambda = \frac{1}{4}\left(\frac{A-B}{A+B}\right)^2$

$$P \approx \pi(a+b)\left[1 + \frac{3\left(\frac{a-b}{a+b}\right)^2 + \lambda^4/(\sin\theta+\cos\theta)^{16}}{10+\sqrt{4-3\left(\frac{a-b}{a+b}\right)^2}}\right]$$

Srinivasa Ramanujan

$$P \approx \pi(a+b)\left[1 + \frac{3\left(\frac{a-b}{a+b}\right)^2}{10+\sqrt{4-3\left(\frac{a-b}{a+b}\right)^2}}\right]$$

http://www.paulbourke.net/geometry/ellipsecirc/



Perimeter $\approx$ 4 AE

Note : $\varnothing$ in degrees

$\sin\theta = \frac{\sqrt{A^2-B^2}}{A}$, $\cos\theta = \frac{B}{A}$ $\therefore (\sin\theta+\cos\theta)^{16} = \left(\frac{B+\sqrt{A^2-B^2}}{A}\right)^{16}$

http://www.paulbourke.net/geometry/ellipsecirc/HassanAbed1.gif

The GRAN Method – Ellipse Perimeter

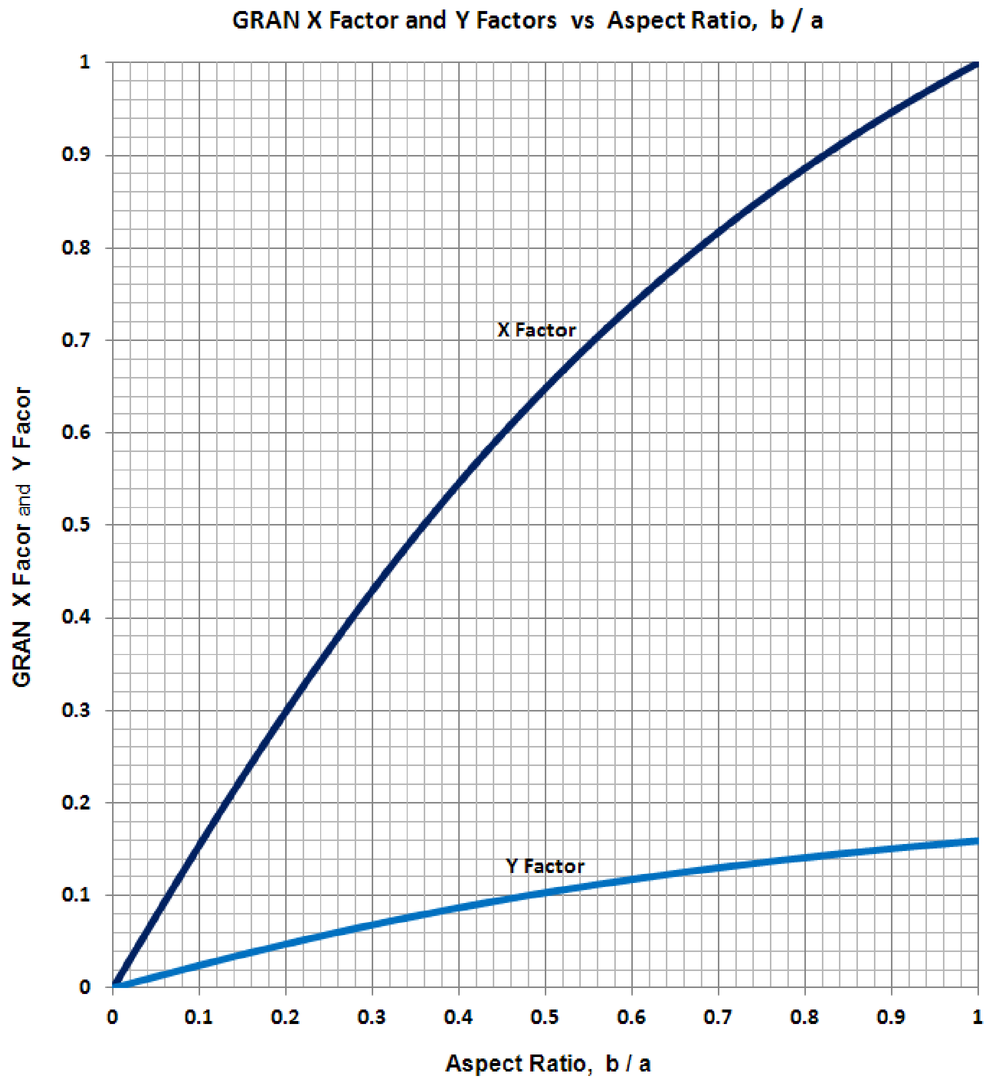
Ellipse Perimeter

$$P_{ellipse} = \frac{A_{ellipse}}{\left(\frac{A_{ellipse}}{A_{circle}}\right)\left(\frac{A_{ellipse}\ /\ P_{ellipse}}{A_{circle}\ /\ C_{circle}}\right)} \approx \frac{A_{ellipse}}{\left(\frac{A_{ellipse}}{A_{circle}}\right)\ X_{factor}}$$

$$Y_{factor} = \frac{X_{factor}}{2\,\pi}$$

Ratio b / a	GRAN X Factor	GRAN Y Factor	Ratio b / a	GRAN X Factor	GRAN Y Factor
0.05	0.0781083	0.0124313	**0.55**	0.6949549	0.1106055
0.10	0.1545889	0.0246036	**0.60**	0.7384163	0.1175226
0.15	0.2284164	0.0363536	**0.65**	0.7790898	0.1239960
0.20	0.2990564	0.0475963	**0.70**	0.8171553	0.1300543
0.25	0.3662205	0.0582858	**0.75**	0.8527866	0.1357252
0.30	0.4297755	0.0684009	**0.80**	0.8861516	0.1410354
0.35	0.4897002	0.0779382	**0.85**	0.9174104	0.1460104
0.40	0.5460528	0.0869070	**0.90**	0.9467127	0.1506740
0.45	0.5989453	0.0953251	**0.95**	0.9741991	0.1550486
0.50	0.6485234	0.1032157	**1.00**	1	0.1591549

The GRAN Method – Ellipse Perimeter



The GRAN Method – Ellipse Perimeter

$$P \approx \frac{b}{Y_{factor}}$$

The GRAN Method – Ellipse Perimeter

Ratio b / a	GRAN Y Factor
0.05	0.0124313
0.10	0.0246036
0.15	0.0363536
0.20	0.0475963
0.25	0.0582858
0.30	0.0684009
0.35	0.0779382
0.40	0.0869070
0.45	0.0953251
0.50	**0.1032157**
0.55	0.1106055
0.60	0.1175226
0.65	0.1239960
0.70	0.1300543
0.75	0.1357252
0.80	0.1410354
0.85	0.1460104
0.90	0.1506740
0.95	0.1550486
1.00	0.1591549

$$P \approx \frac{b}{Y_{factor}}$$

Example a = 10 inch b = 5 inch For b / a = 0.50 Y factor = 0.1032157

$$P \approx \frac{b}{Y_{factor}} \approx \frac{5\ inch}{0.1032157} \approx 48.4422\ inch$$

GRAN Functions – Ellipse Perimeter

Third Order Polynomial

$$Y_{factor} \approx A\left(\frac{b}{a}\right)^3 + B\left(\frac{b}{a}\right)^2 + C\left(\frac{b}{a}\right) + D$$

where A = 0.0241754181163 B = -0.1318547008372 C = 0.2671995647558 D = -0.0005822344755

R-squared: 0.999988668288 R-squared adjusted: 0.999987944987

Sixth Order Polynomial

$$Y_{factor} \approx A\left(\frac{b}{a}\right)^6 + B\left(\frac{b}{a}\right)^5 + C\left(\frac{b}{a}\right)^4 + D\left(\frac{b}{a}\right)^3 + E\left(\frac{b}{a}\right)^2 + F\left(\frac{b}{a}\right) + G$$

where A = 0.055506415895 B = -0.218717429777 C = 0.346788209235 D = -0.246381274132

E = -0.029316702018 F = 0.251297160058 G = -0.000013699159

R-squared: 0.999999991349 R-squared adjusted: 0.999999990169

Legendre Polynomial – Tenth Degree

$$Y_{factor} \approx 0.64237059137 - 1.49695312010\left(\frac{b}{a}\right)$$

$$+ 2.116143704\left\{\frac{1}{2}\left[3\left(\frac{b}{a}\right)^2 - 1\right]\right\}$$

$$- 1.958802974\left\{\frac{1}{2}\left[5\left(\frac{b}{a}\right)^3 - 3\left(\frac{b}{a}\right)\right]\right\}$$

$$+ 1.377147227\left\{\frac{1}{8}\left[35\left(\frac{b}{a}\right)^4 - 30\left(\frac{b}{a}\right)^2 + 3\right]\right\}$$

$$- 0.774611287\left\{\frac{1}{8}\left[63\left(\frac{b}{a}\right)^5 - 70\left(\frac{b}{a}\right)^3 + 15\left(\frac{b}{a}\right)\right]\right\}$$

$$+ 0.351812114\left\{\frac{1}{16}\left[231\left(\frac{b}{a}\right)^6 - 315\left(\frac{b}{a}\right)^4 + 105\left(\frac{b}{a}\right)^2 - 5\right]\right\}$$

$$- 0.126498659\left\{\frac{1}{16}\left[429\left(\frac{b}{a}\right)^7 - 693\left(\frac{b}{a}\right)^5 + 315\left(\frac{b}{a}\right)^3 - 35\left(\frac{b}{a}\right)\right]\right\}$$

$$+ 0.034217125\left\{\frac{1}{128}\left[6{,}435\left(\frac{b}{a}\right)^8 - 12{,}012\left(\frac{b}{a}\right)^6 + 6{,}930\left(\frac{b}{a}\right)^4 - 1{,}260\left(\frac{b}{a}\right)^2 + 35\right]\right\}$$

$$- 0.006255655\left\{\frac{1}{128}\left[12{,}155\left(\frac{b}{a}\right)^9 - 25{,}740\left(\frac{b}{a}\right)^7 + 18{,}018\left(\frac{b}{a}\right)^5 - 4{,}620\left(\frac{b}{a}\right)^3 + 315\left(\frac{b}{a}\right)\right]\right\}$$

$$+ 0.000585935\left\{\frac{1}{256}\left[46{,}189\left(\frac{b}{a}\right)^{10} - 109{,}395\left(\frac{b}{a}\right)^8 + 90{,}090\left(\frac{b}{a}\right)^6 - 30{,}030\left(\frac{b}{a}\right)^4 + 3{,}465\left(\frac{b}{a}\right)^2 - 63\right]\right\}$$

R-squared: 0.999999999998 R-squared adjusted: 0.999999999997

The GRAN Method – Ellipse Perimeter

INPUT

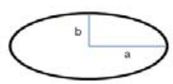
a	10	units
b	5	units

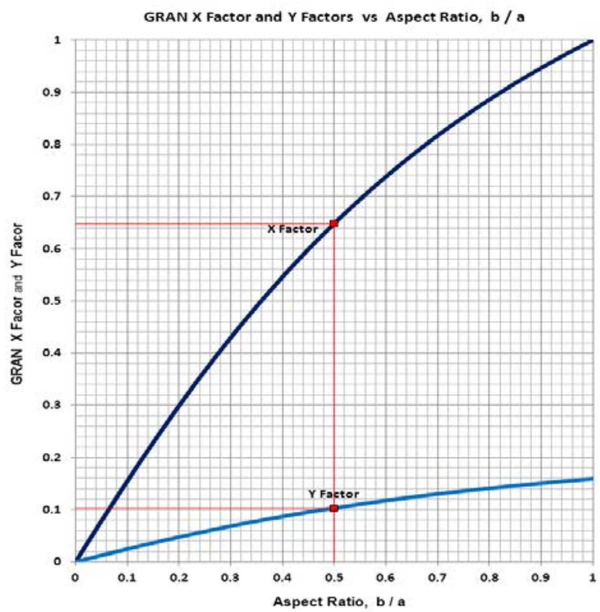
DATA

b / a	0.500
e	0.8660254
Y Factor	0.103216

OUTPUT

P	48.4422	units





Ellipse Perimeter

$$P_{ellipse} = \frac{A_{ellipse}}{\left(\frac{A_{circle}}{C_{circle}}\right)\frac{\left(A_{ellipse}/P_{ellipse}\right)}{\left(A_{circle}/C_{circle}\right)}} \approx \frac{A_{ellipse}}{\left(\frac{A_{circle}}{C_{circle}}\right) X_{factor}}$$

$$P_{ellipse} \approx \frac{b}{Y_{factor}}$$

Third Order Polynomial

Y Factor	0.103076
P	48.4423

Sixth Order Polynomial

Y Factor	0.103215
P	48.4423

Legendre Polynomial

Y Factor	0.103216
P	48.4423

b / a	GRAN Y Factor	b / a	GRAN Y Factor
0.05	0.0124401	0.55	0.1106055
0.10	0.0246066	0.60	0.1175226
0.15	0.0363540	0.65	0.1239960
0.20	0.0475963	0.70	0.1300543
0.25	0.0582858	0.75	0.1357252
0.30	0.0684009	0.80	0.1410354
0.35	0.0779382	0.85	0.1460104
0.40	0.0869070	0.90	0.1506740
0.45	0.0953251	0.95	0.1550486
0.50	0.1032157	1.00	0.1591549

The Solution to Everything

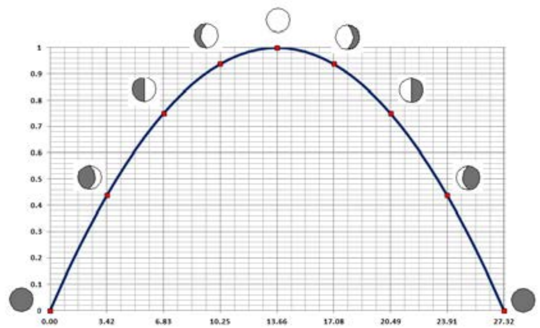
$$P_{ellipse} = \frac{\cancel{A_{ellipse}}}{\left(\cancel{\frac{A_{circle}}{C_{circle}}}\right)\left(\frac{\cancel{A_{ellipse}} / P_{ellipse}}{\cancel{A_{circle} / C_{circle}}}\right)}$$

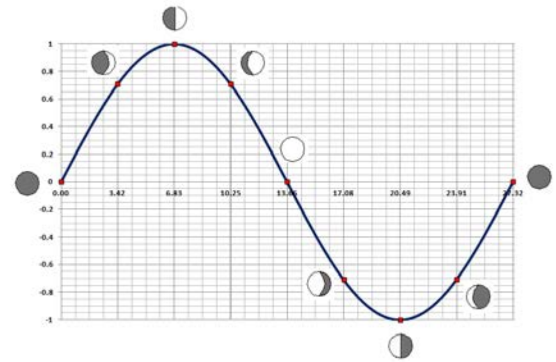
$$P_{ellipse} = P_{ellipse}$$

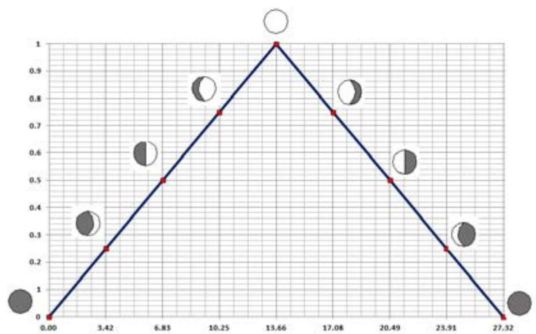
Plot the Dow Jones or Standard and Poor's Index versus Phases of the Moon.

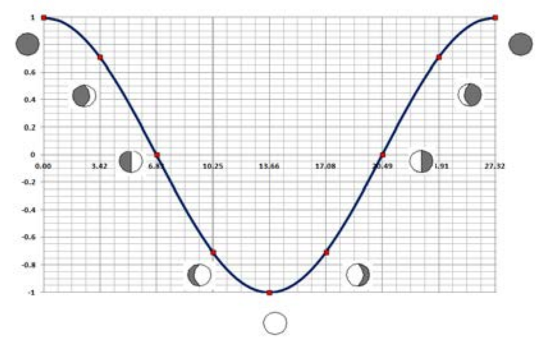
Phases of the Moon

Possible Weighting Factors









Eight Phases

New	Waxing Crescent	First Quarter	Waxing Gibbous
Full	Waning Gibbous	Third Quarter	Waning Crescent

The GRAN Method – Ellipse Perimeter

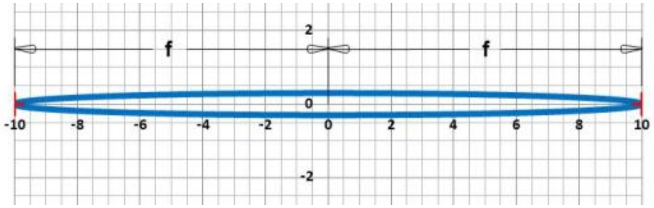
Not “Extreme Perfect”

For a Circle

$$P \approx \frac{b}{Y_{factor}} \approx \frac{r}{1\,/\,2\,\pi} = 2\,\pi\,r$$

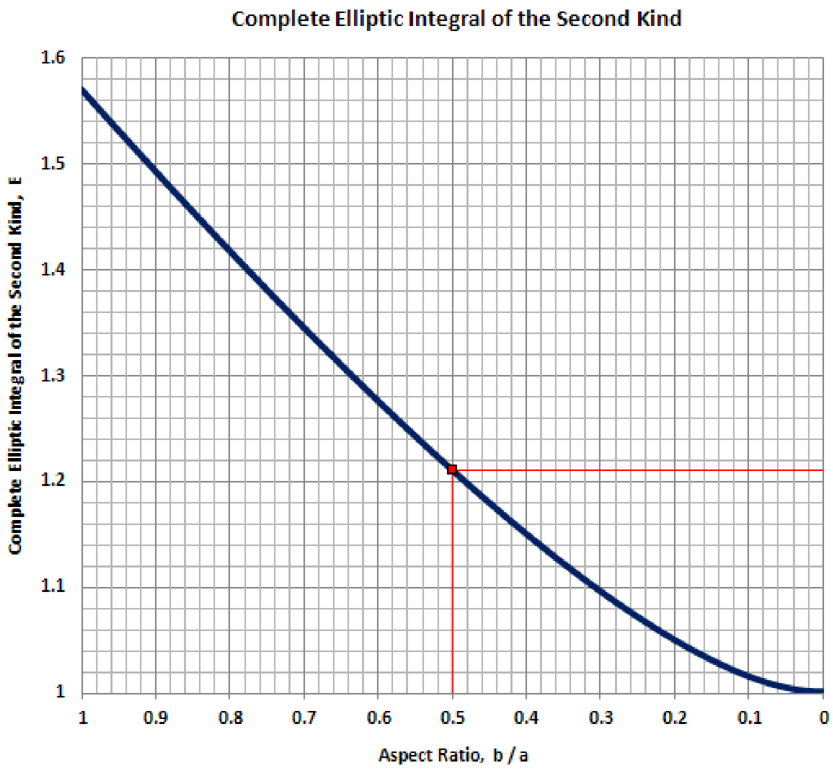
For small aspect ratios the perimeter approaches P = 4 a

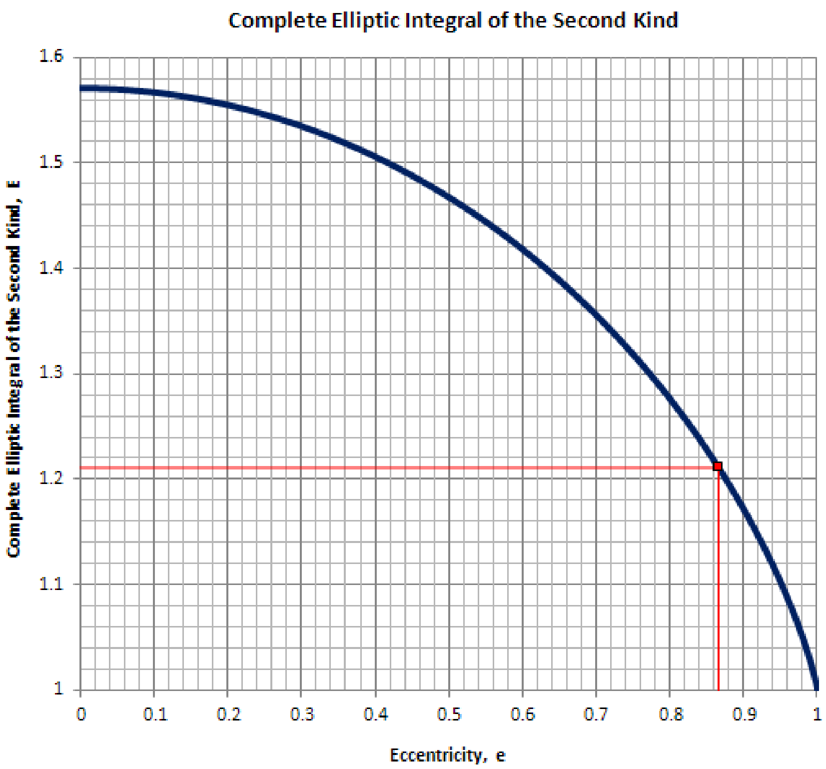
But the Y factor goes to zero which means you are dividing by zero.



The GRAN Formula – Ellipse Perimeter

Plotting the Complete Elliptic Integral versus Aspect Ratio, (b / a) instead of Eccentricity, ε



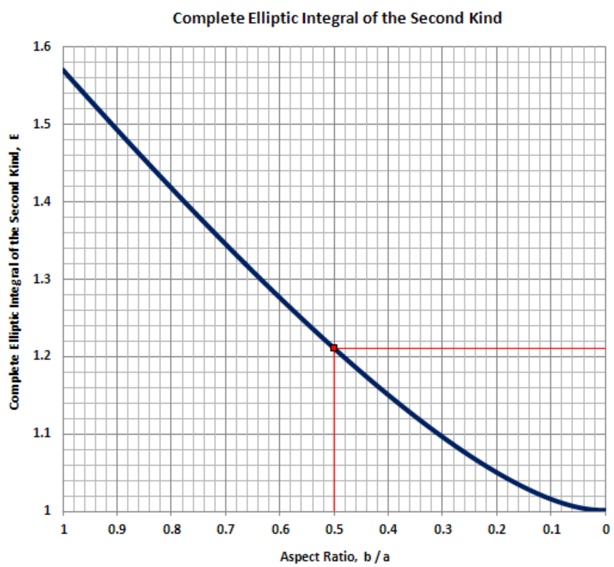


The GRAN Formula – Ellipse Perimeter

Bleasdale – Nelder Power Curve

$$E \approx \left[A + B \left(\frac{b}{a} \right)^{C} \right]^{-1/D} + Offset \approx \left[A + B \left(\frac{b}{a} \right)^{C} \right]^{-1/D} + 1$$

where A = 0.518083 B = 0.538323 C = -0.263801 D = 0.097480 Offset = 1.001249



Example a = 10 inch b = 5 inch

$$P \approx 4 \ a \ E \approx 4 \ (10 \ inch) \ 1.2110639 \approx 48.4426 \ inch$$

The GRAN Formula – Ellipse Perimeter

Simplified

The Complete Elliptic Integral of the Second Kind

$$E \approx \left[0.518 + 0.538 \left(\frac{b}{a} \right)^{-0.264} \right]^{-10.26} + 1$$

where A = 0.518 B = 0.538 C = -0.264 D = 0.09748 - 1 / D = -10.26 Offset = 1.00

Ellipse Perimeter

$$P \approx 4 \ a \left\{ \left[0.518 + 0.538 \left(\frac{b}{a} \right)^{-0.264} \right]^{-10.26} + 1 \right\}$$

Example a = 10 inch b = 5 inch

$$P \approx 4 \ a \ E \approx 4 \ (10 \ inch) \ 1.21106 \approx 48.4 \ inch$$

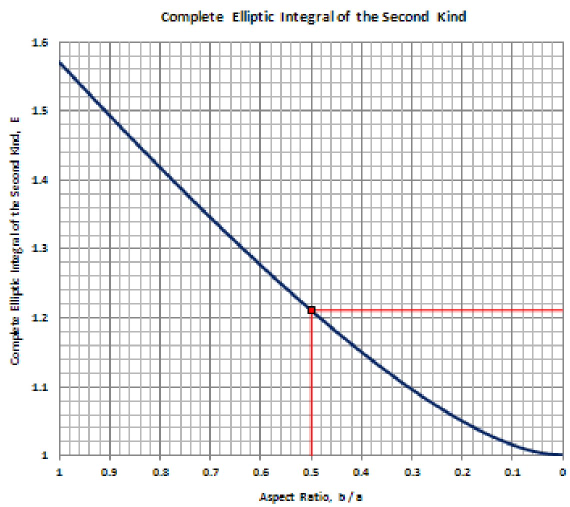
The GRAN Formula – Ellipse Perimeter

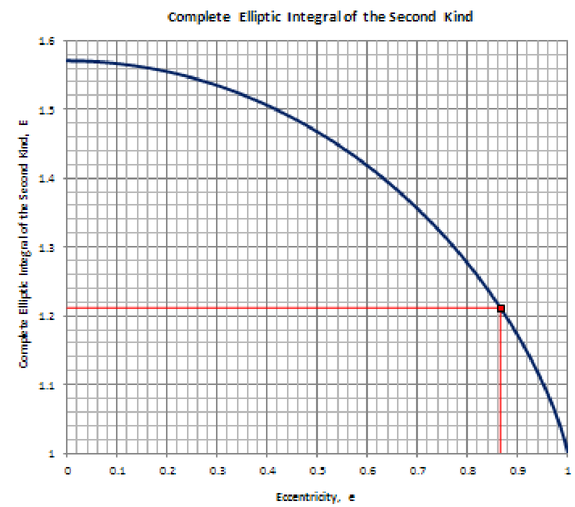
INPUT

a	10	units
b	5	units

DATA

b / a	0.5	
δ	0.36	
ε	0.8660254	
f	8.66025404	units
h	0.11111111	
γ	1.53492854	
Y Factor	0.1032157	
E	1.21105603	





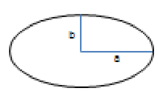
OUTPUT

$$P = 4\,a\,E[\pi/2, \varepsilon] = 4\,a \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\; d\theta$$

Movie

"Exact" Expressions

P	48.4422411	units	**Colin Maclaurin**	$E(\pi/2, \varepsilon) = \int_0^{\pi/2} \sqrt{1 - \varepsilon^2 \sin^2\theta}\; d\theta$
P	48.4422411	units	**Leonhard Euler**	
P	48.4422411	units	**James Ivory**	
P	48.4422411	units	**Arthur Cayley**	



Approximate Formulas

P	48.44224	units	**GRAN Method**	$P \approx \frac{b}{Y_{factor}}$
P	48.44225	units	Legendre Polynomial	
P	48.4427	units	Sixth Order Polynomial	
P	48.5	units	Third Order Polynomial	
P	48.4426	units	**GRAN Formula**	
P	48.4	units		

$$E \approx \left[A + B\left(\frac{b}{a}\right)^{C}\right]^{-1/D} + 1$$

A 0.518083
B 0.538323
C -0.2638
D 0.09748

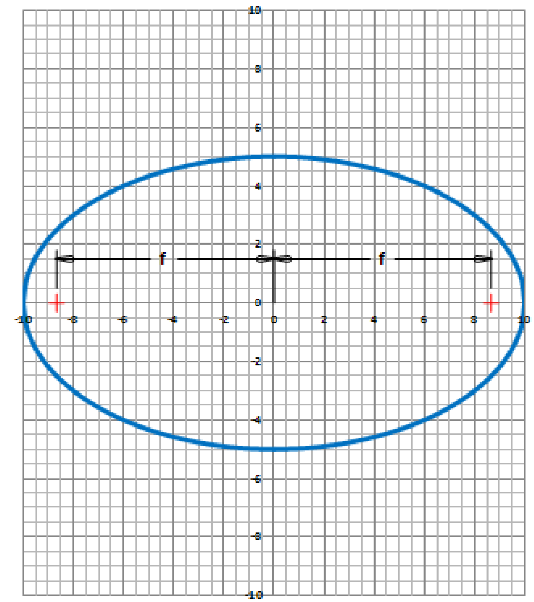
Simplified

$$P \approx 4\,a\left[0.518 + 0.538\left(\frac{b}{a}\right)^{-0.264}\right]^{-10.26} + 1$$

Ratio b / a	GRAN Y Factor
0.05	0.0124313
0.10	0.0246036
0.15	0.0363536
0.20	0.0475963
0.25	0.0582858
0.30	0.0684009
0.35	0.0779382
0.40	0.0869070
0.45	0.0953251
0.50	**0.1032157**
0.55	0.1106055
0.60	0.1175226
0.65	0.1239960
0.70	0.1300543
0.75	0.1357252
0.80	0.1410354
0.85	0.1460104
0.90	0.1506740
0.95	0.1550486
1.00	0.1591549

Perimeter – 48.4422 units

$$P \approx \frac{b}{Y_{factor}}$$



The GRAN Formula – Ellipse Perimeter

INPUT

a	10	units
b	5	units

DATA

b / a	0.5	
δ	0.36	
ε	0.866025	
f	8.660254	units
h	0.111111	
γ	1.534929	
Y Factor	0.103216	
E	1.211056	

OUTPUT

Colin Maclaurin

P	48.44224	units

Leonhard Euler

P	48.44224	units

James Ivory

P	48.44224	units

Arthur Cayley

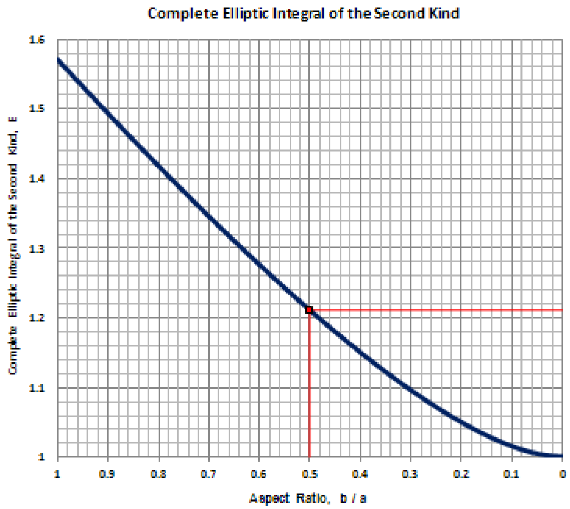
P	48.44224	units

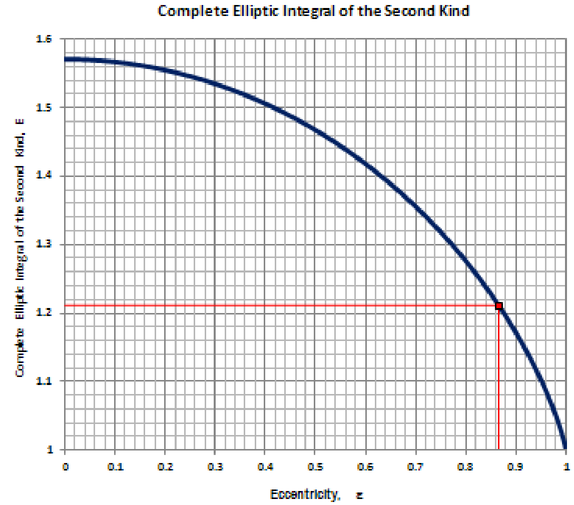
GRAN Method

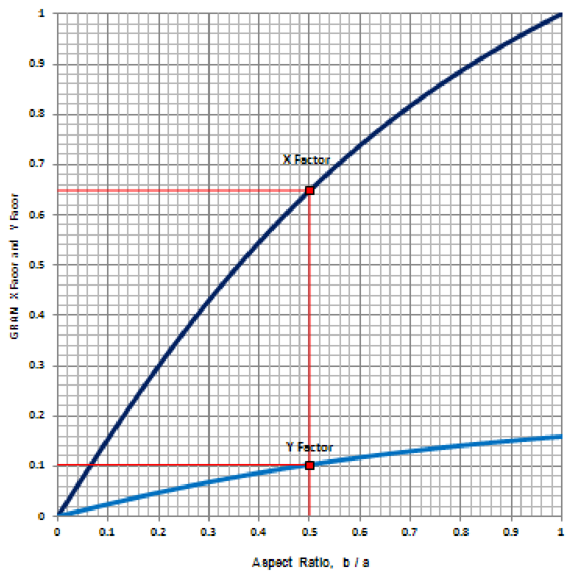
P	48.44224	units

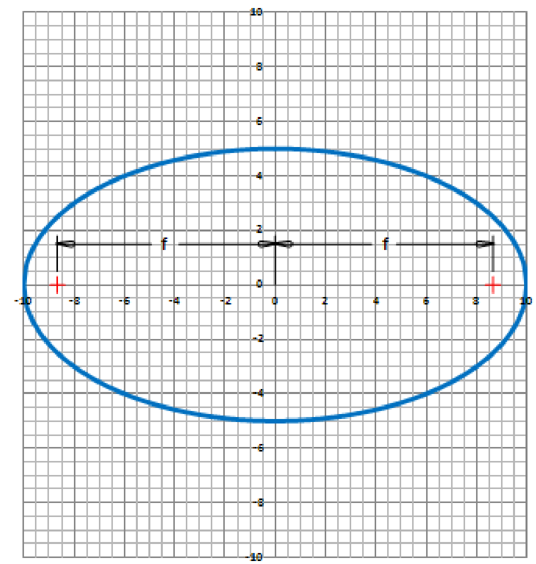
GRAN Formula

P	48.4426	units



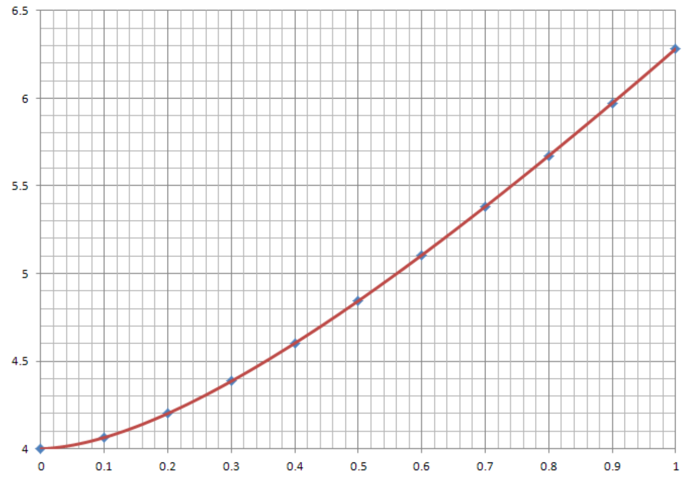






Another Way – Ellipse Perimeter

Plotting the perimeter of an ellipse with a = 1 inch and b = 0 to 1 inch …



Third Order Polynomial

$$P \approx a\left[-0.9812\left(\frac{b}{a}\right)^3 + 2.6221\left(\frac{b}{a}\right)^2 + 0.6511\left(\frac{b}{a}\right) + 3.9808\right]$$

a = 10 inch b = 5 inch Perimeter = 48.4 inch

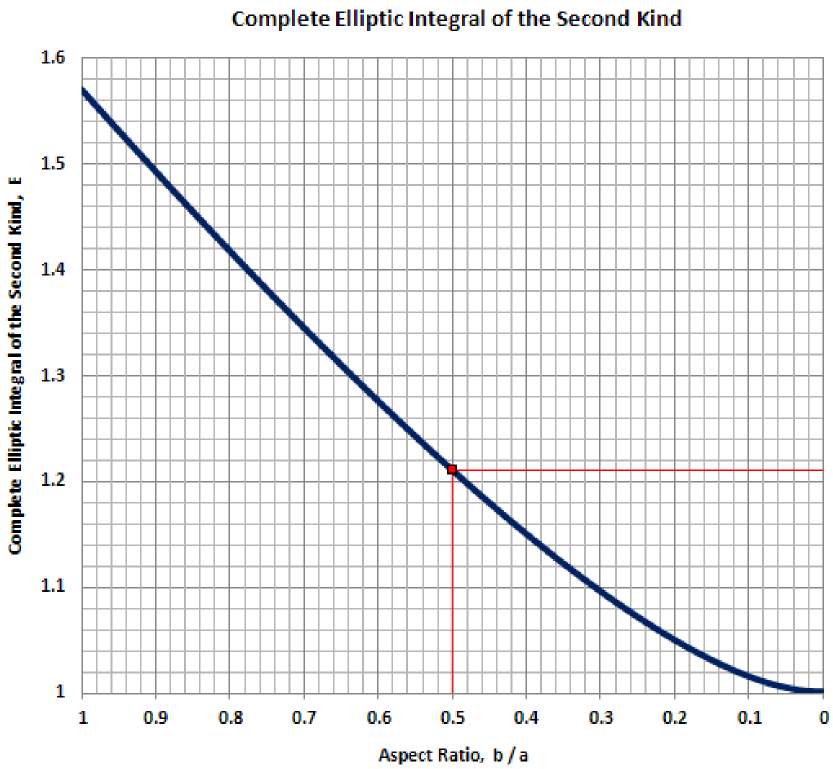
Sixth Order Polynomial

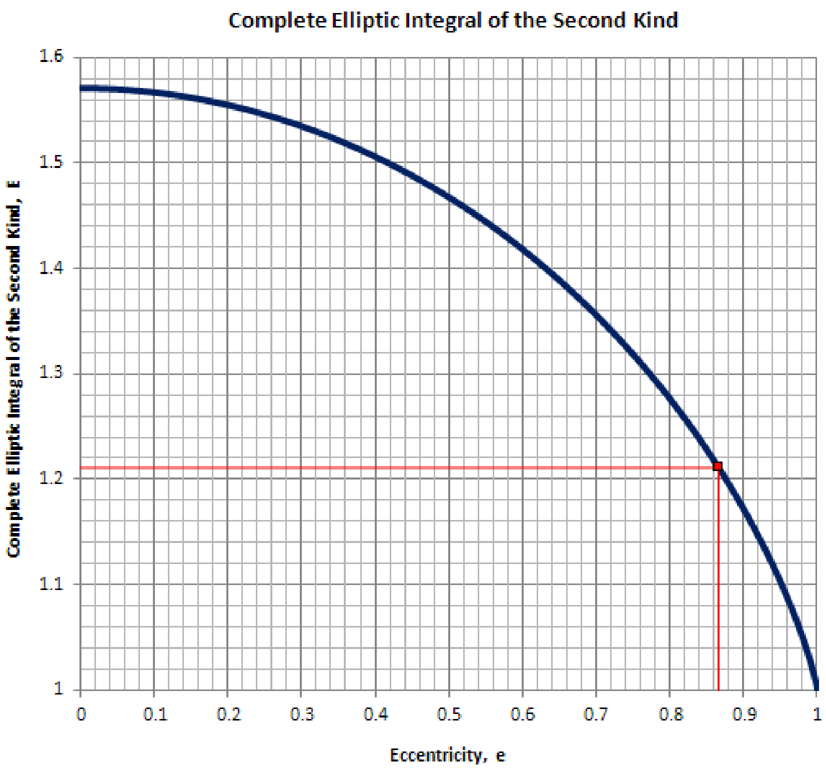
$$P \approx a\left[1.561786\left(\frac{b}{a}\right)^6 - 5.90478\left(\frac{b}{a}\right)^5 + 9.355009\left(\frac{b}{a}\right)^4 - 8.4884\left(\frac{b}{a}\right)^3 + 5.596493\left(\frac{b}{a}\right)^2 + 0.1634155\left(\frac{b}{a}\right) + 3.999961\right]$$

a = 10 inch b = 5 inch Perimeter = 48.44 inch

Simplest Exact Equation – Ellipse Perimeter

Plotting the Complete Elliptic Integral versus Aspect Ratio, (b / a) instead of Eccentricity, ε





Divide by four and we're back to where we started.

Plotting the data versus eccentricity instead of aspect ratio yields the Complete Elliptic Integral of the Second Kind.

Ellipse Perimeter

Back to where we started:

Exact – Not Simple

$$P = 4a \int_0^{\pi/2} \sqrt{1 - \left(\frac{a^2 - b^2}{a^2}\right) \sin^2\theta} \; d\theta$$

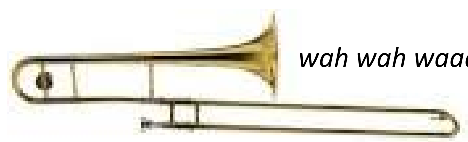
Simple – Not Exact

Ratio b / a	GRAN Y Factor
0.30	0.0684009
0.35	0.0779382
0.40	0.0869070
0.45	0.0953251
0.50	0.1032157
0.55	0.1106055
0.60	0.1175226
0.65	0.1239960
0.70	0.1300543
0.75	0.1357252
0.80	0.1410354

$$P \approx \frac{b}{Y_{factor}}$$

In the End

FAILURE



wah wah waaaahhhhhhhh

Even Wile E. Coyote, Super Genius …

has things blow up in his face.

He never quits.

Wile E. Coyote, Super Genius … I.Q. 207

Adventures of the Road-Runner (1962)

Ralph Phillips:

The thing I don't understand is why he wants the Road Runner in the first place.

Wile E. Coyote:

A legitimate question, young man, deserving a legitimate answer.

[he shows a picture of the Road Runner during this whole scene as he says:]

*Now then, I can easily understand why it should puzzle you that a person of my intelligence, **I.Q. 207 Super Genius**, should devote his valuable time chasing this ridiculous road runner, this bird that appears to be so skinny, scrawny, stringy, unappetizing, anemic, ugly and misbegotten. Ah, but how little you know about road runners. Actually, the road runner is to the taste buds of a coyote, what caviar, champagne, filet mignon and chocolate fudge are to the taste buds of a man.*

http://www.imdb.com/character/ch0029626/quotes

High IQ & Genius IQ

Genius or near-genius IQ is considered to start around 140 to 145. Less than 1/4 of 1 percent fall into this category.

Here are some common designations on the IQ scale:

115-124 - Above Average
125-134 – Gifted
135-144 - Very Gifted
145-164 – Genius
165-179 - High Genius
180-200 - Highest Genius

http://blog.getiq.net/content/iq-scales-what-does-iq-scale-measure

Unlock the Super Genius in YOU!

A Simple, Exact Solution Awaits

Martin Gran

Be honest, work hard … love everyone.

Mange Takk

and

God Bless America!

Legal Department

www.GranCorporation.com

Bill Gran

President, CEO & Super Genius

GRAN Corporation